On the use of rational-function fitting methods for the solution of 2D Laplace boundary-value problems

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ABSTRACT

A computational scheme for solving 2D Laplace boundary-value problems using rational functions as the basis functions is described. The scheme belongs to the class of desingularized methods, for which the location of singularities and testing points is a major issue that is addressed by the proposed scheme, in the context of the 2D Laplace equation. Well-established rational-function fitting techniques are used to set the poles, while residues are determined by enforcing the boundary conditions in the least-squares sense at the nodes of rational Gauss–Chebyshev quadrature rules. Numerical results show that errors approaching the machine epsilon can be obtained for sharp and almost sharp corners, nearly-touching boundaries, and almost-singular boundary data. We show various examples of these cases in which the method yields compact solutions, requiring fewer basis functions than the Nyström method, for the same accuracy. A scheme for solving fairly large-scale problems is also presented.

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1. Introduction

Laplace boundary-value problems (BVPs) are prevalent in the applied sciences, and methods for their solution have been the subject of study for decades. Among these, prominent approaches are finite-difference and finite-element methods, integral-equation methods, and, in 2D, conformal-mapping techniques. While the Laplace equation is the simplest elliptic partial differential equation (PDE), improving methods for its solution is nevertheless of continued interest. At the focus of recent research are problems with small radii of curvature, nearly-touching boundaries, and singular, or almost-singular boundary data [1–6].

The technique described in this paper belongs to a class of methods known as desingularized methods, which are used for the solution of linear BVPs. What characterizes these methods is that the basis functions are elementary solutions of the homogenous PDE, and hence the singularities of the basis functions lie outside the BVP domain. The simplest example of a desingularized method is the method of images, which is only applicable in a handful of special cases for which the exact locations and amplitudes of the image sources are known analytically. For more general problems, an approximate solution can be obtained by setting the locations of the sources heuristically and solving for the amplitudes so that the boundary conditions are satisfied at a set of testing points. Various types of image sources and heuristics have been proposed over the years. Vekua [7] and Kupradze [8] are usually credited as the pioneers of these methods, and other early works include those of Fox et al. [9], Bergman [10], and Eisenstat [11]. Nowadays, the methods in this category go by various names, such as

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