Low-Frequency Characteristic Modes for Aperture Coupling Problems

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Abstract — In this paper, a recently suggested general procedure which leads to an eigenvalue equation for aperture problems is specialized to the range of frequencies for which the maximum linear dimension of the aperture is much smaller than the wavelengths, known as the Rayleigh region. As $kl$ asymptotically approaches zero, we arrive at a set of two eigenvalue equations which, aided by the edge condition, constitutes an alternative set of equations for a derivation of the quasi-static distributions characterizing the aperture.

I. Introduction

Modal solutions have long been used for the analysis and synthesis of radiating systems. The most familiar case is when the regions of the source and the field coincide with coordinate surfaces in which the Helmholtz equation is separable. For bodies of arbitrary shape, similar modes can be defined. In these cases, the modal solutions are eigenvectors of a generalized or weighted eigenvalue equation. Garbacz [1] approached the problem by diagonalizing the scattering matrix of the body, and his results for wire objects are given in [2]. Harrington and Mautz [3] dealt with the problem by diagonalizing the generalized impedance matrix of the body, and their results for wire objects as well as bodies of revolution are given in [4]. Harrington et al. [5] subsequently extended the formulation to encompass dielectric, magnetic, and both dielectric and magnetic bodies. Other related work includes that of Inagaki and Garbacz [6] and Eftimiu and Huddleston [7]. Inagaki and Garbacz extended Parseval's relation and arrived at an eigenvalue equation for which the eigensources and corresponding eigenfields are complete and orthogonal over the source and field regions, respectively. Then they applied the theory to arrays and to a two-dimensional aperture problem. Eftimiu and Huddleston developed a useful approximate analytic expression for both eigenvalues and eigenvectors for the case of a long finite circular cylinder. Recently, Harrington and Mautz [8] suggested a procedure similar to that of [3] which leads to an eigenvalue equation for equivalent magnetic currents in three-dimensional aperture problems.

The coupling of electromagnetic energy through an aperture in a conducting wall is an important problem in the theory of electromagnetic compatibility and interference. A model used in recent years is that of two regions separated by an infinitely thin, perfectly conducting wall in which an aperture is cut. The method of solution is briefly as follows. The equivalence principle is used to divide the original problem into two parts; this is done by replacing the aperture by a perfect conductor and providing for the tangential electric field originally present in the aperture by attaching postulated magnetic current sheets to both sides of the aperture. Continuity of the tangential magnetic field across the aperture gives an integral equation for the unknown magnetic current. To solve the integral equation via the method of moments, the unknown magnetic current is expressed as a linear combination of a selected set of expansion vector functions. This linear combination is then substituted into the integral equation, which in turn is tested with each element of a set of testing functions. Obviously, the success and simplicity of the moment solution depend, often crucially, on a suitable choice of both expansion and testing functions. In this context, the eigenfunctions yielded by the eigenvalue equation possess the following desirable properties. They are equiphasal and can be chosen real; they are orthogonal in some sense over the aperture region; and their radiation fields are Hermitian orthogonal over the sphere at infinity. Hence, using these functions can unquestionably render moment solutions for aperture problems very simple. This assured simplification would come of course at the expense of the indirect step of determining the eigencurrents. However, it is expected that for electrically small apertures, only a few modes, which can be ordered according to their relative importance, are required for accurate solutions.

In this paper, we specialize the procedure of [8] to the Rayleigh region, i.e., the range of frequencies for which the maximum linear dimension of the aperture is much smaller than the wavelength. The eigenvalue equation is first ordered in ascending powers of $kl$, where $k$ is the wave-number and $l$ is the maximum linear dimension of the aperture. Then, we allow $kl$ to approach zero, thereby obtaining two low-frequency eigenvalue equations which, aided by the edge condition, constitute an alternative set of equations for the derivation of the quasi-static distributions characterizing the aperture. Specifically, the two eigencurrents of the first low-frequency eigenvalue equation give rise, by means of the equation of continuity, to the two quasi-static magnetic charge densities, while the solenoidal eigencurrent of the second eigenvalue equation is the actual quasi-static magnetic current.