Communication

A Source-Model Technique for Analysis of Waveguiding Across an Array of Arbitrary Smooth Cylinders Partially Buried in a Penetrable Substrate

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Abstract—A computational technique for modal analysis of an array of penetrable cylinders with smooth arbitrary cross section is described. The cylinders are partially buried between two penetrable half-space media, as may occur in fabricated devices. Our suggested method is a rigorous full-wave frequency-domain source-model technique. The corner-like intersections of the cylinders with the substrate and superstrate are addressed with particular care, by intricately locating properly modulated fictitious sources, so that the rapid spatial variations of the fields can be effectively modeled. The spurious-free modal analysis scheme is sped up with an efficient serial mode-tracking scheme, which is based on the physical perturbation theory. The respective software tool is robust to the choice of materials, geometric parameters, and wavelength. Sample results are presented for circular and triangle-like cylinders. We demonstrate a red shift of the cut-off frequency of these potential waveguides as they are more deeply buried in a high refractive index medium, yet in general, the eigenmodes change rapidly but not necessarily monotonously as a function of the burial depth.

Index Terms—Buried structures, computational electromagnetics, frequency-domain analysis, modal analysis, optical waveguides, periodic structures, source-model technique (SMT).

I. INTRODUCTION

The development of nanofabrication techniques in the last decades has enabled the fabrication of nanometer-scale arrays of elongated metallic particles, e.g., rods or wires, which can serve as waveguides of plasmonic-type waves across the direction of periodicity [1]. Nearby materials have an important effect on the EM response and waveguiding characteristics of particle arrays [2]–[8]. Partial burial of the particles into the supporting substrate, whether as a side effect of the fabrication process [9] or as an intentional feature for improved robustness or flexibility [10], [11], should also affect the waveguiding properties. In scattering analyses, for example, a red shift of the main resonances of the absorption and reflection spectra, and of the inferred dispersion curves, is observed as the arrays approach and gradually become buried into a substrate with higher refractive index [11]–[15].

Methods for modal analysis of nanoparticle arrays include the popular coupled dipole (or filament) approximation method [16], [17], which is applicable for particles that are much smaller than the wavelength and the periodicity, yet cannot account for arbitrary-sized particles that are partially buried; multidisciplinary approaches based on methods in quantum mechanics [18], [19], which can account for burial in a multilayered substrate, yet assume a complex-frequency time-harmonic variation of the fields; and full-wave differential methods, such as finite-difference time-domain [6] and finite-element method [13]–[15], which are very general, yet are presumed less apt than integral-equation (IE)-based methods for tailoring efficient multiscale reliable modal solvers of lossy open waveguides. IE methods for analysis of periodic structures, e.g., arrays of 3-D particles [20], [21], are somewhat laborious, and benefit from integrating an additional acceleration method [21], [22]. Off-surface IE methods offer a more speedy analysis of periodic structures. The generalized-multipole technique [23] is applied for modal analysis of particles with circular cross section in free space [24]–[26]. The source-model technique (SMT) [27] offers modal analysis schemes for arrays of arbitrary cylinders with smooth cross sections in free space [28], or in a triple layered geometry [29]. References [28]–[30] discuss the principles and the benefits of the SMT for modal analysis of periodic structures like those discussed here. We present an SMT-based method for modal analysis of partially buried arrays of cylinders.

The partially buried cylinders are smooth themselves, but the “edges” of the superstrate and substrate at their intersection with the cylinders are “sharp” from a computational point of view, and some components of the fields diverge there [31], [32]. Unsurprisingly, most IE methods suffer from singularities of the integrand when applied to partially buried particles in stratified media, and only an intricate singularity subtraction can assist [33]–[36]. In a recently published SMT-based analysis of scattering by an array of partially buried cylinders, we suggested a built-in numerical remedy for the divergence of the field representations near the sharp intersection of the cylinders, substrate, and superstrate [30]. We reported an order-of-magnitude improvement in the accuracy of the field representations in the scattering analysis. That remedy is reapplied here in the context of modal analysis. For increased efficiency of the modal analysis, a novel serial eigenmode detection scheme is suggested: first, large computational efforts are invested in detecting a mode when no a priori knowledge regarding the expected modal content is available, and then, for small perturbation-like variations of the external parameters, the evolution of each mode is tracked.

II. SMT SCHEME FOR MODAL ANALYSIS

We consider the following geometry: an infinite linear array of 2-D cylinders of arbitrary smooth cross section. The cylinders are oriented parallel to the z-direction, as shown in Fig. 1, and characterized by permittivity \( \varepsilon_c \). The linear chain is periodic in the x-direction and the period is denoted by \( L \). The cylinders are anchored to a penetrable...