Towards a Stable Two-Dimensional Time-Domain Source-Model Solution by Use of a Combined Source Formulation

Alon Ludwig, Student Member, IEEE, and Yehuda Leviatan, Fellow, IEEE

Abstract—The application of the source-model technique to the solution of two-dimensional (2-D) transient scattering problems directly in the time domain is described. Special attention is given to the accuracy and the stability of the numerical solution scheme. A detailed comparison is made between the more often used single-type-source solution scheme and a newly proposed combined-source solution scheme. It is found that the combined-source scheme can offer a significant advantage in terms of stability, without any appreciable loss in accuracy, over the single-type-source scheme.

Index Terms—Electromagnetic transient scattering, integral equations, numerical stability, time-domain analysis.

I. INTRODUCTION

THE problem of late-time instabilities in time-domain integral equation (TDIE)-based numerical solvers for transient scattering problems has been a subject of research for many years [1]–[11]. This problem occurs both in two-dimensional (2-D) and three-dimensional (3-D) scattering problems. However, it is much more crucial when solving 2-D scattering problems like the one considered in this paper. The added difficulty stems from the fact that the 2-D time-domain Green’s function has an infinite tail. This implies that even currents from the distant past still have an effect on the present fields. Hence, one should keep on adding contributions from these currents, which makes the 2-D integral equation solution more prone to accumulative errors that lead to late-time instabilities.

The generally accepted cause of late-time instabilities of the kind encountered in the 2-D problem that is considered in this paper is attributed to an erroneous shift of the Laplace transform poles of the transfer function between the incident field excitation and the current in terms of which the scattering problem is formulated [12], [13]. The location of these poles in the Laplace space implies the temporal behavior of the homogeneous solutions of the scattering problem, where poles giving rise to non decaying homogeneous solutions, which are associated with resonant modes of the scatterer, are located on the imaginary axis of the Laplace space. The erroneous shift of the poles stems from the insufficiently accurate discretization of the differential and integral operators that form the

function [14], [15]. However, simply increasing the spatial discretization mesh or the temporal discretization resolution for a given scheme does not guarantee improved stability, and in many cases may even act to destabilize the scheme. In recent years, a considerable effort has been devoted to rigorously analyze the stability of different schemes by means of Fourier stability analysis methods [16], [17]. Due to their complexity, these methods are often used to analyze simplified schemes for which they can give an exact prediction of stability and help in the development of reliable techniques to improve the stability. Other techniques to rectify the instability problem are based on a few common observations that were made throughout the years. Some of these techniques attempt to improve the accuracy of the discretization scheme while avoiding the excitation of higher-frequency homogeneous solutions, which usually leads to the formation of more exponentially increasing instabilities. Of these techniques, the most commonly used has been the time averaging method [18], [19]. In this technique, past currents undergo low-pass temporal filtering before their contribution to the present time is taken into account. This technique has clearly proved useful in stabilizing the solution in many cases [20]–[22]. However, there were cases [23] in which the use of this approach was somewhat less successful. Another, more recent technique to improve the accuracy without introducing higher frequencies has used bandlimited temporal basis functions together with a bandlimited extrapolator to produce stable results [24]. A different technique to gain improvement in stability relays on the combined field formulation [24], [25]. The combined field integral equation formulation is known to produce a unique solution even at the resonance frequencies of the scatterer [28]. Thus, in a combined formulation there will be no poles located on the imaginary axis of the Laplace space. Other poles located on the left-hand side of the imaginary axis of the Laplace space will be less prone to an erroneous shift towards the right-hand side of the Laplace space, where they give rise to late-time instabilities.

In this paper, instead of using standard surface formulations, we resort to the source-model technique (SMT) [29]–[32]. The SMT is also known as the method of fictitious sources (MFS) [33], [34] and the method of auxiliary sources (MAS) [35]–[37], and is related to the generalized multipole technique (GMT) [38]–[41]. The SMT was found to be an efficient and versatile computational tool for the analysis of frequency domain wave scattering problems. Here, we formulate it in the time-domain and, with a view towards improving stability, consider the use of a combined source (CS) scheme [42]. The idea behind