On the Relationship Between the Transmitting and Receiving Properties of Antennas

ALON SCHATZBERG AND YEHUDA LEVIATAN

Abstract—This paper outlines an alternative way of presenting the relationship between the transmitting and receiving properties of antennas, which we think is more deductive. We suggest a direct method to calculate the effective area and the polarization mismatch factor without resorting to restricting assumptions or to the use of a specific antenna. This exposition was class-tested several times in our classes on antennas at the Technion. Our experience reveals that this sequence is better accepted in class than the sequences that can be found in existing antenna textbooks.

I. INTRODUCTION

When antennas are utilized for reception of electromagnetic radiation, the widely used parameter which characterizes their performance is the antenna effective receiving area. To determine this parameter, most textbooks first establish the equivalence between the transmitting and receiving characterizing parameters by proving that the ratio between the directivity and the effective area is a constant (referred to as the universal value) and subsequently evaluate this constant by analyzing an arbitrary simple antenna (usually an infinitesimal electric dipole). The few books that do not follow this line of presentation, develop approaches which often necessitate simplifying assumptions, particularly about the polarization of the antenna [1], [2], or involve the subject of impedance mismatch losses which actually is not material to the issue under study as it stems directly from circuit theory and can be put off to a separate discussion. This paper presents a rigorous way for evaluating the effective area and the polarization mismatch factor. We assume that the antenna, which serves as a coupling device between free space and the electrical network of the receiver, has a pair of well-defined terminals through which it is connected to that network and at which it can be replaced by a Thévenin equivalent circuit. This enables us to treat ohmic and impedance mismatch losses separately. Thus, we make the simplifying assumption that the antenna under consideration is lossless and is matched to its load. It is important to stress that this assumption is by no means a restricting one. The only restricting assumption we make is that the antenna is composed of linear and reciprocal matter.

II. TRANSMITTING ANTENNA PARAMETERS EXPRESSED IN TERMS OF THE COMPLEX VECTOR LENGTH

The fact that the transmitting and receiving properties of an antenna are related is known to stem from the Lorentz reciprocity theorem. Assuming harmonic $e^{j\omega t}$ time dependence, we use the concept of the complex effective vector length of an antenna, introduced by Sinclair [3], as the parameter connecting between the transmitting and receiving properties of the antenna. The complex effective vector length of the antenna is denoted by $h(\hat{r})$. Here, the argument $\hat{r}$ denotes a unit radius vector (in a coordinate system whose origin is within the antenna region) pointing towards an observation point $P$ in the ambient free space (Fig. 1). The argument $\hat{r}$ indicates the angular dependence of the vector function $h$, say $(\theta, \phi)$-dependence in spherical coordinates. The vector length $h(\hat{r})$ is defined by means of the current distribution $J$ induced on the antenna when a current source of amplitude $I_o$ is applied to the antenna terminals. We have

$$h(\hat{r}) = \frac{1}{I_0} \int J(r', \hat{\theta}) e^{j2\pi \hat{r} \cdot r'} d' \hat{r}'$$

(1)

where the integration is extended over the antenna region, $r'$ is a vector to a source point, $J_{r'}$ is the component of $J$ transversal to $r$, $J_{r'}(r', \hat{\theta}) = J(r') - \hat{r}(r' \cdot J(r'))$, and $k$ is the wave number in free space.

A. The Antenna Radiation Field

The field arising from the antenna that varies as $1/r$ at points remote from the antenna is called the antenna radiation field and is denoted by $E_{rad}(r)$. The region in which the radiation fields predominate is called the far-field region. When a current source of amplitude $I_o$ is applied to the antenna terminals, the radiation field of the antenna can be expressed in terms of $h(\hat{r})$ given by (1). We have

$$E_{rad}(r) = \frac{-j \kappa \eta I_o h(\hat{r}) e^{j2\pi r}}{4\pi r}$$

(2)

where $\eta = (\mu_0/\epsilon_0)^{1/2}$ is the intrinsic impedance of free space.

B. The Antenna Radiation Resistance

The radiation resistance is the resistance to which the terminal current $I_o$ will deliver the total power $P_{rad}$ radiated by the antenna. This radiated power can be evaluated through integration of the power density $S_{rad}$ over any surface enclosing the antenna. Specifically, this integration can be made over a large sphere of constant radius extending into the far-field region. Since the radiation field (2) is locally a spherical TEM (transverse electromagnetic) wave traveling radially outward, the power density over this sphere is

$$S_{rad}(r) = \frac{1}{2\eta} \left| E_{rad}(r) \right|^2 = \frac{\eta I_o^2 }{8\pi r^2} \left| h(\hat{r}) \right|^2$$

(3)

Thus, we have

$$P_{rad} = r^2 \int_S S_{rad}(r) d\Omega = \frac{1}{2} \left| I_o \right|^2 \int_S \left| h(\hat{r}) \right|^2 d\Omega$$

(4)

in which the integration is carried over the sphere and $d\Omega$ is a differential element of solid angle. From (4), the radiation resistance...