# On Achievable Rate Regions for the Gaussian Interference Channel

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**Abstract**

The complete characterization of the capacity region of a two-user Gaussian interference channel is still an open problem unless the interference is strong. In this work, we derive an achievable rate region for this channel. It includes the rate region which is achieved by time/ frequency division multiplexing (TDM/ FDM), and it also includes the rate region which is obtained by time-sharing between the two rate pairs where one of the transmitters sends its data reliably at the maximal possible rate (i.e., the maximal rate it can achieve in the absence of interference), and the other transmitter decreases its data rate to the point where both receivers can reliably decode its message. The suggested rate region is easily calculable, though it is a particular case of the celebrated achievable rate region of Han and Kobayashi whose calculation is in general prohibitively complex. In the high power regime, a lower bound on the sum-rate capacity (i.e., maximal achievable total rate) is derived, and we show its superiority over the maximal total rate which is achieved by the TDM/ FDM approach for the case of moderate interference. For degraded and one-sided Gaussian interference channels, we rely on some observations of Costa and Sato, and obtain their sum-rate capacities. We conclude our discussion by pointing out two interesting open problems.

## 1. Model and Definition of Capacity Region

An interference channel (IFC) models the situation where a number ($M$) of unrelated senders try to communicate their separate information to $M$ different receivers via a common channel. Transmission of information from each sender to its corresponding receiver interferes with the communication between the other senders and their receivers. A two-user (i.e, $M = 2$) discrete, memoryless IFC consists of four finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$, and conditional probability distributions $p(\cdot, \cdot|x_1, x_2)$ on $\mathcal{Y}_1 \times \mathcal{Y}_2$, where $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$. For coded information of block length $n$, the two-user discrete, memoryless IFC is denoted by

$$(\mathcal{X}_1^n \times \mathcal{X}_2^n, p^n(\mathbf{y}_1, \mathbf{y}_2|\mathbf{x}_1, \mathbf{x}_2), \mathcal{Y}_1^n \times \mathcal{Y}_2^n),$$

where

$$p^n(\mathbf{y}_1, \mathbf{y}_2|\mathbf{x}_1, \mathbf{x}_2) = \prod_{k=1}^{n} p(y_{1,k}, y_{2,k}|x_{1,k}, x_{2,k}), \qquad \mathbf{x}_1 \in \mathcal{X}_1^n, \mathbf{x}_2 \in \mathcal{X}_2^n, \mathbf{y}_1 \in \mathcal{Y}_1^n, \mathbf{y}_2 \in \mathcal{Y}_2^n.$$

In the model of a two-user IFC, there are two independent and uniformly distributed sources. Senders 1, 2 produce two integers: $W_1 \in \{1, \ldots, 2^{nR_1}\}$, $W_2 \in \{1, \ldots, 2^{nR_2}\}$, respectively. A $(2^{nR_1}, 2^{nR_2}, n)$ code for a two-user IFC consists of two encoding functions

$$e_1 : \{1, \ldots, 2^{nR_1}\} \rightarrow \mathcal{X}_1^n, \quad e_2 : \{1, \ldots, 2^{nR_2}\} \rightarrow \mathcal{X}_2^n$$

and two decoding functions

$$d_1 : \mathcal{Y}_1^n \rightarrow \{1, \ldots, 2^{nR_1}\}, \quad d_2 : \mathcal{Y}_2^n \rightarrow \{1, \ldots, 2^{nR_2}\}.$$

Since there is no co-operation between the two receivers in this channel, the average probabilities of error are

$$P_{e,i}^{(n)} = \frac{1}{M_1 M_2} \sum_{w_1, w_2} \text{Prob}\, \{d_i(\mathbf{y}_i) \neq w_i | W_1 = w_1, W_2 = w_2\}, \quad i = 1, 2.$$

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A rate pair $(R_1, R_2)$ is said to be *achievable* if there exists a sequence of $(\lceil 2^{nR_1} \rceil, \lceil 2^{nR_2} \rceil, n)$ codes, such that $P_{e,1}^{(n)} \to 0$ and $P_{e,2}^{(n)} \to 0$ as $n \to \infty$. The rates are expressed here in terms of bits per channel use. The *capacity region* of an IFC is defined as the closure of the set of all its achievable rate pairs.

In this correspondence, we derive an achievable rate region for the two-user Gaussian IFC. The derivation of this region is based on a modified time (or frequency) division multiplexing approach that was originated by Sato for the degraded Gaussian IFC, and which is studied here in a general setting. This achievable rate region includes the achievable rate region by time/ frequency division multiplexing (TDM/ FDM), and it also includes the rate region that is obtained by time-sharing between the two rate pairs where one of the transmitters sends its data reliably at the maximal possible rate (i.e., the maximum rate it can achieve in the absence of interference), and the other transmitter decreases its data rate to the point where both receivers can reliably decode its message. Yet, it is still a particular case of the Han and Kobayashi (HK) achievable rate region. In the high power regime, an improved lower bound on the sum-rate capacity is derived as a particular case of the general HK achievable rate region. We analyze some of the properties of this lower bound, and show its superiority over the maximal total rate that is obtained by the TDM/ FDM approach. For degraded and one-sided Gaussian IFCs, we rely on some observations of Costa and Sato, and derive their sum-rate capacities.

The structure of the correspondence is as follows: Earlier results which are related to the derivation of the new results here are presented in Section 2. New results are provided in Section 3, and proved in Section 4. Numerical results are presented and explained in Section 5. Finally, concluding remarks appear in Section 6, where we also two interesting open problems.

## 2. Earlier Results

Similar to broadcast channels, since there is no co-operation between the receivers, the capacity region of a two-user discrete, memoryless IFC *only* depends on the following marginal probability distributions

$$p_1(y_1|x_1, x_2) = \sum_{y_2 \in \mathcal{Y}_2} p(y_1, y_2|x_1, x_2), \qquad p_2(y_2|x_1, x_2) = \sum_{y_1 \in \mathcal{Y}_1} p(y_1, y_2|x_1, x_2).$$

Hence, the capacity region of a discrete, memoryless IFC is therefore identical to the capacity region of any other discrete, memoryless IFC whose marginal probability distributions are the same.

The information-theoretic characterization of the capacity region of a discrete memoryless IFC is in general unknown yet, except for some special cases (see [15] and references therein). The capacity region of a discrete memoryless IFC was expressed in [1] by the following limiting expression

$$C_{\text{IFC}} = \lim_{n\to\infty} \text{closure} \left( \bigcup_{X_1^n, X_2^n \text{ independent}} \left\{ (R_1, R_2) : R_1 \leq \frac{I(X_1^n; Y_1^n)}{n},\ R_2 \leq \frac{I(X_2^n; Y_2^n)}{n} \right\} \right), \tag{1}$$

which unfortunately does not lend itself to feasible computation. Unlike the case of a general discrete, memoryless MAC whose capacity region is expressible by a single letter formula, the limiting expression in Eq. (1) can not be written in general by a single-letter expression (see [11]). Unfortunately, it was also demonstrated in [4] that the restriction to Gaussian inputs in the limiting expression Eq. (1) for the capacity region of a Gaussian memoryless IFC falls short of achieving capacity, even if the inputs are allowed to be dependent and non-stationary.

In 1981, Han and Kobayashi (HK) [9] have derived an achievable rate region for a general discrete memoryless IFC. It encompasses the achievable rate regions that were earlier established, and is still the best one known to date. However, the computation of the full HK achievable rate region for a general discrete, memoryless IFC is in general prohibitively complex, because of the huge number of degrees of freedom which are involved in the computation of its sub-regions (see [9]). We refer the interested reader to a comprehensive survey paper on the IFC [15].

We focus here on the Gaussian IFC, which was extensively treated in the literature (e.g, [2]–[5], [9]–[15]). The input and output alphabet of a memoryless Gaussian IFC is the field of real numbers ($\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y}_1 = \mathcal{Y}_2 = \mathbb{R}$), and the probability density function ($p$) of this channel is derived from the following linear relations between its inputs and outputs: $y_1^* = c_1 x_1^* + c_2 x_2^* + n_1^*, \quad y_2^* = d_1 x_1^* + d_2 x_2^* + n_2^*$, where $x_1^*, x_2^*, y_1^*, y_2^*$ are reals, and $n_1^*, n_2^*$

are additive Gaussian noises with zero mean and variances $N_1, N_2$ respectively. The following power constraints are imposed on the transmitted signals ($n$-length codewords) $\mathbf{x}_1^* \in \mathcal{X}_1^n$, and $\mathbf{x}_2^* \in \mathcal{X}_2^n$:

$$\frac{1}{n}\sum_{k=1}^{n}(x_{1,k}^*)^2 \le P_1^*, \quad \frac{1}{n}\sum_{k=1}^{n}(x_{2,k}^*)^2 \le P_2^*.$$

The capacity region of this channel is identical to the capacity region of the following Gaussian IFC in its *standard* form

$$y_1 = x_1 + \sqrt{a_{12}}\, x_2 + n_1, \quad y_2 = \sqrt{a_{21}}\, x_1 + x_2 + n_2, \tag{2}$$

where $a_{12} = \frac{c_2^2 N_2}{d_2^2 N_1}, a_{21} = \frac{d_1^2 N_1}{c_1^2 N_2}$, and $n_1, n_2$ are additive Gaussian noises with zero mean and unit variance. This equivalence is verified by the transformation

$$y_1 = \frac{y_1^*}{\sqrt{N_1}},\ y_2 = \frac{y_2^*}{\sqrt{N_2}},\ x_1 = \frac{c_1 x_1^*}{\sqrt{N_1}},\ x_2 = \frac{d_2 x_2^*}{\sqrt{N_2}},\ n_1 = \frac{n_1^*}{\sqrt{N_1}},\ n_2 = \frac{n_2^*}{\sqrt{N_2}}.$$

Since the capacity region of an IFC only depends on the marginal probability distributions, it is irrelevant whether the noise terms $n_1$ and $n_2$ are statistically dependent of each other or not. The power constraints in the standard form of the Gaussian IFC are

$$\frac{1}{n}\sum_{k=1}^{n}(x_{1,k})^2 \le P_1, \quad \frac{1}{n}\sum_{k=1}^{n}(x_{2,k})^2 \le P_2, \tag{3}$$

where $P_1 = \frac{c_1^2 P_1^*}{N_1}, P_2 = \frac{d_2^2 P_2^*}{N_2}$. We note that in the high SNR regime, $P_1, P_2$ may attain rather high values, and in the broad band and power-limited scenario, the values of $P_1, P_2$ are typically moderate or low. Throughout this paper, we confine ourselves to Gaussian IFCs in their *standard form*. We assume here perfect synchronization between the transmitters and their corresponding receivers, which implies that the capacity region of the IFC is convex.

An IFC is called *degraded* if there exists a conditional probability $p'(y_2|y_1)$ such that the following equality holds

$$p_2(y_2|x_1,x_2) = \sum_{y_1 \in \mathcal{Y}_1} p'(y_2|y_1) p_1(y_1|x_1,x_2), \quad \forall\, y_2 \in \mathcal{Y}_2,\ (x_1,x_2) \in \mathcal{X}_1 \times \mathcal{X}_2.$$

The last equality implies that one of the output terminals ($Y_2$) is a degraded version of the other output terminal ($Y_1$). Since the capacity region of a degraded IFC only depends on its marginal probability distributions, then the capacity region of a degraded IFC is identical to the capacity region of the IFC whose conditional probability distribution is

$$p(y_1,y_2|x_1,x_2) = p'(y_2|y_1)\, p_1(y_1|x_1,x_2), \quad \forall\, (x_1,x_2) \in \mathcal{X}_1 \times \mathcal{X}_2,\ (y_1,y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2.$$

The latter IFC forms a serial concatenation of two channels $p_1(y_1|x_1,x_2)$ and $p'(y_2|y_1)$ which are combined in cascade. The full characterization of the capacity region of a discrete, memoryless and degraded IFC is still an open problem. In its standard form (2), a two-user Gaussian IFC is degraded if and only if $a_{12} \cdot a_{21} = 1$ [3]. Inner and outer bounds on the capacity region of a degraded Gaussian IFC were derived in [13].

A two-user Gaussian IFC is called *one-sided* if either $a_{12} = 0$ or $a_{21} = 0$. In [5], Costa has demonstrated that the class of degraded Gaussian IFCs are equivalent to the class of the one-sided Gaussian IFCs (from the view point of their capacity regions). More specifically, if $0 < c < 1$, then it follows from [5] that the one-sided Gaussian IFC whose characterization in the standard form is

$$y_1 = x_1 + n_1, \quad y_2 = cx_1 + x_2 + n_2,$$

has the same capacity region as the degraded Gaussian IFC

$$y_1 = x_1 + \frac{x_2}{c} + n_1, \quad y_2 = cx_1 + x_2 + n_2.$$

In both cases $n_1, n_2$ are Gaussian random variables with zero mean and unit variance, and the same power constraints are imposed on the transmitted signals $\mathbf{x}_1, \mathbf{x}_2$ for the two channels. The degraded Gaussian IFC above is the standard form of the IFC which was depicted in [5, Fig. 6(d)]. Later, we will rely on the equivalence between one-sided and degraded Gaussian IFCs, and derive an exact expression for the sum-rate capacities of both channels.

The capacity region of a Gaussian IFC with strong interference was independently determined by Han and Kobayashi [9, Theorem 5.2] and Sato [14]. The capacity region of the Gaussian IFC with very strong interference was determined by Carleial [2]. For the latter case of a Gaussian IFC with very strong interference (which is characterized by the inequalities: $a_{12} \geq 1 + P_1$ and $a_{21} \geq 1 + P_2$), it was surprisingly demonstrated in [2] that the interference *does not* harm the capacity region. However, the capacity region of the Gaussian IFC was not determined yet if either $a_{12}$ or $a_{21}$ lie in the open interval $(0, 1)$. The complete characterization of the capacity region of a one-sided Gaussian IFC whose non-vanishing interference coefficient is between zero and unity is also still unknown. For interference parameters in the interval $(0, 1)$, weak and moderate interference are defined in the symmetric case as follows: If $a_{12} = a_{21} = a$ and $P_1 = P_2 = P$ in (2) and (3), respectively, then by treating the interfering signals as an additive gaussian noise, one can achieve a total rate of $\log_2\left(1 + \frac{P}{1+aP}\right)$ bits per channel use. On the other hand, if time or frequency division multiplexing (TDM/ FDM) are applied, then the maximal achievable total rate is independent of the interference coefficients and equals $\frac{1}{2}\log_2(1 + 2P)$ bits per channel use. Weak/ moderate interference is defined as the range of values of $a$ between zero and unity for which the former/ latter approach yields a larger value of maximal achievable total rate. Following this definition, one obtains that weak interference in a two-user, symmetric Gaussian IFC refers to the condition: $0 < a < \frac{\sqrt{1+2P}-1}{2P}$ , and moderate interference refers to the complementary condition, i.e., $\frac{\sqrt{1+2P}-1}{2P} \leq a < 1$. For a two-user Gaussian IFC with weak or moderate interference, the gap between the reported upper and lower bounds on the sum-rate capacity is rather large (see [10]), though it tends to zero in the case where $a \to 0$ (i.e., no interference) or $a \to 1$ (i.e., a Gaussian MAC).

## 3. New Results

*Theorem 1 (A Computable Achievable Rate Region for the Two-User Gaussian Interference Channel):* The set of rate pairs

$$D = \bigcup_{\alpha,\beta,\lambda\in[0,1]} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \lambda \cdot \gamma\left(\frac{\alpha P_1}{\lambda}\right) + (1-\lambda)\cdot \min\left\{\gamma\left(\frac{(1-\alpha)P_1}{1-\lambda+a_{12}(1-\beta)P_2}\right), \gamma\left(\frac{a_{21}(1-\alpha)P_1}{1-\lambda+(1-\beta)P_2}\right)\right\} \\ R_2 \leq (1-\lambda)\cdot\gamma\left(\frac{(1-\beta)P_2}{1-\lambda}\right) + \lambda\cdot\min\left\{\gamma\left(\frac{\beta P_2}{\lambda+a_{21}\alpha P_1}\right), \gamma\left(\frac{a_{12}\beta P_2}{\lambda+\alpha P_1}\right)\right\} \end{array} \right\} \quad (4)$$

where

$$\gamma(x) \triangleq \frac{1}{2}\log_2(1+x), \quad (5)$$

is achievable for the standard two-user Gaussian IFC (2) under the power constraints in (3). The rate region $D$ in Eq. (4) is included in the general Han and Kobayashi (HK) achievable rate region (see [9, Theorem 3.2]).

If $0 \leq a_{12}, a_{21} < 1$, then $D$ has the following properties:

1) It includes the straight line which connects the two points

$$(R_1, R_2) = \left(\gamma(P_1), \gamma\left(\frac{a_{12}P_2}{1+P_1}\right)\right) \;, \quad (R_1, R_2) = \left(\gamma\left(\frac{a_{21}P_1}{1+P_2}\right), \gamma(P_2)\right). \quad (6)$$

2) It includes the achievable rate region by TDM/ FDM, but in both cases, the maximal achievable total rate stays the same.
3) In the symmetric case where $a_{12} = a_{21} \triangleq a$, and $P_1 = P_2 \triangleq P$:
   - The calculation of the achievable rate region in (4) can be simplified, so that it only involves the two parameters $(\beta, \lambda) \in [0, 1] \times [0, 1]$.
   - For moderate interference (i.e., if $\frac{\sqrt{1+2P}-1}{2P} < a < 1$), the region $D$ includes the particular achievable sub-region $\mathcal{G}'$ of the full HK achievable rate region (see [9], Eq. (5.9)). The maximal total rate which is achieved by the region $D$ is also strictly larger than the one achieved by $\mathcal{G}'$.
   - For weak interference, the maximal total rate which is achieved by $\mathcal{G}'$ is strictly larger than the one achieved by the region $D$ (with an equality if $a = \frac{\sqrt{1+2P}-1}{2P}$).

*Theorem 2 (Sum-Capacity of Degraded and One-Sided Gaussian Interference Channels):* For a *degraded* Gaussian IFC which is expressed in the standard form (2) with the power constraints (3), the sum-rate capacity is

$$\max(R_1 + R_2) = \begin{cases} \gamma(P_1) + \gamma\left(\frac{P_2}{1+a_{21}P_1}\right) & \text{if } a_{12} \geq 1 \\ \gamma(P_2) + \gamma\left(\frac{P_1}{1+a_{12}P_2}\right) & \text{if } a_{21} \geq 1. \end{cases} \quad (7)$$

For a *one-sided* Gaussian IFC, the sum-rate capacity for the case where $a_{12} = 0$ is

$$\max(R_1 + R_2) = \begin{cases} \gamma(P_1) + \gamma\left(\frac{P_2}{1+a_{21}P_1}\right) & \text{if } 0 \leq a_{21} \leq 1 \\ \gamma(a_{21}P_1 + P_2) & \text{if } 1 \leq a_{21} \leq 1 + P_2 \\ \gamma(P_1) + \gamma(P_2) & \text{if } a_{21} \geq 1 + P_2 \end{cases} \tag{8}$$

with a similar expression for the case where $a_{21} = 0$ (by switching the indices of users 1 and 2).

*Corollary 1:* For a one-sided Gaussian IFC with weak or moderate interference (i.e., when the interference coefficient is not above unity), the sum-rate capacity is achieved if the transmitter which is not interfered sends its data at the maximal achievable rate of a single-user, and the second transmitter sends its data at the maximal possible rate where the interfering signal is treated as an additive Gaussian noise.

*Theorem 3 (Achievable Sum-Rate for the Gaussian Interference Channel):* Consider a two-user symmetric Gaussian IFC in the standard form (2) where $a_{12} = a_{21} \triangleq a$, and $P_1 = P_2 \triangleq P$ is the common power constraint in Eq. (3). Then, for large enough values of $P$, the sum-rate capacity is larger than the maximal total rate which is achieved by TDM/ FDM.
In particular, let $a = a_0$ be the *single* root of the polynomial equation

$$P^2a^4 + 2P(P+1)a^3 + (2P+1)a^2 - 2Pa - (1+P) = 0 \tag{9}$$

which lies in the interval $\frac{\sqrt{4P+1}-1}{2P} \leq a \leq 1$ (i.e., it is included in the range of values of moderate interference). Then, if $P \geq 17$ dB, the maximal total rate which is attained by this sub-region is a decreasing function of $a \in [a_0, 1]$; it decreases from its maximal value at $a = a_0$, which is at least

$$\log_2\left(\frac{a_0 + a_0P + a_0^2P}{1 - a_0 + a_0^2(Pa_0 + 1)}\right) \tag{10}$$

to the value $\frac{1}{2}\log_2(1+2P)$ at $a = 1$ (i.e., the maximal total rate that is achieved by TDM/ FDM).
*Corollary 2:* For a symmetric Gaussian IFC with moderate interference, TDM or FDM are *not optimal* in the *high* power regime or in the *narrow* band regime.

## 4. Proofs of the New Results

### A. Proof of Theorem 1

In [12, Theorem 5], Sato has proved that the following rate-region is achievable for a general two-user channel:

$$G_{\text{B}} = \text{convex hull}\left\{G_{B_1} \cup G_{B_2}\right\}$$

where

$$G_{B_1} = \text{convex hull} \bigcup_{P_{X_1}, P_{X_2}} \left\{(R_1, R_2) : \begin{array}{l} 0 \leq R_1 \leq I(X_1; Y_1|X_2), \\ 0 \leq R_2 \leq \min\{I(X_2; Y_1), I(X_2; Y_2)\} \end{array}\right\} \tag{11}$$

and

$$G_{B_2} = \text{convex hull} \bigcup_{P_{X_1}, P_{X_2}} \left\{(R_1, R_2) : \begin{array}{l} 0 \leq R_1 \leq \min\{I(X_1; Y_2), I(X_1; Y_1)\}, \\ 0 \leq R_2 \leq I(X_2; Y_2|X_1) \end{array}\right\}. \tag{12}$$

As was noted in Corollary 3.3 of [9], the achievable rate regions (11) and (12) form particular cases of the general Han and Kobayashi achievable rate region.
For a two-user Gaussian IFC in standard form (2) with the power constraints in Eq. (3), consider the situation where during a fraction $\lambda$ of the transmission time, the symbols of $\mathbf{x}_1$ and $\mathbf{x}_2$ are Gaussian distributed with zero mean, and variances $\frac{\alpha P_1}{\lambda}$ and $\frac{\beta P_2}{\lambda}$, respectively:

$$x_{1,i} \sim N(0, \frac{\alpha P_1}{\lambda}), \quad x_{2,i} \sim N(0, \frac{\beta P_2}{\lambda}), \quad 0 \leq \alpha, \beta \leq 1, \quad i = 1, 2, \ldots, n\lambda \tag{13}$$

and during the remaining fraction $\overline{\lambda} \triangleq 1-\lambda$ of the transmission time, the symbols of $\mathbf{x}_1$ and $\mathbf{x}_2$ are Gaussian distributed with zero mean, and variances $\frac{\overline{\alpha}P_1}{\overline{\lambda}}$ and $\frac{\overline{\beta}P_2}{\overline{\lambda}}$, respectively:

$$x_{1,i} \sim N(0, \frac{\overline{\alpha}P_1}{\overline{\lambda}}), \quad x_{2,i} \sim N(0, \frac{\overline{\beta}P_2}{\overline{\lambda}}), \quad i = n\lambda+1, n\lambda+2, \ldots, n. \tag{14}$$

It can be easily verified that the two input codewords to the Gaussian IFC (2) satisfy the power constraints in Eq. (3):

$$\frac{1}{n}E[||\mathbf{x}_1||^2] = \lambda \cdot \frac{\alpha P_1}{\lambda} + \overline{\lambda} \cdot \frac{\overline{\alpha}P_1}{\overline{\lambda}} = P_1, \quad \frac{1}{n}E[||\mathbf{x}_2||^2] = \lambda \cdot \frac{\beta P_2}{\lambda} + \overline{\lambda} \cdot \frac{\overline{\beta}P_2}{\overline{\lambda}} = P_2.$$

Consider now two modes of work: In the first mode, receiver 1 first decodes the message of the second sender, and then uses it as side information for decoding his message (in the Gaussian IFC model (2), receiver 1 subtracts from the received signal $\mathbf{y}_1$, a scaled version of $\mathbf{x}_2$). On the other hand, receiver 2 directly decodes his message ($\mathbf{x}_2$), based on his received signal ($\mathbf{y}_2$). This mode of work corresponds to the achievable rate region in Eq. (11), and will be used here a fraction $\lambda$ of the transmission time with inputs to the Gaussian IFC which are Gaussian distributed according to Eq. (13). In the second mode (which is dual to the first mode), we refer to the mode of work which corresponds to the achievable rate region in Eq. (12), and assume that it is used during the remaining fraction $\overline{\lambda}$ of the transmission time. We will assume here that during the second mode, the two inputs to the Gaussian IFC (2) are Gaussian distributed according to Eq. (14). Let $R_1^{(i)}$ and $R_2^{(i)}$ be the transmission rates in mode no. $i$. Hence, by time-sharing, the transmission rates of the two users are

$$(R_1, R_2) = \lambda(R_1^{(1)}, R_2^{(1)}) + \overline{\lambda}(R_1^{(2)}, R_2^{(2)}) \tag{15}$$

where from Eqs. (11)–(14), and the definition of the function $\gamma$ in (5):

$$0 \le R_1^{(1)} \le \gamma\left(\frac{\alpha P_1}{\lambda}\right), \quad 0 \le R_2^{(1)} \le \min\left\{\gamma\left(\frac{\frac{a_{12}\beta P_2}{\lambda}}{1+\frac{\alpha P_1}{\lambda}}\right), \gamma\left(\frac{\frac{\beta P_2}{\lambda}}{1+\frac{a_{21}\alpha P_1}{\lambda}}\right)\right\} \tag{16}$$

and

$$0 \le R_1^{(2)} \le \min\left\{\gamma\left(\frac{\frac{\overline{\alpha}P_1}{\overline{\lambda}}}{1+\frac{a_{12}\overline{\beta}P_2}{\overline{\lambda}}}\right), \gamma\left(\frac{\frac{a_{21}\overline{\alpha}P_1}{\overline{\lambda}}}{1+\frac{\overline{\beta}P_2}{\overline{\lambda}}}\right)\right\}, \quad 0 \le R_2^{(2)} \le \gamma\left(\frac{\overline{\beta}P_2}{\overline{\lambda}}\right). \tag{17}$$

The combination of Eqs. (15)–(17) provides the achievable rate region in Eq. (4).

If $0 \le a_{12}, a_{21} < 1$, then $D$ in (4) is simplified to

$$D = \bigcup_{\alpha,\beta,\lambda \in [0,1]} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \le \lambda \cdot \gamma\left(\frac{\alpha P_1}{\lambda}\right) + \overline{\lambda} \cdot \gamma\left(\frac{a_{21}\overline{\alpha}P_1}{\overline{\lambda}+\overline{\beta}P_2}\right) \\ R_2 \le \overline{\lambda} \cdot \gamma\left(\frac{\overline{\beta}P_2}{\overline{\lambda}}\right) + \lambda \cdot \gamma\left(\frac{a_{12}\beta P_2}{\lambda+\alpha P_1}\right) \end{array} \right\}. \tag{18}$$

Though the achievable rate region (18) extends the one which corresponds to TDM/ FDM (since the latter region is a particular case of the region (18) for the particular case where $\alpha = 1$ and $\beta = 0$), unfortunately, the former rate region does not increase the maximal total rate which is achievable by TDM/ FDM. From Eq. (18), and from the simple identity $\gamma\left(\frac{x}{y}\right) + \gamma\left(\frac{z}{x+y}\right) = \gamma\left(\frac{x+z}{y}\right)$ which is valid for all positive values of $x, y$ and $z$, we obtain that

$$R_1 + R_2 \le \frac{\lambda}{2} \cdot \log_2\left(1 + \frac{\alpha P_1 + a_{12}\beta P_2}{\lambda}\right) + \frac{\overline{\lambda}}{2} \cdot \log_2\left(1 + \frac{\overline{\beta}P_2 + a_{21}\overline{\alpha}P_1}{\overline{\lambda}}\right).$$

The maximal value of the right-hand side above is achieved by

$$\lambda = \frac{\alpha P_1 + a_{12}\beta P_2}{(\alpha + a_{21}\overline{\alpha})P_1 + (\overline{\beta} + a_{12}\beta)P_2} \ ,$$

which yields that the maximal total rate satisfies the inequality

$$\begin{aligned} R_1 + R_2 &\le \tfrac{1}{2}\log_2\left(1 + (\alpha + a_{21}\overline{\alpha})P_1 + (\overline{\beta} + a_{12}\beta)P_2\right) \\ &\le \tfrac{1}{2}\log_2(1 + P_1 + P_2) \end{aligned}$$

with equality if $\alpha = 1$ and $\beta = 0$. Since the maximal value here is also the maximal total rate which is achievable by optimal TDM/ FDM, it follows that the maximal total rate of the achievable rate region (18) is equal to the one which corresponds to TDM/ FDM. However, in contrast to the achievable rate region by TDM/ FDM, the substitution $\alpha = \beta = \lambda$ reveals that the rate region (18) also includes the achievable rate pairs

$$(R_1, R_2) = \left(\gamma(P_1), \gamma\left(\frac{a_{21}P_1}{1+P_2}\right)\right), \quad (R_1, R_2) = \left(\gamma\left(\frac{a_{12}P_2}{1+P_1}\right), \gamma(P_2)\right)$$

and the straight line which connects these two points. This shows that the achievable rate region (18) necessarily extends the achievable rate region of TDM/ FDM.

For the symmetric case where $P_1 = P_2 \triangleq P$ and $a_{12} = a_{21} \triangleq a$, the achievable rate region in (18) is simplified under the assumption of weak or moderate interference (i.e., $0 \leq a \leq 1$) to

$$D = \bigcup_{\alpha,\beta,\lambda \in [0,1]} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \lambda \cdot \gamma\left(\frac{\alpha P}{\lambda}\right) + (1-\lambda) \cdot \gamma\left(\frac{a(1-\alpha)P}{1-\lambda+(1-\beta)P}\right) \\ R_2 \leq (1-\lambda) \cdot \gamma\left(\frac{(1-\beta)P}{1-\lambda}\right) + \lambda \cdot \gamma\left(\frac{a\beta P}{\lambda+\alpha P}\right) \end{array} \right\}. \tag{19}$$

The simplification in (19) also suggests a simplification in the numerical calculation of the achievable rate region in (4). For values of $R_2$ between $\gamma\left(\frac{P}{1+aP}\right)$ and $\gamma(P)$, calculate for any pair of $(\beta, \lambda) \in [0,1] \times [0,1]$, the value of $\alpha$ which satisfies the equation

$$(1-\lambda) \cdot \gamma\left(\frac{(1-\beta)P}{1-\lambda}\right) + \lambda \cdot \gamma\left(\frac{a\beta P}{\lambda + \alpha P}\right) = R_2$$

which is given in closed form as

$$\alpha \triangleq \alpha(\beta, \lambda) = a\beta \left[\exp\left\{\frac{2}{\lambda} \cdot \left[R_2 - (1-\lambda)\,\gamma\left(\frac{(1-\beta)P}{1-\lambda}\right)\right]\right\} - 1\right]^{-1} - \frac{\lambda}{P}\,, \tag{20}$$

and then to choose the maximal value of the function

$$R_1 \triangleq R_1(\beta, \lambda) = \lambda \cdot \gamma\left(\frac{\alpha P}{\lambda}\right) + (1-\lambda) \cdot \gamma\left(\frac{a(1-\alpha)P}{1-\lambda+(1-\beta)P}\right)$$

over those values of $(\beta, \lambda) \in [0,1] \times [0,1]$ for which $\alpha(\beta, \lambda)$ in Eq. (20) is located inside the interval $[0,1]$. This procedure suggests a simplification in the calculation of the boundary of the achievable rate region (4) under the assumption above (we note that based on Eq. (18), a similar procedure can be done under the assumption that $0 \leq a_{12}, a_{21} \leq 1$, even without the further symmetry assumption above.) Straightforward (though tedious) algebra shows that for moderate interference (i.e., when $\frac{\sqrt{1+2P}-1}{2P} < a < 1$), the sub-region $\mathcal{G}'$ which is given in Eqs. (4.1)–(4.9) and Eq. (5.9) of [9] (with the special setting in Section 5A of [9]) is strictly included in the achievable rate region in (19) (as is also exemplified later in Section 5). However, in the case of weak interference (i.e., when $0 < a < \frac{\sqrt{1+2P}-1}{2P}$), then we have already proved that the maximal total rate which corresponds to the achievable rate region in (4) is equal to that of TDM/ FDM, but on the other hand, the maximal total rate of the region $\mathcal{G}'$ exceeds this common value. The reason for the latter statement is attributed to the fact that since the achievable rate region $\mathcal{G}'$ includes the achievable rate region where the interfering signal is regarded as an additive Gaussian noise, and also in the latter case, the total rate is equal to $\log_2\left(1 + \frac{P}{1+aP}\right)$, then for weak interference (i.e., if $0 < a < \frac{\sqrt{1+2P}-1}{2P}$), it exceeds the total rate of optimal TDM/ FDM which is equal to $\frac{1}{2}\log_2(1+2P)$. It then follows directly that for weak interference, the maximal total rate of the achievable rate region $\mathcal{G}'$ is strictly larger than the maximal total rate which is obtained by the achievable rate region in (18).

### *B. Proof of Theorem 2*

The sum-rate capacity of a degraded Gaussian IFC is obtained as a consequence of the analysis by Sato in [13]. By referring to [13, Fig. 1], the point $A_1$ in this figure was shown to be on the boundary of the capacity region of degraded Gaussian IFCs (by showing that this point lies on the boundary of some inner and outer bounds on this capacity region). Relying on observations in [13], we show that the point $A_1$ in [13, Fig. 1] achieves the sum-rate capacity of this channel (note that the capacity region of a degraded Gaussian IFC is unknown yet). To verify this claim, we refer to the curve connecting points $A_1$ and $A_3$ in [13, Fig. 1], which together with the lines $R_1 = C_1$

and $R_2 = C_2$ forms an outer bound on the capacity region of a degraded Gaussian IFC [13]. The point $A_1$ achieves the maximal total rate w.r.t. this outer bound (and hence, since it also lies on the boundary of the inner bound, it also obtains the sum-rate capacity). The reason for the claim that $A_1$ indeed achieves the maximal total rate of the outer bound follows from [13, Eq. (11)] where it yields that for points on the curve which connects the points $A_1$ and $A_3$ in [13, Fig. 1], the inequality $-1 \leq \frac{dR_2}{dR_1} < 0$ is satisfied. Hence, in the considered case where the output $Y_2$ is degraded w.r.t. the output $Y_1$, the point $A_1$ achieves the maximal value of $R_1 + R_2$ on this curve. By symmetry, if the output $Y_1$ is degraded w.r.t. $Y_2$, then the point which achieves the sum-rate capacity is the one which is symmetric to the point $A_1$ w.r.t. to the line $R_1 = R_2$.

The sum-rate capacity of a one-sided Gaussian IFC follows from the observation which was made by Costa [5] regarding the equivalence between one-sided and degraded Gaussian IFCs (see also Section 2 of this paper). Based on this equivalence, the part of Eq. (8) which provides the sum-rate capacity of a one-sided Gaussian IFC with weak or moderate interference follows from Eq. (7) (which refers to the sum-rate capacity of a degraded Gaussian IFC). The part of Eq. (8) which provides the maximal total rate of a one-sided Gaussian IFC with strong or very strong interference is a known result (and it is included in (8) for the sake of completeness; it follows directly from the discussion in [14, Section 3]). we refer the reader to Remarks 3 and 4 in Section 6 where we discuss a consequence of Theorem 2.

### C. *Proof of Theorem 3*

Based on Theorem 6 in [3], the following rate region is achievable for the two-user Gaussian IFC in the standard form (2)

$$\begin{aligned} R_1 &\leq \min \left\{ \begin{array}{l} \gamma\left(\frac{\delta_1 P_1}{1+(\alpha_1+\beta_1)P_1+a_{12}P_2}\right) + \gamma\left(\frac{(\alpha_1+\beta_1)P_1}{1+a_{12}\alpha_2 P_2}\right), \\ \gamma\left(\frac{a_{21}(\delta_1+\beta_1)P_1}{1+a_{21}\alpha_1 P_1+(\alpha_2+\beta_2)P_2}\right) + \gamma\left(\frac{\alpha_1 P_1}{1+a_{12}\alpha_2 P_2}\right) \end{array} \right\} \\ R_2 &\leq \min \left\{ \begin{array}{l} \gamma\left(\frac{\delta_2 P_2}{1+(\alpha_2+\beta_2)P_2+a_{21}P_1}\right) + \gamma\left(\frac{(\alpha_2+\beta_2)P_2}{1+a_{21}\alpha_1 P_1}\right), \\ \gamma\left(\frac{a_{12}(\delta_2+\beta_2)P_2}{1+a_{12}\alpha_2 P_2+(\alpha_1+\beta_1)P_1}\right) + \gamma\left(\frac{\alpha_2 P_2}{1+a_{21}\alpha_1 P_1}\right) \end{array} \right\} \end{aligned} \tag{21}$$

where $\alpha_i, \beta_i, \delta_i$ (where $i = 1, 2$) are non-negative numbers so that $\alpha_i + \beta_i + \delta_i = 1$, and the function $\gamma$ is introduced in (5). We note that the interference coefficients $a_{12}$ and $a_{21}$ in the standard form (2) are flipped as compared to the notation in [3].

In our case, since we assume that the two-user Gaussian IFC is symmetric (i.e., $a_{12} = a_{21} \triangleq a$ and $P_1 = P_2 \triangleq P$), then it follows that the maximal value of $R_1 + R_2$ in the achievable rate region (21) is attained in the symmetric case where $\alpha_1 = \alpha_2 \triangleq \alpha$, $\beta_1 = \beta_2 \triangleq \beta$, $\delta_1 = \delta_2 \triangleq \delta$, and $\alpha$, $\beta$ and $\delta$ are non-negative numbers whose sum is 1 (i.e., $\alpha + \beta + \delta = 1.$).
We intend to show that for values of $P$ which are *above* a certain threshold, the maximal total rate which is achieved by Carleial's region (21) is above the maximal total rate which is achieved by optimal TDM/ FDM (where the latter value is equal to $\frac{1}{2}\log_2(1+2P)$ bits per channel use). To this end, we will simplify the calculation by showing that for high enough values of $P$, then even for the particular case where $\beta = 0$ in (21), the corresponding maximal total rate of the latter achievable rate region exceeds the maximal total rate which is obtained by optimal TDM/ FDM. Under the assumption that $\beta = 0$, one obtains that $\delta = \overline{\alpha} \triangleq 1 - \alpha$, which yields that the maximization of $R_1 + R_2$ for the special case of the achievable rate region (21) where $\beta = 0$ is performed by maximizing the function

$$f(\alpha) \triangleq \min\left\{\log_2\left(1 + \frac{\overline{\alpha}P}{1 + \alpha P + aP}\right), \log_2\left(1 + \frac{a\overline{\alpha}P}{1 + a\alpha P + \alpha P}\right)\right\} + \log_2\left(1 + \frac{\alpha P}{1 + a\alpha P}\right) \tag{22}$$

over the interval $0 \leq \alpha \leq 1$.

The ratio between the two arguments of the function $\log_2(1 + x)$ inside the minimization in the right-hand side of (22) is

$$\frac{\frac{a\overline{\alpha}P}{1+a\alpha P+\alpha P}}{\frac{\overline{\alpha}P}{1+\alpha P+aP}} = \frac{a + a^2 P + a\alpha P}{1 + \alpha P + a\alpha P}$$

so if $a + a^2P \leq 1$ (i.e., if $0 \leq a \leq \frac{\sqrt{1+4P}-1}{2P}$), then it follows (since also $\alpha \geq 0$) that

$$\begin{aligned} f(\alpha) &= \log_2\left(1 + \frac{a\overline{\alpha}P}{1 + a\alpha P + \alpha P}\right) + \log_2\left(1 + \frac{\alpha P}{1 + a\alpha P}\right) \\ &= \log_2\left(1 + \frac{a\overline{\alpha}P + \alpha P}{1 + a\alpha P}\right). \end{aligned}$$

Since $a + a^2P \leq 1$, then the maximum of the function $f(\alpha)$ over the interval $\alpha \in [0, 1]$ is achieved for $\alpha = 1$, and it is equal to $\log_2\left(1 + \frac{P}{1+aP}\right)$. The latter value corresponds to the maximal total rate which is achieved when the interfering signal is Gaussian distributed, and when the detector treats it as part of the additive Gaussian noise of the received signal (in addition to the AWGN of the channel).
If on the other hand $a + a^2P \geq 1$ and $a \leq 1$ (i.e., $\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$), then we can separate the latter condition into two sub-cases: If $1 \leq a + a^2P \leq 1 + \alpha P$, then same as we had before

$$f(\alpha) = \log_2\left(1 + \frac{a\overline{\alpha}P + \alpha P}{1 + a\alpha P}\right).$$

Otherwise, if $1 \leq 1 + \alpha P \leq a + a^2P$, then it follows from Eq. (22) that

$$f(\alpha) = \log_2\left(1 + \frac{\overline{\alpha}P}{1 + \alpha P + aP}\right) + \log_2\left(1 + \frac{\alpha P}{1 + a\alpha P}\right).$$

Let $\alpha_0 \triangleq \frac{a+a^2P-1}{P}$ (so that $0 \leq \alpha_0 \leq 1$, and $1 + \alpha_0 P = a + a^2P$), then short calculation shows that the maximal total rate which is achieved by the rate region (21) is at least equal to

$$f(\alpha_0) = \log_2\left(\frac{a + aP + a^2P}{1 - a + a^2(aP + 1)}\right).$$

Let us define the function

$$g(a) \triangleq \frac{a + aP + a^2P}{1 - a + a^2(aP + 1)}, \quad \frac{\sqrt{1 + 4P} - 1}{2P} \leq a \leq 1. \tag{23}$$

From the monotonicity of the logarithm function, the maximal value of the total rate in the achievable rate region (21) is lower bounded by the logarithm of $g(a)$ for all values of $a$ within the interval $\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$. We intend to find the maximum of $g$ on the latter interval, and also to find a range of values for $a$, so that the lower bound on the total rate $(R_1 + R_2)$ exceeds the value of the total rate which is obtained by optimal TDM/ FDM (i.e., $\frac{1}{2}\log_2(1 + 2P)$).

It can be shown that the derivative of $g$ is a monotonic decreasing function in the interval $\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$, and it has opposite signs at the endpoints of this interval. Therefore, there exits a single point inside this interval where the derivative of $g$ is equal to zero, which also achieves the maximal value of $g$ inside this interval. By differentiating the function $g$, and setting the derivative to zero, straightforward algebra shows that the maximal value of $g$ inside this interval can be calculated by solving the polynomial equation (9). Let $a = a_0$ be the proper solution of the polynomial equation (9) (see Theorem 3). Then, a lower bound on the maximal total rate which is attained by the achievable region (21) is above the value which corresponds to TDM/ FDM if

$$\log_2\left(\frac{a_0 + a_0P + a_0^2P}{1 - a_0 + a_0^2(a_0P + 1)}\right) \geq \frac{1}{2}\log_2(1 + 2P) \tag{24}$$

where it can be verified numerically that inequality (24) is satisfied for values of $P$ above 17 dB.

*Discussion*: We will now consider a generalization of the achievable rate region of Carleial considered above (based on Theorem 6 in [3]), which also includes the two approaches of treating the weak signal as additive noise (for weak interference) or the TDM/ FDM approach (for moderate interference). To this end, we will consider the particular case of the achievable rate region of Han and Kobayashi [9] where the size of the alphabet of the time-sharing parameter $Q$ is four, the random variables $U_1, U_2, W_1$ and $W_2$ are conditionally independent given $Q$ (their distributions are given in Table I for every possible value of $Q$), and where the random variables $X_1$ and $X_2$ are given by

$$X_1 = U_1 + W_1, \quad X_2 = U_2 + W_2. \tag{25}$$

| Time-sharing parameter | $U_1$ | $W_1$ | $U_2$ | $W_2$ |
|---|---|---|---|---|
| Prob $(Q=0)=\delta$ | $\sim N(0, 2\alpha\delta P)$ | $\sim N(0, 2\overline{\alpha}\delta P)$ | $\sim N(0, 2\beta\delta P)$ | $\sim N(0, 2\overline{\beta}\delta P)$ |
| Prob $(Q=1)=\delta$ | $\sim N(0, 2\beta\delta P)$ | $\sim N(0, 2\overline{\beta}\delta P)$ | $\sim N(0, 2\alpha\delta P)$ | $\sim N(0, 2\overline{\alpha}\delta P)$ |
| Prob $(Q=2)=\lambda(1-2\delta)$ | $\sim N(0, \frac{(1+2\delta)P}{\lambda})$ | 0 | 0 | 0 |
| Prob $(Q=3)=\overline{\lambda}(1-2\delta)$ | 0 | 0 | $\sim N(0, \frac{(1+2\delta)P}{\overline{\lambda}})$ | 0 |

TABLE I
THE PARAMETERS $Q, U_1, W_1, U_2, W_2$ WHICH ARE USED TO DEFINE A PARTICULAR SUB-REGION OF THE ACHIEVABLE RATE REGION OF HAN AND KOBAYASHI [9]. THE RANDOM VARIABLES $U_1, U_2, W_1$ AND $W_2$ ARE CONDITIONALLY INDEPENDENT GIVEN $Q$, AND THE RANDOM VARIABLES $X_1$ AND $X_2$ ARE GIVEN IN EQ. (25). WE ASSUME HERE THAT $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, $0 \leq \lambda \leq 1$, AND $0 \leq \delta \leq \frac{1}{2}$.

Based on Table I and Eq. (25)

$$E[x_1^2] = \delta \cdot 2\delta P + \delta \cdot 2\delta P + \lambda(1-2\delta) \cdot \frac{(1+2\delta)P}{\lambda} = P$$

and similarly $E[x_2^2] = P$, so both inputs of the two-user Gaussian IFC satisfy the common power constraint (3) where $P_1 = P_2 \triangleq P$. By symmetry considerations, it is clear that the maximal total rate of the considered achievable rate region is achieved for $\lambda = \frac{1}{2}$.

The achievable rate region by TDM/ FDM results in as a particular case of the HK region with the setting in Table I when $\delta = 0$. The special achievable rate region where the interfering signal is treated as an additive Gaussian noise results in as a particular case of the HK region with the setting in Table I for $\delta = \frac{1}{2}$ and $\alpha = \beta = 1$. Finally, The particular achievable rate region of Carleial (see [3, Theorem 6]) for the symmetric two-user Gaussian IFC results in from the setting in Table I when $\delta = \frac{1}{2}$ (which is verified from the superposition in Eq. (25)). These simple observations yield that the maximal total rate of the achievable rate region of Han and Kobayashi in the setting of Table I is not below the maximal total rate which one obtains by treating the interfering signal as an additive noise (in the case of weak interference), and it is also not below the maximal total rate which is obtained by optimal TDM/ FDM (in the case of moderate interference). From the discussion above regarding Carleial's achievable rate region (21), we obtained that for high values of $P$, the maximal total rate of the latter region (and hence, the maximal total rate which corresponds to the Han and Kobayashi (HK) region with the setting in Table I) exceeds the maximal total rate which is obtained by the two aforementioned methods (for weak and moderate interference). In the sequel, we will calculate the maximal total rate which corresponds to the HK region with the specific setting in Table I, and numerical results of the resulting expression are presented in Section 5. The discussion in [9] yields that the maximal total rate of the achievable rate region by Han and Kobayashi is given by

$$\max(R_1 + R_2) = \rho_{12} \tag{26}$$

where

$$\rho_{12} \triangleq \sigma_{12} + I(Y_1; U_1 | W_1, W_2, Q) + I(Y_2; U_2 | W_1, W_2, Q) \tag{27}$$

and

$$\sigma_{12} \triangleq \min\Big\{ \begin{array}{l} I(Y_1; W_1, W_2 | Q),\ I(Y_2; W_1 W_2 | Q), \\ I(Y_1; W_1 | W_2, Q) + I(Y_2; W_2 | W_1, Q), \\ I(Y_2; W_1 | W_2, Q) + I(Y_1; W_2 | W_2, Q) \Big\}. \end{array} \tag{28}$$

Based on the definition of the function $\gamma$ in Eq. (5), we obtain that for $\lambda = \frac{1}{2}$

$$\begin{aligned} & I(Y_1; U_1 | W_1, W_2, Q) = I(Y_2; U_2 | W_1, W_2, Q) \\ & \quad = \delta \cdot \gamma\left(\frac{2\alpha\delta P}{1 + 2a\beta\delta P}\right) + \delta \cdot \gamma\left(\frac{2\beta\delta P}{1 + 2a\alpha\delta P}\right) + \left(\frac{1-2\delta}{2}\right) \cdot \gamma\big(2(1+2\delta)P\big), \end{aligned} \tag{29}$$

$$I(Y_1;W_1,W_2|Q) = I(Y_2;W_1,W_2|Q) = \delta\cdot\gamma\left(\frac{2\overline{\alpha}\delta P+2a\overline{\beta}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right)+\delta\cdot\gamma\left(\frac{2\overline{\beta}\delta P+2a\overline{\alpha}\delta P}{1+2\beta\delta P+2a\alpha\delta P}\right) \tag{30}$$

$$I(Y_1;W_1|W_2,Q) = I(Y_2;W_2|W_1,Q) = \delta\cdot\gamma\left(\frac{2\overline{\alpha}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right)+\delta\cdot\gamma\left(\frac{2\overline{\beta}\delta P}{1+2\beta\delta P+2a\alpha\delta P}\right) \tag{31}$$

$$I(Y_2;W_1|W_2,Q) = I(Y_1;W_2|W_1,Q) = \delta\cdot\gamma\left(\frac{2a\overline{\alpha}\delta P}{1+2a\alpha\delta P+2\beta\delta P}\right)+\delta\cdot\gamma\left(\frac{2a\overline{\beta}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right). \tag{32}$$

By substituting Eqs. (27)–(32) in (26), we obtain that the maximal total rate of the achievable region that corresponds to Table I is equal to the maximum of the function

$$\begin{aligned}
\rho_{12} &\triangleq \rho_{12}(\alpha,\beta,\delta)\\
&= 2\delta\cdot\gamma\left(\frac{2\alpha\delta P}{1+2a\beta\delta P}\right)+2\delta\cdot\gamma\left(\frac{2\beta\delta P}{1+2a\alpha\delta P}\right)+(1-2\delta)\cdot\gamma\big(2(1+2\delta)P\big)\\
&\quad+\min\Bigg\{\delta\cdot\gamma\left(\frac{2\overline{\alpha}\delta P+2a\overline{\beta}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right)+\delta\cdot\gamma\left(\frac{2\overline{\beta}\delta P+2a\overline{\alpha}\delta P}{1+2\beta\delta P+2a\alpha\delta P}\right),\\
&\qquad 2\delta\cdot\gamma\left(\frac{2\overline{\alpha}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right)+2\delta\cdot\gamma\left(\frac{2\overline{\beta}\delta P}{1+2\beta\delta P+2a\alpha\delta P}\right),\\
&\qquad 2\delta\cdot\gamma\left(\frac{2a\overline{\alpha}\delta P}{1+2a\alpha\delta P+2\beta\delta P}\right)+2\delta\cdot\gamma\left(\frac{2a\overline{\beta}\delta P}{1+2\alpha\delta P+2a\beta\delta P}\right)\Bigg\}
\end{aligned} \tag{33}$$

where the function $\gamma$ is defined in Eq. (5), and the maximization of $\rho_{12}$ is carried over the parameters $(\alpha,\beta,\delta)$ where $0\leq\alpha\leq 1$, $0\leq\beta\leq 1$ and $0\leq\delta\leq\frac{1}{2}$.

Corollary 2 follows directly from the scaling of the input signals in order to obtain the standard form (2) for the two-user Gaussian IFC.

## 5. Numerical Results

We present here numerical results which illustrate the theorems in Section 3, regarding achievable rate regions and bounds on the sum-rate capacity of a two-user Gaussian IFC.

Fig. 1 compares the achievable rate region for a two-user Gaussian IFC where $a_{12}=a_{21}=0.5$ in the standard form (2), and under the common power constraint $P_1=P_2=6$ in (3). As expected from Theorem 1, the achievable rate region by TDM/ FDM (curve 1) is included in the rate region which is achieved by Theorem 1 (curve 4), but the maximal total rate in both cases stays the same. Based on Theorem 1, the achievable rate region whose boundary is curve 2 is included in the achievable rate region which is obtained in Theorem 1. Since in our setting $P=6$, $a=0.5$, and $a>\frac{\sqrt{1+2P}-1}{2P}=0.2171$, then it follows from Theorem 1 that the particular achievable rate region $\mathcal{G}'$ (see curve 3) is included in the achievable rate region of Theorem 1 (see curve 4), as is illustrated in Fig. 1. Moreover, the maximal achievable total rate in the former region is strictly smaller than the one which corresponds to the latter achievable rate region.

Fig. 2 compares upper and lower bounds on the sum-rate capacity of a two-user symmetric Gaussian IFC. We consider here the symmetric case where in the standard form, $P_1=P_2\triangleq P$, and $a_{12}=a_{21}\triangleq a$ (where $a$ designates the square of the magnitude of the interference coefficient in (2), and it is the horizontal axis in Fig. 2). As an example, for the case where $P=30$ dB, it follows that $a_0=0.0821$ is the appropriate solution of the polynomial equation in Theorem 3. As can be verified from Fig. 2, if the common value of $a_{12}$ and $a_{21}$ (under the symmetry assumption) is between 0.0821 and 1, then the maximal total rate which is achieved by curve 5 is strictly larger than the achievable region by TDM/ FDM (curve 2). In particular, if $a_{12}=a_{21}=0.0821$, the values of the maximal total rate which are achieved by curves 2 and 5 are 5.483 and 5.911 bits per channel use, respectively (furthermore, the latter value coincides with the lower bound on the maximal total rate which is given in Eq. (10), as compared to an upper bound of 6.774 bits per channel use, which follows from Kramer's upper bound [10] and is depicted in curve 1 of Fig. 2). Moreover, the solution of inequality (24) (where $a_0$ in (24) is now replaced with the arbitrary common value of the interference coefficient $a$) yields that $0.0448\leq a\leq 0.1382$. Clearly, as reflected in Fig. 2, it

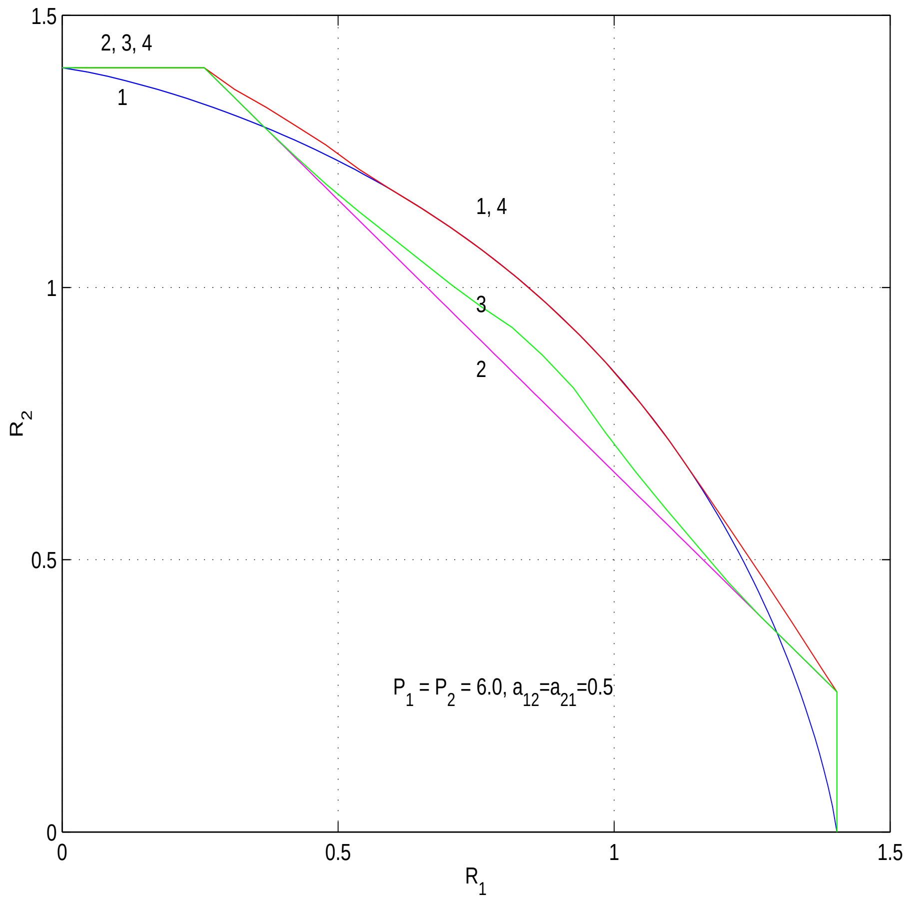


Fig. 1. Achievable rates for the two-user Gaussian IFC with weak/ moderate interference coefficients. Curve 1 is the boundary of the achievable rate region by TDM/ FDM. Curve 2 is the boundary of the achievable rate region which is obtained by time-sharing between the two rate pairs where one of the transmitters sends its data reliably at the maximal possible rate (i.e., the maximal rate it can achieve in the absence of interference), and the other transmitter decreases its data rate to the point where both receivers can reliably decode its message. The boundary of curve 2 includes the straight line that connects the two rate pairs specified in Eq. (6). Curve 3 is the boundary of the achievable rate region $\mathcal{G}'$ which was derived by Han and Kobayashi as a particular case of their general achievable rate region (see Eqs. (4.1)–(4.9), (5.9), and the setting in Section 5.A of [9]). Curve 4 is the achievable rate region in Theorem 1 which in the symmetric case where $P_1 = P_2$ and $a_{12} = a_{21}$, it is simplified to the rate region in Eq. (19).

is a partial interval as compared to the interval for which curve 5 in Fig. 2 gets values above the horizontal line in curve 2 (where the latter corresponds to the maximal total rate which is obtained by TDM/ FDM). The reason for this observation is related to the simplifications which finally led to the derivation of inequality (24), referring to a looser achievable rate region as compared to one which corresponds to the particular Han and Kobayashi rate region with the setting in Table I and Eq. (25). However, it is interesting to note the lower bound to $a$ (i.e., 0.0448) approximates well the lower limit of $a$, for which the total rate in curves 3, 4, and 5 is above the value which corresponds to curve 2 (see Fig. 2). This phenomenon was verified also for other values of $P$ above 17 dB. This value is the threshold on $P$ for which there exists a bump in Fig. 2 which in turn signals a total rate above the maximal total rate which is obtained by TDM/FDM.

The sum-rate capacity of a degraded or a one-sided Gaussian IFC are provided in Eqs. (7) and (8), respectively. The sum-rate capacity of a one-sided Gaussian IFC is shown in Fig. 3, where we assume a common power constraint of $P = 6$. It is a monotonic decreasing function of the interference coefficient ($a$) in the range $0 \leq a \leq 1$, and a monotonic increasing function of $a$ for strong interference (i.e., for $1 \leq a \leq 1 + P$) [14]. For very strong interference, the sum-rate capacity stays constant because the interference does not harm, as was demonstrated by Carleial [2]. The two limit cases in Fig. 3 where $a = 0$ and $a = 1$ correspond to two separate AWGN channels and to a Gaussian multiple-access channel, and therefore, the sum-rate capacity is equal to $\log_2(1 + P) = 2.807$ and $\frac{1}{2}\log_2(1 + 2P) = 1.850$ bits per channel use, respectively (see Fig. 3).

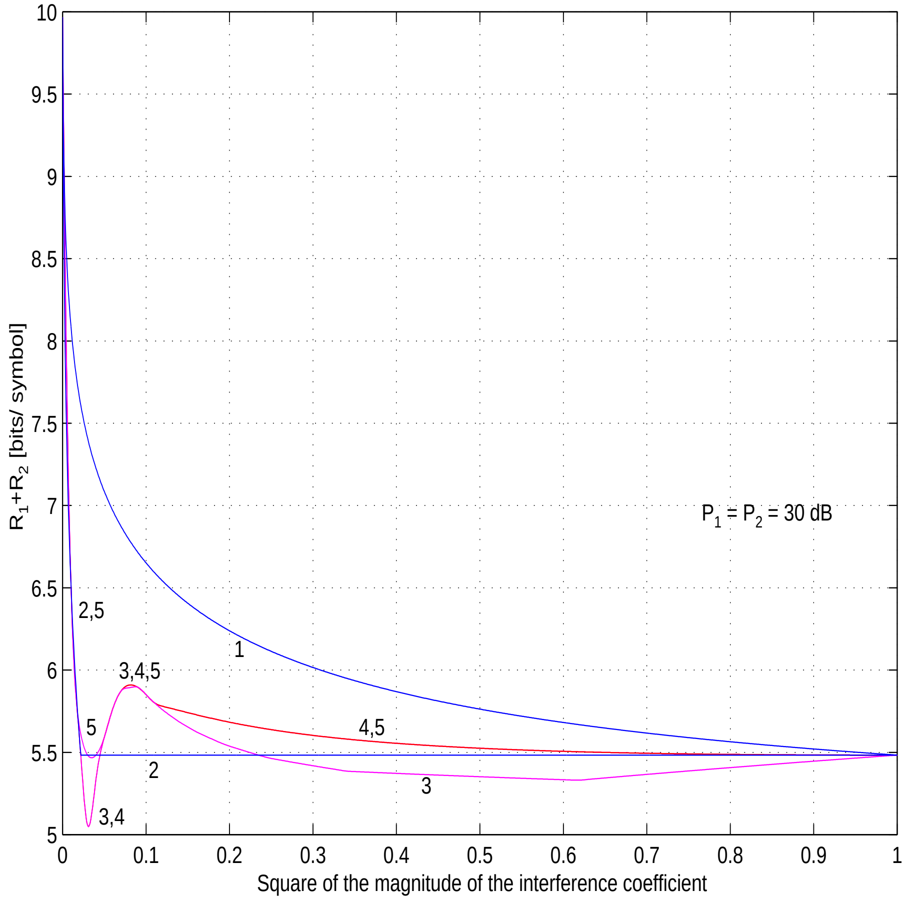


Fig. 2. Upper and lower bounds on the sum-rate capacity for the two-user Gaussian IFC. The curves which are depicted in Fig. 2 correspond to the following bounds: Curve 1 is Kramer's upper bound on the maximal achievable total rate [10, Theorem 2]. Curve 2 is the simple version of Carleial's lower bound [3] which treats the interfering signal as an additive Gaussian noise for weak interference, and which relies on the TDM/ FDM approach for moderate interference. Curve 3 is the maximal total rate which is achieved with the particular sub-region of Han and Kobayashi ($\mathcal{G}'$) in Section 5.A of [9] (where in the latter case, time-sharing is not performed). Curves 4 and 5 refer to the maximal total rates which are obtained for the particular cases of the general achievable rate region of Han and Kobayashi [9] with the specific setting in Table I and in Eq. (25); curve 4 refers to the case where $\delta = \frac{1}{2}$ in Table I (so, the time-sharing parameter $Q$ is binary), and curve 5 of Fig. 2 corresponds to the case where the maximization of the total rate is carried over the additional parameter $\delta \in [0, \frac{1}{2}]$ (so, in the latter case, $Q$ is a random variable whose alphabet is of size four). Curves 4 and 5 are therefore calculated by the maximization of the right-hand side of Eq. (33) where for the calculation of curve 4, we set $\delta = \frac{1}{2}$ and maximize $\rho_{12}$ over the parameters $(\alpha, \beta) \in [0, 1] \times [0, 1]$, and the calculation of curve 5 is performed by the numerical maximization of $\rho_{12}$ in (33) over the three parameters $\alpha, \beta$ and $\delta$ (where $0 \leq \delta \leq \frac{1}{2}$).

## 6. Concluding Remarks

1) In [9, Theorem 3.1], Han and Kobayashi derived an achievable rate region for a general discrete memoryless IFC. It is the best reported achievable rate region, and it yields as particular cases all the previously reported achievable rate regions by Sato, Carleial and others (including the achievable rate region that is specified in Theorem 1 here). However, it is in general prohibitively complex to calculate the HK rate region. For a two-user Gaussian IFC with weak or moderate interference, the calculation of the achievable region in [9, Theorem 3.1] does not seem to be feasible. The achievable rate region $D$ in Theorem 1 is feasible for calculation, and for a two-user Gaussian IFC with *moderate* interference (i.e., if $\frac{\sqrt{1+2P}-1}{2P} < a \leq 1$), it includes the particular rate region $\mathcal{G}'$ which was derived by Han and Kobayashi in [9, Section V-A] (note that, as opposed to their full achievable rate region, the particular rate region $\mathcal{G}'$ is feasible for calculation, and it also coincides with the capacity region of a two-user Gaussian IFC with *strong* interference). For moderate interference, the maximal total rate which is achieved by the rate region $D$ in Theorem 1 is strictly larger than the maximal total rate which corresponds to $\mathcal{G}'$. We note that although for a two-user Gaussian IFC with *weak* interference (i.e., for $0 < a < \frac{\sqrt{1+2P}-1}{2P}$), the maximal total rate which is obtained by $\mathcal{G}'$ is strictly

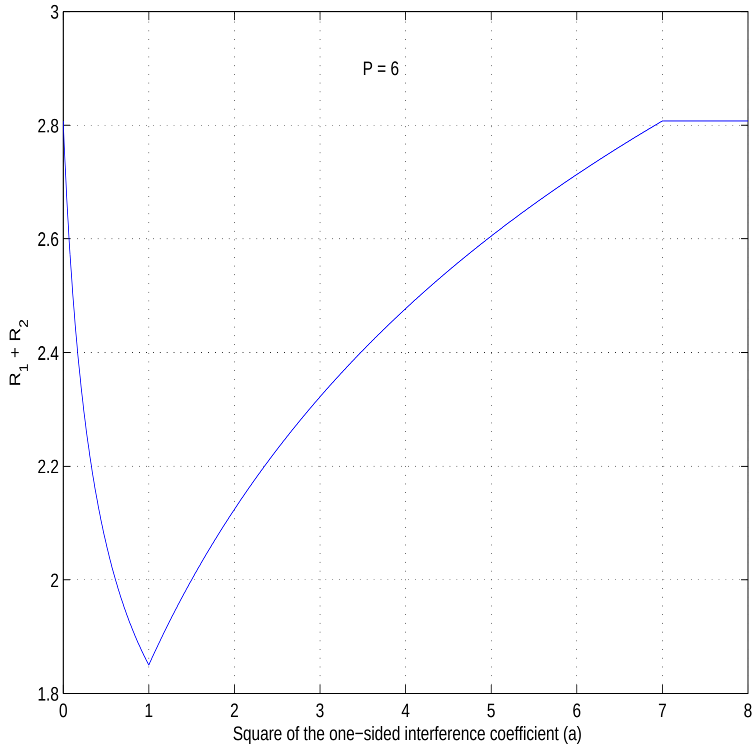


Fig. 3. The sum-rate capacity of a one-sided Gaussian IFC with a common power constraint $P_1 = P_2 = 6$.

larger than the one which corresponds to the achievable rate region $D$ in Theorem 1, the region $D$ is not necessarily included in $\mathcal{G}'$ for the case of weak interference; on the hand, Theorem 1 ensures that $\mathcal{G}' \subset D$ for a two-user Gaussian IFC with moderate interference.

2) It is not clear whether the sum-rate capacity should be a decreasing function of the common interference coefficient when its value varies between zero and unity (since there is no co-operation between the receivers). Nonetheless, we believe that the bump which is observed in curves 3, 4 and 5 of Fig. 2 is an artifact of the inner bounds on the capacity region: for one-sided or degraded Gaussian IFC, the sum-rate capacity is a monotonic decreasing function in the range of weak/ moderate interference (as follows from Theorem 2, and exemplified in Fig. 3), it is likely to be the case in general for a two-user Gaussian IFC.

3) It is noted that the sum-capacity of a one-sided Gaussian IFC with is provided in Eq. (8) implies the upper bound on the sum-rate capacity of a two-user Gaussian IFC in [10, Theorem 1]. To see that, let us consider a two-user Gaussian IFC in the standard form (see (2)) where the power constraints on the first and second inputs are $P_1$ and $P_2$, respectively. We will consider in the following the case where the interference coefficients $a_{12}$ and $a_{21}$ in the standard form (2) are between zero and one, though the bound follow in the same way for all the other cases. If one of the interferences between the two senders in canceled (i.e., if we set $a_{12} = 0$ or $a_{21} = 0$), then the capacity region of the resulting one-sided Gaussian IFC does not shrink as compared to the capacity region of the original Gaussian IFC. Hence, the minimal value among the two sum-rate capacities of the two resulting one-sided Gaussian IFCs forms an upper bound on the sum-rate capacity of the original Gaussian IFC (whose interference coefficients $a_{12}$ and $a_{21}$ are both nonzero). From the expression for the exact sum-rate capacity of a one-sided Gaussian IFC in (8), it follows that the sum-rate capacity of the original Gaussian IFC satisfies the inequality

$$R_1 + R_2 \leq \min\left\{\gamma(P_1) + \gamma\Big(\frac{P_2}{1 + a_{21}P_1}\Big),\ \gamma(P_2) + \gamma\Big(\frac{P_1}{1 + a_{12}P_2}\Big)\right\}. \tag{34}$$

Since $\gamma(x) = \frac{1}{2}\log_2(1+x)$ and we examine here the case where $0 \leq a_{12}, a_{21} \leq 1$, then it follows that

$$\gamma(P_1) + \gamma\Big(\frac{P_2}{1 + a_{21}P_1}\Big) = \frac{1}{2}\log_2\Big((P_2 + a_{21}P_1 + 1)\Big(\frac{P_1 + 1}{\min\{1, a_{21}\}P_1 + 1}\Big)\Big) \tag{35}$$

and

$$\gamma(P_2) + \gamma\Big(\frac{P_1}{1 + a_{12}P_2}\Big) = \frac{1}{2}\,\log_2\Big((P_1 + a_{12}P_2 + 1)\Big(\frac{P_2 + 1}{\min\{1, a_{12}\}P_2 + 1}\Big)\Big) \tag{36}$$

so the combination of (34)–(36) provides the upper bound on the sum-rate capacity of a two-user Gaussian IFC which appears in [10, Theorem 1]. It also proves that the upper bound on the sum-rate capacity in [10, Theorem 2] is always better than the one in [10, Theorem 1], as was demonstrated in [10, Theorem 3]. It is here that our notation for the interference coefficients in (2) are consistent with those in the paper by Han and Kobayashi (see [9, Eqs. (5.3), (5.4)]), but $a_{12}$ and $a_{21}$ in [10] need to be reversed to be consistent with the notation used here.

4) In continuation to item 3, it is noted that the upper bounds on the sum-rate capacity of a two-user Gaussian IFC which are presented in [10] coincide asymptotically in the limit where we let $P$ tend to infinity. For simplicity, it is shown in the symmetric case where $a_{12} = a_{21} = a$ and $P_1 = P_2 = P$. In this case, the upper bounds on the sum-rate capacity of a two-user Gaussian IFC are

$$R_1 + R_2 \leq \frac{1}{2}\,\log_2(1 + P) + \frac{1}{2}\,\log_2\Big(1 + \frac{P}{1 + aP}\Big) \tag{37}$$

and

$$R_1 + R_2 \leq \log_2\left(1 + \frac{-(1+a) + \sqrt{(1+a)^2 + 4a(1+a)P}}{2a}\right) \tag{38}$$

so, it can be verified that the ratio of the right-hand sides of (37) and (38) tends to 1 as we let $P$ tend to infinity. For finite $P$, as was mentioned in [10] and in item 3 above, the upper bound on the same-rate capacity in (38) is always tighter than the upper bound in (37).

5) For a two-user Gaussian IFC, it was stated in [5, Theorem 1] that if one of the senders is transmitting at its maximal possible rate (i.e., the maximal rate which is achievable for a single-user, in the absence of interference), then the other sender is obliged to decrease its data rate to the point where both receivers can reliably decode its message. The implication of the proof of this converse theorem is that these two rate-pairs form the corner points of the capacity region of a two-user Gaussian IFC. For a two-user Gaussian IFC with strong interference whose characterization of its capacity region is completely known (see [9, Theorem 5.2] and [14]), this is indeed the case, as the capacity region in the latter case is the intersection of the capacity regions of the two Gaussian multiple-access channels which are induced by the two-user Gaussian IFC. Unfortunately, it is not clear whether in general these are the two corner points of the capacity region of a two-user Gaussian IFC due to a problem in a certain step of the proof to the converse theorem in [5, Appendix B] (which was confirmed by the author of [5], though we could not fix this problematic step in the proof). By restating this step as a mathematical problem, it considers the following issue: Let $X$ and $Y$ be two $n$-dimensional random vectors, where $X$ is a Gaussian random vector with i.i.d. components of zero mean and variance $\sigma^2$. The differential entropy of $X$ attains its maximal value under the above power constraint, and it is equal to

$$h(X) = \frac{n}{2}\,\log(2\pi e\sigma^2).$$

Let $Y$ be an $n$-dimensional random vector whose components satisfy the following constraints:

$$E[Y_i] = 0, \quad \frac{1}{n}\sum_{i=1}^{n}(Y_i)^2 \leq \sigma^2. \tag{39}$$

Unlike $X$, the components of $Y$ may be correlated, and $Y$ may *not* be Gaussian. However, $Y$ is considered to be ”almost Gaussian” in the sense that

$$h(X) - h(Y) \leq n\varepsilon \tag{40}$$

for some positive $\epsilon$ (we note that under the conditions in (39), the left-hand side of (40) is necessarily non-negative.) We introduce now an arbitrary $n$-dimensional random vector $Z$ where in probability 1, the vector $Z$ is inside a sphere of radius $\sqrt{nP}$ for a positive constant $P$, i.e.,

$$\frac{1}{n}\sum_{i=1}^{n}(z_i)^2 \leq P \tag{41}$$

and $Z$ is known to be statistically independent of $X$ and $Y$. The question is if under the assumptions in (39)–(41), one can prove an inequality of the form

$$h(Y+Z) - h(X+Z) \leq \delta(\varepsilon, P) \cdot n$$

where $\delta(\varepsilon, P) \rightarrow 0$ as $\varepsilon \rightarrow 0$ ?
In essence, the proof in [5, Appendix B] shows that

$$D(P_Y || P_X) \leq n\varepsilon,$$

where $P_X$ and $P_Y$ designate the probability distributions of $X$ and $Y$, respectively, and $D(P||Q)$ stands for the divergence between the probability distributions $P$ and $Q$. Then, by the data processing theorem, it follows that

$$D(P_{Y+Z} || P_{X+Z}) \leq D(P_Y || P_X) \leq n\varepsilon.$$

The proof in [5, Appendix B] relies on Pinsker's inequality which forms a lower bound on the divergence in terms of the $L_1$ distance between the two probability measures. The problematic issue in this proof is related to the link between the $L_1$ distance and the difference between the two differential entropies. Specifically, [5] utilizes a parameter $\beta$ which depends exponentially in $n$, but is treated after [5, Eq. (B.11)] as if it was a constant. This yields that $\log(\beta)$ in Eq. (B.14) of [5] grows linearly in $n$, and hence the right-hand side of (B.14) grows like $n^{\frac{3}{2}}$ (and not linearly, as desired).

6) One possible direction in trying to fix the problematic issue which is addressed in item 5 relied on some recent refinements of Pinsker's inequality [8]. One of these refinements (see [8, Theorem 7]) states that

$$D(P||Q) \geq \frac{1}{2}V^2 + \frac{1}{36}V^4 + \frac{1}{270}V^6 + \frac{221}{340,200}V^8 \tag{42}$$

where $V$ designates the $L_1$ distance between $P$ and $Q$, i.e.,

$$V \triangleq \int_{-\infty}^{\infty} |P(x) - Q(x)| \, \mathrm{d}x.$$

Inequality (42) enables to reduce the power of $n$ in the right hand-side of [5, Inequality (B.14)]) from $\frac{3}{2}$ to $\frac{9}{8}$. A further refinement of Pinsker's inequality (see [8, Section 4]) enables to further reduce the power of $n$ to $\frac{49}{48}$, but it is still not the desired *linear dependence* in $n$ which is required to complete the proof in [5]. Solving this problem has a direct implication on the characterization of the two corner points of the capacity region of a general Gaussian interference channel (it is likely that the two rate pairs which were specified in Theorem 1 of [5] are indeed the two corner points of the capacity region of a two-user Gaussian IFC.)

7) The derivation of an achievable rate region for the two-user Gaussian IFC refers to vanishing *average decoding error probability*. The maximal-error and average-error capacity regions are *not necessarily identical* for general multi-user channels; In [7], Dueck has shown by examples that for two-way channels and for multiple-access channels, the capacity regions depend on the error concept used. Though these two capacity regions are identical for a general broadcast channel ([6], [16]), this is not necessarily the case for general multi-user channels. In particular, these two capacity regions are not necessarily identical for interference channels (otherwise, this would also be the case for the multiple-access channel, in contradiction to [7]).

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