Impedance Matrix Compression (IMC) Using Iteratively Selected Wavelet Basis

Zachi Baharav, Student Member, IEEE, and Yehuda Leviatan, Fellow, IEEE

Abstract—In this paper, we present a novel approach for the incorporation of wavelets into the solution of frequency-domain integral equations arising in scattering problems. In this approach, we utilize the fact that when the basis functions used are wavelet-type functions, only a few terms in a series expansion are needed to represent the unknown quantity. To determine these dominant expansion functions, an iterative procedure is devised. The new approach combined with the iterative procedure yields a new algorithm that has many advantages over the presently used methods for incorporating wavelets. Numerical results which illustrate the approach are presented for three scattering problems.

Index Terms—Electromagnetic scattering, wavelet transforms.

I. INTRODUCTION

Wavelet expansions have been employed recently in numerical solutions of commonly used frequency-domain integral equations [1]–[5]. In the conventional approach to the solution of these integral equations [1], the unknown quantity of interest (usually the current on the scatterer) is first expanded in terms of a set of wavelet basis functions. Then the difference between the two sides of the equation is forced to be orthogonal to a set of wavelet testing functions. This amounts to describing the operator, which is a convolution integral of the unknown quantity with the Green’s function, in a wavelet basis. In many cases, the wavelet testing functions are nearly orthogonal to the fields due to the wavelet basis functions. Hence, the resultant matrix representation of the operator (the impedance matrix) is highly localized and becomes diagonally dominant as the wavelet functions get spatially narrower. In these cases, the impedance matrix can undergo a thresholding operation, which renders the matrix sparse. However, this virtue of being localized is liable to be scatterer geometry dependent. Moreover, once thresholding has been applied there is no systematic way (other than trivially using smaller threshold levels) leading to a more accurate solution.

In this paper, a different approach is proposed. Rather than resorting to the sparseness of the operator in the wavelet expansion, we utilize the sparse representation of the (yet unknown) quantity in the wavelet expansion. It is well known that wavelets can represent nonstationary signals with only a few terms; namely, when one expands such a signal in a wavelet series, only few terms are dominant and constitute the major part of the signal energy. This fact has mainly been applied for compression purposes in signal processing [6]–[8], but recently it has also been used in computational electromagnetics [9], [10]. In [9], [10], instead of solving for all the coefficients in the wavelet expansion of the unknown induced current, only those expected to be dominant based on the physical optics approximation of the current have been solved for.

In this paper, the determination of the dominant coefficients is affected systematically using an iterative procedure. The iterative procedure allows to zoom in on the fine details of the signal in any region of interest. It also provides a means for gradually attaining higher accuracy level. The matrices involved are much smaller and, hence, the solution requires significantly less memory and run time. Clearly, the subset of basis functions providing good approximation to the current on the scatterer for a certain excitation may not be best suited for other excitations. Hence, the iterative basis selection procedure should be repeated over again each time the incident field changes.

The organization of the paper is as follows. In the next section, the problem under study is specified and formulated using a wavelet expansion. Section III embodies the description of the iterative compression algorithm, comprising an iterative wavelet basis selection that is followed by a matrix compression (as opposed to thresholding) procedure. Numerical results are described in Section IV. Finally, summary and conclusions are given in Section V.

II. FORMULATION

Without loss of generality, let us consider the scalar problem of computing the current \( J_z \) on the perimeter of a perfectly conducting \( z \)-directed cylinder excited by a TM\(_z\) wave, as described in Fig. 2. This scattering problem can be formulated in various ways, but here we resort to the \( E \)-field integral equation formulation. To overcome the difficulties associated with the integration of the wavelet functions, \( N \) conventional pulse-basis functions are used initially to expand the current.

We have

\[
J_z = \sum_{i=1}^{N} I_i P_i \tag{1}
\]

where \( P_i \) denotes the pulse-function centered about the \( i \)-th source point on the cylinder perimeter and \( I_i \) is the yet