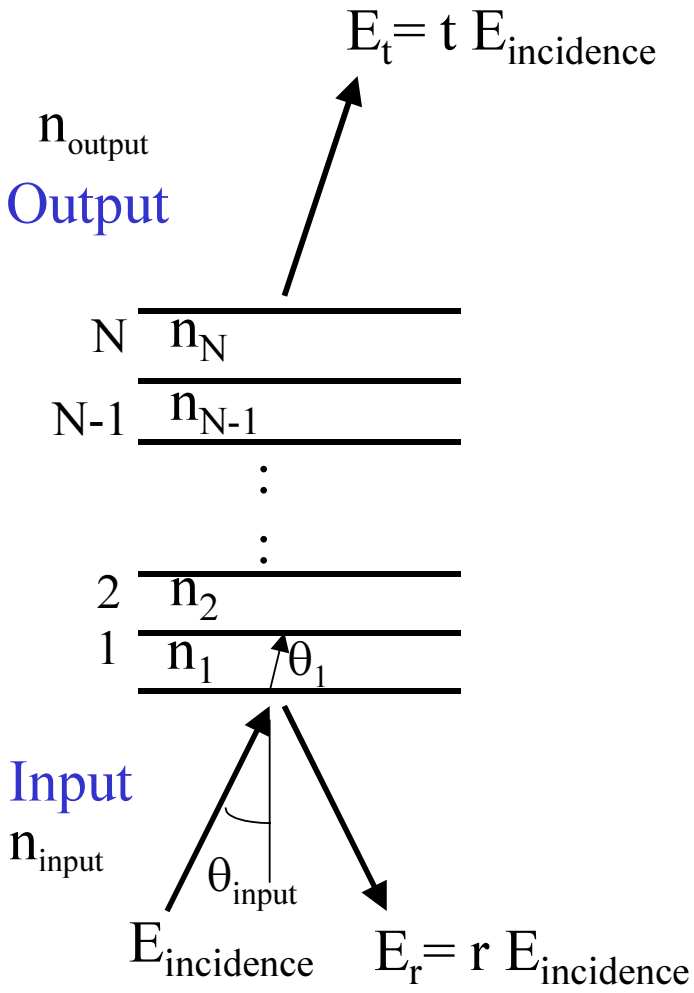
Transmission and Reflection in a Multi-Layer optical System:

Based on the book by Born&Wolf (Principles of optics p. 52).



The idea is that the propagation of the electromagnetic field through a structure is described by a characteristic matrix (M). M relates the field (E and H) at the input plane to that at the output plane (input=M*output). If a structure is complex (like multi-layer system) the equation becomes: input=M_1*M_2*….M_{N-1}*M_N*output, where M_N describes the layer that is closest to the output plane or in other words M= M_1*M_2*….M_{N-1}*M_N.

The format of Mj (j=1÷N) is:

$$M_j = \begin{bmatrix} M_j(1,1) & M_j((1,2) \\ M_j((2,1) & M_j((2,2) \end{bmatrix} = \begin{bmatrix} \cos(\beta_j) & \frac{-i}{p_j}\sin(\beta_j) \\ -ip_j \sin(\beta_j) & \cos(\beta_j) \end{bmatrix} \qquad (1)$$

β_j is the propagation coefficient defined as:

$$\beta_j = \frac{2\pi}{\lambda_0} L_j n_j \cos(\theta_j) \qquad (2)$$

L_j is j^{th} layer width, n_j is the complex refractive index, θ_j is the propagation angle in this layer

$$n_j \cos(\theta_j) = n_j \sqrt{1 - \sin(\theta_{ref})^2 n_{ref}^2 n_j^{-2}} \qquad (3)$$

n_{ref} is the refractive index in the medium which defines the angle (θ_{ref}) we are interested in. Usually this would be the input medium (like air) and θ_{ref} is the incidence angle.

For TE (S) wave p_j is: $p_j = n_j \cos(\theta_j)$

For TM (P) wave p_j is: $p_j = n_j^{-1} \cos(\theta_j)$

Reflection (amplitude):

$$r_{TE} = \frac{(M_{1,1} + M_{1,2}P_{output})P_{input} - (M_{2,1} + M_{2,2}P_{output})}{(M_{1,1} + M_{1,2}P_{output})P_{input} + (M_{2,1} + M_{2,2}P_{output})} \qquad (4)$$

$$r_{TM} = -\frac{(M_{1,1} + M_{1,2}P_{output})P_{input} - (M_{2,1} + M_{2,2}P_{output})}{(M_{1,1} + M_{1,2}P_{output})P_{input} + (M_{2,1} + M_{2,2}P_{output})}$$

{$R=r^2$}

(note that the difference between TE and TM is not only in the sign but also the p_j values are different)

Transmission:

$$t = 2 * p_{input} / \ldots$$
$$((M(1,1)+M(1,2)* p_{output})* p_{input} + (M(2,1)+M(2,2)* p_{output}))$$