magnitude due to the surface wave field, especially for the case of larger ϵ_r and μ_r. Note that as the oblique incidence becomes normal incidence (i.e., $\theta' \rightarrow 90°$), the result obtained here reduces to the Maliuzhinets-based solution developed by Ly and Rojas [4] when the expressions in [4] are simplified to the special case of equal coatings on both faces of the half-plane. The accuracy of the two-part material slab solution from which the present solution is obtained has been verified in [1] using a dependent moment-method solution as a reference.

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PERIODIC WAVELET EXPANSIONS FOR ANALYSIS OF SCATTERING FROM METALLIC CYLINDERS

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KEY TERMS

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ABSTRACT

The recent application of wavelet transforms in method-of-moments solutions for scattering problems is extended to cases involving metallic cylinders whose periphery contain a variety of length scale features ranging from smoothly varying large-scale features to rapidly varying small-scale ones. The basic idea is to first consider a periodic extension of the equivalent current in the arc-length variable with a period identical to the scatterer circumference, and then to expand this representation, using a set of periodic wavelets derived from a conventional basis of wavelets by a periodic extension. Using a Galerkin method and subsequently applying a threshold procedure, a substantial reduction in the number of elements of the moment-method matrix is attained without virtually affecting the solution accuracy. The proposed extension is illustrated by a numerical study of TM (transverse magnetic) scattering from a cylinder of elliptic cross section. © 1994 John Wiley & Sons, Inc.

I. INTRODUCTION

The numerical solution of electromagnetic scattering problems formulated as integral equations is often effected by the method of moments. For the method of moments to be efficient when analyzing scattering from objects that contain a variety of length scale features ranging from smoothly varying large-scale features to rapidly varying small-scale ones, a judicious choice of basis functions should be made. A set of functions comprising translated and dilated forms of a suitable wavelet is naturally a good a candidate, because since it facilitates a basis for representing multiscale functions.

The application of wavelet-transform representations to the solution of integral equations in general [1, 2] and to the solution of electromagnetic scattering problems in particular [3–5] has been a subject of recent research efforts. However, as indicated in [4], a difficulty is encountered when one is pursuing a complete and orthonormal set of basis functions that is made only of translation and dilation of a given smooth function. The difficulty lies in the inability to span a function of finite support by means of such a set, unless the domain of definition of some of the basis functions is allowed to extend beyond that finite support. In other words, the wavelet expansion for a given function of finite support requires that some of the wavelet functions reside outside that support. The equivalent current in a problem of scattering from a finite-size perfectly conducting body is defined on the body surface, which is inherently of finite size, and the question of the number of wavelets and their proper location outside the scatterer surface naturally arises [4]. In this Letter, we propose a way to overcome this difficulty in scattering problems involving two-dimensional cylinders. Denoting the arc-length variable along the periphery by s, the basic idea is to consider a periodic extension of the equivalent current in s, with a period identical to the circumference, and then to expand this representation, which coincides with the equivalent current on the fundamental interval, using a set of periodic wavelets [6, 7]. Such a set can be constructed from a conventional basis of wavelets by a periodic extension, which is described as follows.

Let $\{\psi_{mn}\}_{m,n\in\mathbb{Z}} = \{2^{m/2}\psi(2^m\bar{s} - n)\}_{m,n\in\mathbb{Z}}$, with $\bar{s}$ denoting the arc-length variable normalized with respect to the circumference, be a complete set of orthogonal nonperiodic wavelets derived from a multiresolution analysis [6, 8]. Then, for $m \geq 1$, $0 \leq n < 2^m$, the new set of periodic functions $\{\psi^p_{mn}\}_{m,n\in\mathbb{Z}}$, defined as

$$\psi^p_{mn}(\bar{s}) = \sum_l \psi_{mn}(\bar{s} - l), \qquad l \in \mathbb{Z}, \tag{1}$$

forms, along with a constant unit function, a complete orthonormal basis in $L_2[0, 1]$. Although the new set $\{\psi^p_{mn}\}_{m,n\in\mathbb{Z}}$ has lost the dilation-invariance property of the conventional wavelet set, it still possesses many of the other