A Numerical Methodology for Efficient Evaluation of 2D Sommerfeld Integrals in the Dielectric Half-Space Problem

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Abstract—The analysis of 2D scattering in the presence of a dielectric half-space by integral-equation formulations involves repeated evaluation of Sommerfeld integrals. Deformation of the contour to the steepest-descent path results in a well-behaved integrand, that can be readily integrated. A well-known drawback of this method is that an analytical expression for the path is available only for evaluation of the reflected fields, but not for the evaluation of the transmitted fields. A simple scheme for numerical determination of the steepest-descent path, valid for both cases, is presented. The computational cost of the numerical determination is comparable to that of evaluating the analytical expression for the steepest-descent path for reflected fields. When necessary, contributions from branch-cut integrals and a second saddle point are taken into account. Certain ranges of the input parameters, which result in integrands that vary rapidly in the neighborhood of the saddle point, require special treatment. Alternative paths and specialized Gaussian quadrature rules for these cases are also proposed. An implementation of the proposed Numerically Determined Steepest-Descent Path (ND-SDP) method is freely available for download.

Index Terms—Sommerfeld Integrals, Green functions, Moment methods, Integral equations, Nonhomogeneous media.

I. INTRODUCTION

The determination of the fields of an elementary source radiating in plane-stratified media is a canonical problem in electromagnetics. Even though some variants of this problem have been the subject of research since the beginning of the 20th century [1], [2], they are still of interest today. Comprehensive references are [3]–[5], and reviews of computational aspects can be found in [6]–[8]. Nowadays, interest is largely motivated by integral-equation formulations for scattering and propagation problems, as they entail repeated evaluation of the fields of elementary sources that constitute the Green’s function. The starting point for the evaluation of the various Green’s functions is an integral representation of the fields (or potentials), of the Sommerfeld Integral (SI) type. Although the literature on SI evaluation is vast and the procedures are varied, most methods include some or all of the following steps:

• Contour Deformation: The integration contour is deformed from the real axis to a contour on the complex plane. The purpose of this step is to obtain a more well-behaved integrand by avoiding pole and branch-point singularities and possibly also minimizing phase variation along the path. Some possible paths are given in [9]–[11].
• Singularity Subtraction: Singular terms of the integrand are subtracted and then added back after analytical integration. This step has been used together with contour deformation [12], [13], or as an alternative to it [14].
• Numerical Integration: The value of the integral is estimated from a finite number of samples of the integrand. When this is done by a quadrature rule, the estimate is a linear combination of the samples of the integrand. In a more sophisticated scheme, the integrand (or some part of it) is approximated by a superposition of complex exponentials and this approximation is then integrated analytically [15]. This so-called Discrete Complex Image Method (DCIM), which has found widespread use [16], [17], is closely related to the continuous complex image method [18]. In a similar technique [19], the integrand is approximated by a superposition of rational functions, and the resulting approximation is then integrated analytically.

Among the possible integration contours, the Steepest-Descent Path (SDP) passing through a saddle point is considered, in some respects, the optimal choice [20].

Another aspect of using the SDP is that if, in the process of deforming the original path to the SDP, a branch point is intercepted, a path surrounding the intercepted branch point must be added. Although this entails some book-keeping, it also highlights an appealing feature of the method, namely, that the integral is obtained as a sum of distinct, physically meaningful, contributions. From a computational point of view, as simple quadrature is used, this method can potentially outperform the popular DCIM which involves finding the complex images by more computationally intensive methods such as Prony’s method [12], or the Matrix Pencil Method [21]. Moreover, evaluating the Green’s function along the SDP is essential for the Fast Inhomogeneous Plane Wave Algorithm [22] which can be used to solve electromagnetically large layered-media problems in $O(N \log N)$ computational complexity.

When the observation point and the source point are in the same medium, an analytical expression for the SDP is available, and it has been used extensively [6], [23], [24]. In contrast, when the source and observation points are not in the same medium, an analytical expression for the SDP is not available. One option, in this latter case, is to determine the path numerically, but this was deemed too computationally expensive, or otherwise impractical [23], [25].