Abstract—Motivated by the lack of scalability of the source-model technique (SMT) to many-scatterer problems, a new SMT-based method for 3-D many-scatterer problems is developed. The large time complexity of the SMT is reduced by deriving an implicit compact representation of the scattering problem via a fast multipole formulation, which divides the interactions into far and near ones while grouping and approximating the far interactions. This results in significant acceleration of classical SMT, which is demonstrated in a variety of 3-D many-body scattering problems. The improvement is notable already at the minimal number of unknowns necessary, and grows with the size of the problem.

Index Terms— Computational electromagnetics, fast multipole method, frequency domain analysis, source-model technique (SMT).

I. INTRODUCTION

Many-scatterer problems arise in various scenarios in electromagnetics. The common approach to solving these scattering problems utilizes various integral equation methods, such as the method of moments (MoM) [1], [2]. For \( N \) unknowns, a direct solution of the MoM system of equations requires \( O(N^3) \) operations, thus for large \( N \) an iterative method may be preferable. In each iteration, a dense matrix is multiplied by a generated vector which requires \( O(N^2) \) operations. Several numerical techniques have been proposed to reduce the number of multiplications necessary in each iteration [3]–[6], including the fast multipole method (FMM) as introduced in [7], and applied to MoM in [8]. The FMM accelerates the matrix-vector product by partitioning the problem domain into regions, where the interactions between distant regions are approximated by a far-field expansion, reducing the complexity of the matrix-vector product from \( O(N^2) \) to \( O(N^{1.5}) \). The Multilevel Fast Multipole Algorithm (MLFMA), [9]–[11], recursively applies this procedure and asymptotically reduces the complexity to \( O(N \log N) \).

Another integral equation method is the source-model technique (SMT) [12] that has been effectively applied to various scattering problems [13], [14]. The SMT offers many advantages over moment method solutions. In particular, the SMT is meshless, has a simple error metric, and its matrix fill-in does not involve any costly integral calculations; these factors significantly reduce computational costs [14]. In addition, in many instances the SMT requires fewer unknowns in comparison to other methods, which further lowers the time and memory complexities. The resulting impedance matrix often has a higher condition number than that of MoM which prohibits the application of iterative methods. This property was shown in detail for the case of a 2-D circular cylinder in [15]. Therefore, this system of equations is solved directly requiring \( O(N^3) \) operations that make the SMT impractical for problems with large \( N \), such as many-scatterer problems.

In this communication, we accelerate the SMT for many-scatterer problems by incorporating a fast multipole formulation. Our method, the fast multipole SMT (FMSMT), retains the advantages of SMT while providing a more efficient representation of the impedance matrix. As in the FMM, far fields that are similar enough are grouped together and approximated, while near fields are computed directly. This minimizes the redundancies inherent to SMT formulations of many-scatterer problems. We establish the FMSMT as an attractive tool for solving many-scatterer problems by presenting a collection of such problems in which the FMSMT considerably improves upon the SMT. Although this communication presents results for some selected examples, the method is applicable in settings where an SMT formulation is applicable.

An FMM formulation was incorporated in a method of auxiliary sources (SMT-like, see [16]) solution in [17], where run time was compared with that of iterative method of auxiliary sources, showing an improvement in the run time of each iteration. However, the number of iterations needed for convergence was not addressed. Since we are striving to improve upon the much faster direct SMT, we are interested not in the time per iteration but in the total run time of our FMSMT in comparison with the direct SMT. In [18], an FMM-type formulation was applied to the 2-D method of fundamental solutions (SMT-like, see [19], [20]) for acoustic problems, where an improvement was achieved when using a number of sources which is sufficiently large, but not minimal. We note that our method achieves improvement over SMT already for the minimal number of sources required to satisfy the boundary conditions, and even more so when more sources than necessary are used. Finally, we noted that all previous related work was set in two dimensions and, to the best of our knowledge, this communication is the first to apply an FMM-type formulation to SMT in three dimensions.

The remainder of the communication is organized as follows. In Section II, we provide background on the SMT and the FMM. Section III presents the mathematical formulation of our method, and in Section IV numerical results for several scattering problems are given. Finally, we present the conclusions and discuss the future work in Section V.

II. BACKGROUND

The scattering problems considered in this communication consist solely of PEC bodies. This restriction allows a clearer presentation of FMSMT; however, it is not a substantial limitation and the method can be readily applied to penetrable scatterers with minor changes. An application of SMT to penetrable bodies can be found in [14], [21]. Throughout this communication, a time-harmonic variation \( \exp(j\omega t) \) is assumed and suppressed.

In an SMT treatment of a PEC scattering problem, each scatterer is replaced by a group of discrete sources of unknown amplitude located on a mathematical surface \( S' \) enclosed in the original boundary \( S \). While a variety of source types have been used in different SMT formulations, in this communication each source is comprised of...