A Numerical Methodology for Efficient Evaluation of 2D Sommerfeld Integrals in the Dielectric Half-Space Problem

Amit Hochman *Member, IEEE*, and Yehuda Leviatan, *Fellow, IEEE*

***Abstract*— The analysis of 2D scattering in the presence of a dielectric half-space by integral-equation formulations involves repeated evaluation of Sommerfeld integrals. Deformation of the contour to the steepest-descent path results in a well-behaved integrand, that can be readily integrated. A well-known drawback of this method is that an analytical expression for the path is available only for evaluation of the reflected fields, but not for the evaluation of the transmitted fields. A simple scheme for numerical determination of the steepest-descent path, valid for both cases, is presented. The computational cost of the numerical determination is comparable to that of evaluating the analytical expression for the steepest-descent path for reflected fields. When necessary, contributions from branch-cut integrals and a second saddle point are taken into account. Certain ranges of the input parameters, which result in integrands that vary rapidly in the neighborhood of the saddle point, require special treatment. Alternative paths and specialized Gaussian quadrature rules for these cases are also proposed. An implementation of the proposed Numerically Determined Steepest-Descent Path (ND-SDP) method is freely available for download.**

***Index Terms*— Sommerfeld Integrals, Green functions, Moment methods, Integral equations, Nonhomogeneous media.**

singularities and possibly also minimizing phase variation along the path. Some possible paths are given in [9]–[11].

- *Singularity Subtraction:* Singular terms of the integrand are subtracted and then added back after analytical integration. This step has been used together with contour deformation [12], [13], or as an alternative to it [14].
- *Numerical Integration:* The value of the integral is estimated from a finite number of samples of the integrand. When this is done by a quadrature rule, the estimate is a linear combination of the samples of the integrand. In a more sophisticated scheme, the integrand (or some part of it) is approximated by a superposition of complex exponentials and this approximation is then integrated analytically [15]. This so-called Discrete Complex Image Method (DCIM), which has found widespread use [16], [17], is closely related to the continuous complex image method [18]. In a similar technique [19], the integrand is approximated by a superposition of rational functions, and the resulting approximation is then integrated analytically.