Localization in frequency for periodically kicked light propagation in a dispersive single-mode fiber

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We show a special effect of localization in the temporal frequency domain of light pulses that propagate in a dispersive single-mode fiber in the presence of a time-periodic phase modulation that is repeatedly applied at equally spaced locations along the fiber. The effect is analogous to the dynamical localization that occurs for the quantum kicked rotor, which is similar to Anderson localization in disordered solids. The wave behavior eliminates the diffusive spread of sidebands (harmonics). The light propagation, which is described by a Schrödinger-like propagation equation, can provide a new testing ground for the investigation of localization besides shedding light on technologically important pulse propagation in fibers and mode-locked lasers.

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In this Letter we consider propagation in a dispersive optical fiber of broad light pulses that are repeatedly kicked by a sinusoidal rf phase modulation at equally spaced locations along the fiber. The naïve expectation concerning the evolution of the spectrum and the buildup of sidebands (harmonics) is that their number diffusively increases with the number of kicks, so that the spectrum continuously broadens with propagation. It is shown here that this expectation is not fulfilled; instead a special mechanism of localization causes the spectrum to be confined. This mechanism is similar to localization in the phase space of a quantum kicked rotor,[1] which is related to Anderson localization for electrons in one-dimensional disordered solids.[2,3] The localization commonly occurs after a few kicks and has an exponential signature, as can be seen from Fig. 1.

The kicked rotor is an important prototype in the field of quantum chaos[4,5] for the study of the role of quantum mechanics in systems that are chaotic in the classical limit. The classical dynamics of chaotic systems resembles random motion, although it is generated by deterministic equations, and if the phase space is unbounded, its randomlike motion leads to diffusion in this space. In the quantum mechanics case, however, the diffusion can be suppressed by quantum interference, leading to dynamical localization.[1] This localization is actually a wave phenomenon, and it is expected to take place also for classical waves, such as light waves.[6,7] However, there was, so far, only one type of experimental verification for localization in the quantum kicked rotor, with laser-cooled Na and Cs atoms in a magneto-optic trap.[8,9] Recently, we added a first realization of such a system in optics with free-space-propagating light that passes a series of phase gratings.[10]

Our optical system can provide a new testing ground for the investigation of localization. In addition, it illuminates important new aspects of the technologically important modulated pulse behavior in fibers, lasers, and other dispersive media.

The equation for the electric-field amplitude, ψ, of a pulse that is propagating in dispersive single-mode fibers, in the slowly varying amplitude approximation, satisfies the following Schrödinger-like equation,[11] with a potential that results from the periodic modulation (kicks):

$$i\,\frac{\partial\psi}{\partial z}=\frac{\beta_2}{2}\,\frac{\partial^2\psi}{\partial T^2}+\kappa\,\cos(\Omega T)\sum_N\delta(z-Nz_0)\psi\,. \quad (1)$$

The propagation is along z, $T=t-z/v_g=t-\beta_1 z$ is the internal pulse-time variable (relative to the center of the pulse), where $v_g=1/\beta_1$ is the group velocity, and β_2 is the group-velocity dispersion (responsible for pulse broadening), which serves here as the Planck constant in the quantum case. The potential results from perturbation by the time-dependent refractive-index change δn, or the phase modulation, which appears in the wave equation with an amplitude $\kappa=\omega\Delta n l_m/c$, where l_m is the modulator length, ω is the light frequency, and c is the speed of light. The distance between successive kicks is z_0, and κ and Ω are the strength and the frequency of the modulation, respectively. The phases of the modulators (kicks) have to be synchronized. We do not include absorption in Eq. (1), as it can be compensated for, if needed, by amplifiers, as described below. We also ignore in the present analysis nonlinearities, such as the Kerr effect. This is a reasonable assumption for experiments in single-mode fibers with low light intensities.

As discussed above, the properties of the kicked fiber system are similar to those of the quantum rotor.[1] The Hamiltonian-like operator is $\mathcal{H}=-(1/2)\gamma\hat{n}^2+V(T)$, where $V(T)=\kappa\cos(\Omega T)\sum_m\delta(z-Nz_0)$, $\gamma=\beta_2 z_0\Omega^2$, and $\hat{n}=-i\partial/\partial(\Omega T)$. The direction of propagation z corresponds to the time of the kicked rotor, time ΩT corresponds to the angle, and the sideband n corresponds to angular momentum. Note that the time variable ΩT is unbounded, whereas the angle variable

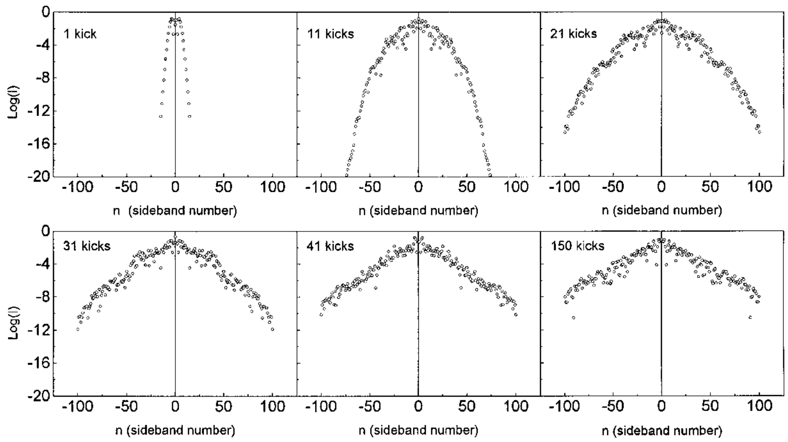

Fig. 1. Evolution of the frequency spectrum according to Eq. (1) (the sideband intensity I is on a 10-based logarithmic scale versus the number of sidebands, n), showing the convergence to the exponential profile as localization occurs. $\beta_2 = 20\ \mathrm{ps}^2/\mathrm{km}$, $\kappa = 5$, $z_0 = 40$ km, and $\Omega/2\pi = 8$ GHz.

of the rotor is confined to the interval $(0, 2\pi)$. Therefore the frequency, unlike the angular momentum, is not quantized, but the transitions that are due to $V(T)$ are only between frequencies that differ by integer multiples of Ω. The properties of the system are governed by the properties of the one-period evolution operator, $\hat{U} = \exp(i\gamma\hat{n}^2/2)\exp[-i\kappa\cos(\Omega T)]$, that transforms ψ after the Nth kick at $z = Nz_0$ to ψ after $z = (N+1)z_0$. $\hat{U}$ is the transfer operator along the propagation direction. The solutions for ψ can be expanded in the quasi-energy states that are the eigenstates of the evolution operator. Here these are the Bloch–Floquet states in the z direction. The nature of these states determines the dynamics. For rational γ/π the states are extended, whereas if γ/π is irrational the states are exponentially localized.[1] In the localized case the envelope of the wave functions behaves as $\exp(-|n|/\xi)$, where ξ is the localization length. The analysis shows that in the diffusive regime the spectrum width follows a $\sigma_N \approx \kappa(N/2)^{1/2}$ dependence. The transition from diffusion to localization occurs after $\sim\kappa^2/8$ kicks, and the localization width is $\xi \sim \kappa^2/4$ harmonics orders, in specific parameter regimes.[1] Other features of the system, such as resonances and antiresonances,[10] will be discussed in the future.

In Fig. 1 the typical evolution behavior of the frequency spectrum (sideband intensity versus the number of sidebands or harmonics, n) is presented. We start from a narrow (single) frequency that represents a very broad pulse and show the evolution that is due to the propagation and the effect of the kicks, as described in Eq. (1). We can see the convergence to an exponential profile as localization occurs. Figure 2 gives the frequency width (standard deviation of n) as a function of the number of kicks, N, for ordered phase modulation, showing confinement behavior. Also shown is the classical case, with diffusive behavior, in which we eliminate phases and propagate intensities rather than amplitudes.

In the propagation along the fiber of a broad input pulse with a relatively narrow spectral width, the pulse undergoes successive sinusoidal phase modulations. In this process the repeated kicks tend to broaden the number of sidebands (harmonics). Each kick can be analytically described by a Bessel function distribution, $J_{n-m}(\kappa)$, for the coupling of the amplitudes of the diffraction orders n and m. Nevertheless, the propagation between kicks adds extra phases, $\exp(i\gamma n^2/2)$, depending quadratically on the sideband order n, and for large n this factor behaves as a random number.[1] Therefore, a specific nth-order spectral component after the Nth kick is composed of many successive former modulations (kicks) plus propagation in the fiber between them. Each path accumulates a series of random phases. It turns out that the overall contribution is weakened, resulting in exponential localization.[1–4] Consequently, low-n sidebands are mostly composed of former low-order harmonics that add constructively. It is crucial that the propagator $\hat{U}$ is identical for all kicks. Although $\exp(i\gamma n^2/2)$ behave as random numbers, they are identical for all intervals of free motion. If for some reason $\hat{U}$ depends on the position z along the fiber (for example, if the kicks are not exactly spaced), this imperfection is equivalent to noise in the original Anderson model. This noise leads to destruction of localization on a length scale corresponding to a similar time scale for the kicked rotor, which is usually inversely proportional to the variance of the noise. For longer lengths diffusion takes place. If this noise is sufficiently strong that

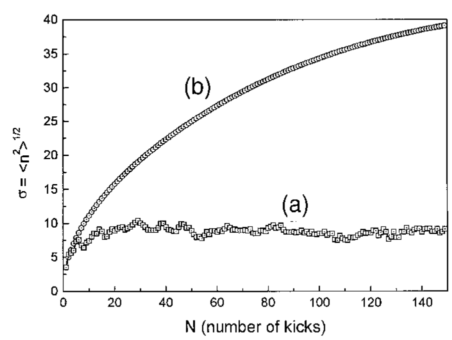

Fig. 2. Route to localization, showing the frequency width according to Eq. (1) (standard deviation of n) as a function of the number of kicks, N, for (a), ordered system modulation and (b), diffusive behavior of the classical case, where we treat intensities rather than amplitudes or where the modulation phase of the kicks is random. $\beta_2 = 20\ \mathrm{ps}^2/\mathrm{km}$, $\kappa = 5$, $z_0 = 40$ km, and $\Omega/2\pi = 8$ GHz.

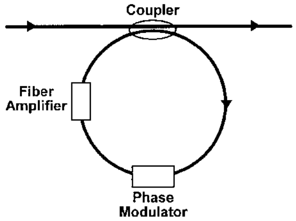

Fig. 3. Schematic of an experimental realization of the fiber kicked system in a loop configuration as repeated modulations and kicks are applied as a pulse circulates in the loop.

this length scale is equal to the length for the onset of localization, classical diffusion is recovered,[9] as demonstrated in Fig. 2.

The outcome of this work from the point of view of fiber optics is interesting. For a signal with a frequency near ω_0 with a bandwidth of $\Delta\omega$, a modulation of the phase with an amplitude κ at a frequency Ω forms sidebands at distances of $n\Omega$, for the nth sideband. If such a modulation is repeatedly applied, one could naïvely expect a steady broadening, namely, an increase of the number of sidebands, and thus an increased overall frequency bandwidth, in a diffusive manner similar to the classical kicked rotor. The present study teaches us that because of the wave behavior and the dispersive propagation between the successive kicks the unbounded bandwidth broadening of the pulse is eliminated.

We now discuss the requirements for an experimental demonstration of localization in a fiber system. It seems that the simplest way to form a periodic propagation with the same kick would be to use a ring structure, as shown in Fig. 3. This structure can be a passive recirculating loop, into which a pulse of light is coupled from outside by a laser source, such as a diode laser with a wavelength of ~1550 nm.

The group-velocity dispersion for a standard single-mode fiber in this regime is $\beta_2 = \sim 20\ \mathrm{ps}^2/\mathrm{km}$. For the quadratic phase factor that results from free propagation between the kicks, γn^2, to generate pseudorandom numbers for the various orders, we take γ of order unity and have a spacing of ~40 km between kicks, for a modulation frequency of 8 GHz. We emphasize the need for synchronization of the modulation of the phases of the kicks. This synchronization can be achieved with continuous modulation when the modulation frequency is adjusted to the loop length such that $\Omega/2\pi = mv_g/l$, where l is the cavity length and m is an integer. This modulation fits the mth-order cavity-resonance frequency. To follow the evolution of the pulse we can tap a single pulse that circulates in the fiber loop with a fiber coupler. Because of the long propagation length in the fiber, it might also be necessary to add an amplifier in the loop. This can easily be done with erbium-doped fibers. Nevertheless, losses or amplifications do not alter any of the localization features if they do not have a significant effect on the spectrum.

An alternative experimental realization can be achieved with an active cavity, in which we have a ring or a linear fiber laser with intracavity phase modulation. With the above phase-synchronization requirement, this is effectively a FM mode-locked laser. Thus this study predicts localization in the long-term spectral structure of the laser and a new constraint on the lower pulse width of such lasers. This special behavior should also occur for AM mode-locked lasers.

In conclusion, we have shown that localization occurs in the temporal frequency domain of light that is periodically kicked as it propagates in a dispersive fiber. We have discussed experiments to verify the effect and stressed the attractiveness of this approach, as it provides a new testing ground for the investigation of localization as well as new insight into light propagation in fibers, pulsed lasers, and other dispersive media.

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