Analysis of Truncated Periodic Array Using Two-Stage Wavelet-Packet Transformations for Impedance Matrix Compression

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Abstract—A novel method of moments procedure is applied to the problem of scattering by metallic truncated periodic arrays. In such problems, the induced current shows localized behavior within the unit cell and at the same time exhibits cell-to-cell periodicity. In order to select a set of expansion functions that may account for such behavior, a two-stage basis transformation, of which the first stage is an ordinary wavelet transformation performed independently on each unit-cell, has been applied to a pulse basis. The resultant basis functions at the first stage are regrouped and retransformed to reveal the periodicity of their coefficients. Expansion functions are then iteratively selected from this newly constructed basis to form a compressed impedance matrix. The compression ratios obtained in this manner are higher than the compression ratio achieved using a basis constructed via an ordinary single-stage wavelet transformation, where compression is the ratio between the number of nonzero elements in the matrix used to solve the problem and the number of elements in the original matrix. An even higher compression is attained by considering, in addition, functions that reveal array-end related features and iteratively selecting the expansion from an overcomplete dictionary.

Index Terms—Electromagnetic scattering by periodic structures, method of moments, wavelet transforms.

I. INTRODUCTION

Among the various categories of scattering problems the case of scattering by a truncated periodic array is of special challenge to be efficiently solved. Such arrays are widely used in both microwave and optical systems owing to their frequency selective properties. Their various applications range from microwave filters to optical focusing devices and reflectors. In a strictly periodic case it is possible to define a unit cell, which once analyzed reveals the entire information about the scattered fields. Unfortunately, in a real case, edge-effects break down the pure periodicity.

Various authors [1]–[3] have proposed methods for solving these scattering problems, which involve some kind of approximation or modification of the Floquet modes in order to compensate for the truncation of the infinite array. The method proposed in this paper is motivated by the fact that in such problems the induced surface current might exhibit complicated localized behavior within the unit-cell while keeping a certain degree of cell-to-cell periodicity. We therefore construct a new set of expansion functions for the scattering equation via a two-stage transformation. Of these two stages, the first one is carried out at the unit-cell level and focuses on the localized features of the current. We have chosen to apply the wavelet transformation at this stage, owing to its multiresolution property that enables efficient expansion of the solution of scattering problems [4]. The transformation is applied to each cell as if it were isolated from the others, therefore excludes wavelet functions that span over more than one cell. The second stage is performed at the array-level and binds together identical basis functions from different cells, thereby taking into account the inherent periodicity of the array. Assuming that the cells are similar (let alone identical), we expect the same basis functions to be the dominant one in each and every cell. Hence, the second stage of transformation is expected to result in a new basis comprising composite functions which are much more suitable for effectively expanding the current.

Once this new basis has been constructed, we can apply the impedance matrix compression (IMC) and more particularly the iterative IMC methods presented in [5]–[8]. These methods solve a reduced (compressed) version of the matrix form of the corresponding electromagnetic field integral equation. The reduced form is obtained via an iterative process that extracts a set comprising a small number of basis elements for expanding the solution at a given accuracy. We further note that the cell-to-cell variation of the coefficients of identical basis functions belonging to different cells is expected to be mainly characterized by phase-alternation. Therefore, it is very likely that a Fourier-like basis would be quite adequate for the second transformation stage. We have chosen to use the windowed-Fourier-transform wavelet-packet (WF) basis ([7], [9]). The elements of the WF basis may be well associated with a different equally-spaced nonoverlapping spatial frequency bands corresponding to the number of zero crossings they make, thus spanning the discrete spatial-frequency domain in a similar way to the discrete windowed (noncyclic) Fourier transform basis. Moreover, this basis has a simple transformation algorithm similar to the one used for wavelet transformation. We further note that edge-effects, due to the fact that the array is finite, call for more localized basis functions. Hence, both a WF and a wavelet transformation are applied independently at this second stage to form an overcom-