CUTOFF FREQUENCIES OF DIELECTRIC WAVEGUIDES USING THE MULTIFILAMENT CURRENT MODEL

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KEY TERMS
Dielectric waveguides, cutoff frequencies

INTRODUCTION
A method-of-moments approach based on the use of a multifilament current model (MCM) has been successful in solving two-dimensional electromagnetic scattering problems [1, 2] and, more recently, in determining the propagation constants of a dielectric waveguide of uniform cross section along the direction of propagation (the z axis) [3]. In the latter case, the wave fields of the propagating mode are approximated by two sets of z-directed electric and magnetic current filaments that vary as $I_z = I_{0z} e^{-\beta z}$ and $K_z = K_{0z} e^{-\beta z}$, where $\beta$ is the propagation constant, and $I_{0z}$ and $K_{0z}$ are the electric and magnetic current amplitudes, respectively. One set of filaments approximates the fields within the core, while the other set approximates them outside the core. The requirement of continuity of both the longitudinal and the transverse tangential field components, $E_z$, $H_z$, $E_t$, and $H_t$ (where $t$ denotes the tangential component in the transverse plane), at a set of matching points on the waveguide boundary defines the matrix equation

\[ [Z] I = 0 \]  

which is frequency dependent only. This quantity gives us a rough criterion whether a given frequency is a solution of the matrix Equation (1) in the cutoff limit, or in other words, to what extent the boundary conditions are satisfied at cutoff for all frequencies. The longitudinal tangential components in the matrix $[Z]$ contain the zero-order Hankel function of the second kind, $H_0^{(2)}(k_{p1} R)$, where $R$ is the distance from a filament to a matching point on the waveguide surface. The transverse tangential components contain the first-order Hankel function of the second kind, $H_1^{(2)}(k_{p1} R)$. When $s = 2$ we shall make use of the following limits:

\[ \lim_{\beta \to k_{p2}} \frac{2j}{\pi} \ln \left( \frac{|k_{p2} R|}{\beta} \right) \]  

where $k_{p2}$ is a negative imaginary number and $\Gamma$ is Euler’s constant.

Let $[Z]$ be a $4N \times 4N$ matrix and let $[Z']$ denote this matrix in its cutoff limit ($k_{p2} \to 0$), which we write in the following manner:

\[ [Z'] = \begin{bmatrix} E_z^{0,e} & 0 & E_z^{0,m} & 0 \\ 0 & H_z^{0,m} & 0 & H_z^{0,e} \\ E_t^{0,e} & E_t^{0,m} & E_t^{0,e} & E_t^{0,m} \\ H_t^{0,e} & H_t^{0,m} & H_t^{0,e} & H_t^{0,m} \end{bmatrix} \]  

Each element of the matrix $[Z]$ describes a tangential component generated by a single current filament of unit amplitude, at one of the matching points at the waveguide boundary. It was shown [3] that the propagation constant can be obtained for values of $\beta$ for which $L(\beta) = \ln|\det[Z]|$ has a local minimum. In this letter we use the MCM to determine the cutoff frequencies of the different modes.

ANALYSIS
Let $k_p$ and $k_{p1}$ denote the wave number and the transverse wave number, respectively, within the core ($s = 1$) and outside it ($s = 2$). Then, $k_1^2 = k_{p1}^2 + \beta^2$ and $k_2^2 = k_{p2}^2 + \beta^2$. In the frequency cutoff limit [4], $k_{p2} \to 0$, $\beta \to k_2$, and $k_{p1} \to (k_1^2 - k_2^2)^{1/2}$. In this limit, $\beta$ and $k_{p1}$ are linearly dependent on the frequency, and so all the matrix elements of $[Z]$, as well as $\det[Z]$, are functions of the frequency. We shall be interested in the limit

\[ \lim_{\beta \to k_{p2}} L(\beta) = \frac{2j}{\pi} \ln \left( \frac{|k_{p2} R|}{\beta} \right) \]  

which is frequency dependent only. This quantity gives us a rough criterion whether a given frequency is a solution of the matrix Equation (1) in the cutoff limit, or in other words, to what extent the boundary conditions are satisfied at cutoff for all frequencies. The longitudinal tangential components in the matrix $[Z]$ contain the zero-order Hankel function of the second kind, $H_0^{(2)}(k_{p1} R)$, where $R$ is the distance from a filament to a matching point on the waveguide surface. The transverse tangential components contain the first-order Hankel function of the second kind, $H_1^{(2)}(k_{p1} R)$. When $s = 2$ we shall make use of the following limits:

\[ \lim_{\beta \to k_{p2}} H_0^{(2)}(k_{p1} R) = \frac{2j}{\pi} \ln \left( \frac{|k_{p2} R|}{\beta} \right) \]  

where $k_{p2}$ is a negative imaginary number and $\Gamma$ is Euler’s constant.

Let $[Z]$ be a $4N \times 4N$ matrix and let $[Z']$ denote this matrix in its cutoff limit ($k_{p2} \to 0$), which we write in the following manner:

\[ [Z'] = \begin{bmatrix} E_z^{0,e} & 0 & E_z^{0,m} & 0 \\ 0 & H_z^{0,m} & 0 & H_z^{0,e} \\ E_t^{0,e} & E_t^{0,m} & E_t^{0,e} & E_t^{0,m} \\ H_t^{0,e} & H_t^{0,m} & H_t^{0,e} & H_t^{0,m} \end{bmatrix} \]  

$E$ and $H$ are $N \times N$ submatrices whose elements are the respective electric and magnetic tangential components, longitudinal ($z$) and transverse ($t$). The superscripts $e$ and $m$ specify the type of current filament (electric or magnetic) that generates the field, while $I$ and $O$ (inner and outer) correspond to interior or exterior fields at the matching points.

We define a matrix $C$ composed of the elements (actually submatrices) $E_t^{0,e}$, $E_t^{0,m}$, $H_t^{0,e}$, and $H_t^{0,m}$, which contain singularities as in Equation (4). We multiply every element in the lower $2N$ rows of $[Z']$ by $k_{p2}$ and denote the resulting matrix by $[Z'']$. The matrices $E_t^{0,e}$, $E_t^{0,m}$, $H_t^{0,e}$, and $H_t^{0,m}$ vanish. Let $C'$ denote the matrix $C$ whose elements have