Scattering by a Groove in a Conducting Plane—A PO-MoM Hybrid Formulation and Wavelet Analysis

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Abstract—A novel method is presented to solve the two-dimensional (2-D) problem of scattering of an electromagnetic plane wave by a groove in a perfectly conducting infinite plane. In this method, the unknown induced current is expressed in terms of the known physical optics solution of the unperturbed problem of scattering by an infinite conducting plane plus a yet to be determined localized correction current placed in the vicinity of the groove. It is then shown that a good approximation of the induced current can be obtained using only a few dominant functions in the wavelet expansion of the correction current. Moreover, the same set of dominant wavelet functions serves the purpose of approximating the induced current at different angles of incidence. A numerical example demonstrates these various features of the proposed method of solution.

Index Terms—Hybrid methods, method of moments (MoM), physical optics (PO), scattering, wavelet analysis.

I. INTRODUCTION

WHEN analyzing the radar cross section of realistic scatterers, it is occasionally required to determine the scattering of electromagnetic waves by grooves in large metallic sheets [1]. To avoid the relatively large number of unknowns expected to be involved in such problems, it has been suggested to apply hybrid methods in which the problem is separated into different regions of interaction. In [2], for example, a finite-element method (FEM) is applied in the vicinity of grooves and other small feature, while the large scale interactions are accounted for via the shooting-and-bouncing-ray (SBR) method. In [3], in a similar way, a hybridization of a method of moment (MoM) procedure and the SBR method is pursued. In this paper, we consider the problem of two-dimensional (2-D) electromagnetic scattering by a groove in a perfectly conducting plane, and apply a hybrid method, which combines the physical optics (PO) solution with the MoM. First, we calculate the PO current induced on an infinite conducting plane that is congruent with the original scatterer. Then, since the groove affects the PO solution only in its vicinity, a correction current is numerically evaluated via the MoM over the groove and finite sections of the plane that spread out several wavelengths in each direction. To reduce the number of unknowns in the MoM expansion for this current, we utilize wavelet functions that have already been proven to be an efficient tool in scattering analysis [4], [5]. Moreover, we show that the use of a small set, comprising the dominant wavelet functions in the expansion of the correction current at a certain incident angle, yields a satisfactory approximation of the induced current for all angles of incidence. Thus, it is possible to span the correction current for all the angles of incidence with the same a priori known small set of wavelet functions.

The paper is organized as follows. In Section II, we formulate the problem and the proposed method of solution. Numerical results that demonstrate the features of this method are illustrated in Section III. It is then shown in Section IV that wavelet functions in the proposed MoM procedure can offer savings in computational resources. Finally, summary and conclusions are given in Section V.

II. PROBLEM FORMULATION AND THE METHOD OF SOLUTION

The problem under consideration is that of scattering of a TM\(_2\) time-harmonic electromagnetic plane wave by a rectangular groove in a perfectly conducting infinite plane. The cross-sectional view of the geometry of the scattering surface and the incident wave are depicted in Fig. 1. The angle of incidence is \(\theta\) measured with respect to the \(y\) axis. The depth and width of the groove are \(h\) and \(d\), respectively.

Since the scatterer is uniform in the \(z\) direction and the excitation is TM\(_2\), the induced current \(\vec{J}\) on the scatterer is \(z\)-directed and independent of \(z\). Hence, we have \(\vec{J} = J_e(x,y)\hat{z}\). To solve this scattering problem, we consider the finite segments \(\ell_2, \ell_3, \ell_4,\) and \(\ell_6\), and the half-infinite segments \(\ell_1\) and \(\ell_5\), defined in Fig. 2, and write \(J_e\) in terms of four current distributions \(J_{PO}, J_{L}^{PO}, J_{C}^{C},\) and \(J_{C}\) as

\[
J_e = J_{PO} - J_{L}^{PO} + J_{C}^{C} + J_{C}^{C}
\]

where

- \(J_{PO}\) known PO’s current solution of the unperturbed problem (the current that would be induced on a perfectly conducting infinite plane that is congruent with \(\bigcup_{i=1}^{5} \ell_i\));
- \(J_{L}^{PO}\) portion of the PO’s current \(J_{PO}\) residing on \(\bigcup_{i=2}^{5} \ell_i\);
- \(J_{C}^{C}\) unknown surface correction current in the groove region on \(\ell_6\) and on the groove’s immediate vicinity, defined by \(\ell_2 \cup \ell_4\);
- \(J_{C}\) unknown surface correction current on \(\ell_1\) and \(\ell_5\).

The widths of \(\ell_2\) and \(\ell_4\) are chosen sufficiently large to ensure that further out on \(\ell_1\) and \(\ell_5\) the induced current is virtually equal to the PO’s current.