Parametric scattering with constructive and destructive light patterns induced by two mutually incoherent beams in photorefractive crystals

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We demonstrate a new type of scattering process induced by two mutually incoherent beams in photorefractive crystals. The input beams may be of different colors. Dark and bright scattering patterns were obtained by control of an external electric field. The effect results from an angular selectivity of the amplified noise. It involves an enhancement of shared gratings, induced by each input beam with a specific part of its noise, in a forward four-wave-mixing process.

A variety of interesting scattering phenomena have been found in photorefractive crystals. Most of them are a result of the strong amplification capability in wave-mixing processes. In this Letter we demonstrate a new type of parametric scattering, induced by two mutually incoherent beams that may be of different colors. This is an isotropic scattering (compared with the known unisotropic scattering in coherently) in the sense that there is no change in the polarization. The interaction between the waves causes dark or bright rings, depending on an applied electric field on the crystal. The scattering is based on forward four-wave mixing and critically depends on phase-matching conditions.

The experimental setup is shown in Fig. 1. Two mutually incoherent beams of an argon-ion laser (514.5 nm) with equal intensities (16 mW) illuminated the photorefractive medium, a poled BaTiO$_3$ or Sr$_{0.6}$Ba$_{0.4}$Nb$_2$O$_3$ (SBN) crystal. Both beams were extraordinarily polarized. Figure 2 shows the three types of scattering pattern that we have found. The pictures were taken on a screen behind the crystals. Figure 2(a) shows a typical pattern obtained with a BaTiO$_3$ crystal without an external electric field. In this case, the c axis was in the plane that contained the wave vectors of the input waves. We can see a dark cone inside the scattered light (fanning), especially of the right beam. When we applied an external electric field (~2.5 kV/cm) in parallel with the c axis of the SBN crystal, we saw a bright ring [Fig. 2(b)] with the same polarization as the input waves. The ring was induced by both beams. However, the light in the right and left parts of the ring originates from the opposite beams, the left and right, respectively. Therefore, when one beam was blocked (e.g., the right one), the opposite (left) half of the ring was erased immediately, but the right part decayed only after several seconds, in the photorefractive erasure time. In this case we used an SBN crystal with a c axis in the plane that contains the wave vectors of the input waves. The apex angle of the bright ring is slightly higher than that of the dark ring described above, which can be seen if no external electric field is applied. If we rotate the crystal in such a way that the c axis becomes normal to the plane that contains the input beams, a new scattered pattern can be seen [the vertical lines in Fig. 2(c)].

We attribute the observed scattering patterns to amplification or deamplification of noises, which is caused by the mixing of four forward propagating waves. The coupling between the mutually incoherent input beams occurs through shared gratings induced by each input with its own scattered light. The condition for phase matching of the waves causes an increase or decrease of the gain in specific directions. These changes in the amplification profile affect the distribution of the output intensity of internal noises, which originate from imperfections and are amplified by the large gain of the crystal. If we assume that $k_1$ and $k_3$ are the wave vectors of the incident waves, $k_2$ and $k_4$ are the wave vectors of the scattered waves, originating from $k_1$ and $k_3$, respectively, and $k_5$ is the grating wave vector, the Bragg condition is described by one of the following equations:

$$k_1 - k_4 = k_3 - k_2 = k_{g1},$$

$$k_4 - k_1 = k_3 - k_2 = k_{g2}.$$  \hspace{1cm} (1)

We start with the first type of grating $k_{g1}$. If we assume that the two input waves have the same color, the Bragg condition is fulfilled by the grating of Fig. 1(b) ($k_g$ is on a circle perpendicular to the input-beam plane), whose projection onto the screen is the vertical lines in Fig. 2(c). Then both beams, each with its own scattering shown in the two patterns of Fig. 1, have the same gratings with a mutual enhancement. The coupled-wave equations in this case describe amplification of noise in the same direction (beams 2 and 4) for both input beams (1 and 3, respectively), with shared induced gratings:
Note the difference of these equations compared with the former case [Eqs. (2)]. The opposite sign of the coupling reflects the opposite amplification direction (i.e., one of the scattered waves is deamplified). Moreover the reversed direction of the scattered waves dictates the complex conjugation of the grating terms. By using the boundary conditions $A_2(z = 0) = A_2(0)$ and $A_4(z = 0) = A_4(0)$, the solution of the equations is given for steady-state conditions in the nondepleted pump approximation ($I_1$ and $I_3$ are much stronger than $I_2$ and $I_4$). For simplicity we did not include a phase mismatch in the equations. It is more significant when the coupling constant is complex as in the next case [Eqs. (4)]. The boundary conditions are $A_2(z = 0) = A_2(0)$ and $A_4(z = 0) = A_4(0)$, and $A_1$ and $A_3$ are constants. Then

$$A_2(z = l) = \frac{1}{I_0} [A_2(0)[I_1 + I_3 \exp(\gamma l)]] + A_4(0)A_3A_1^*[\exp(\gamma l) - 1],$$

where $l$ is the interaction length. From this solution we can conclude that when the Bragg condition is fulfilled, the effective coupling constant for the scattered noise $A_2$ and $A_4$ equals $\gamma$. One can show that outside the Bragg zone the effective gain for beams 2 and 4 is reduced to $\gamma I_3/I_0$ and $\gamma I_1/I_0$, respectively. The vertical lines in Fig. 2(c) are due to this phenomenon.

We now consider the second type of grating $k_{g2}$. By assuming that the two input waves have the same color, the Bragg condition is fulfilled in a cone that contains the wave vectors of the two incident waves as shown in Fig. 1. The observed pattern on the screen of the scattered light is shown in Fig. 2(b). If we permit a small phase mismatch $\Delta k$ such that $k_g = k_1 - k_4$ and $k_g = k_2 - k_3 + \Delta k$, the coupling equations become:

$$\frac{dA_2}{dz} = -\frac{\gamma}{I_0} [A_2A_3^* + A_4A_1^* \exp(-i\Delta kz)]A_3,$$

$$\frac{dA_4}{dz} = \frac{\gamma^*}{I_0} [A_2A_3^* \exp(i\Delta kz) + A_4A_1^*]A_1.$$

Fig. 2. (a) Scattering pattern for BaTiO$_3$: the angles between the input beams and the $c$ axis were 40° and 70° (out of the crystal). (b) Scattering pattern for SBN with an external electric field; the $c$ axis is in the plane that contains the wave vector of the input waves, and the angles between the input beams and the $c$ axis were 84° and 96° out of the crystal. (c) Scattering pattern for SBN, where the $c$ axis is normal to the plane of the input beams. To improve the picture quality, dark absorbing disks were put on the location of the two beam spots on the screen.
intensities:
coupling constant
mismatch

Fig. 3. Theoretical results of the gain versus the phase mismatch. (a) The scattered wave $I_4(z = l)$ for a real coupling constant $\gamma l = 6$ and various ratios of the pump intensities: $I_3/I_1 = 20$ (or 1/20), 10 (or 1/10), 5 (or 1/5), and 1 (stronger gain depletion corresponds to ratios closer to 1). (b) The scattered waves $I_4(z = l)$ (dotted curve) and $I_0(z = l)$ (solid curve) for a complex coupling constant $\gamma l = 2 + 6i$.

solution is

$$A_2(x = l) = \exp(-i\Delta k l/2)\exp(\gamma pl/2) \times [2D_2 \sinh(sl/2) + A_2(0)\exp(-sl/2)],$$

$$A_4^*(x = l) = \exp(i\Delta k l/2)\exp(\gamma pl/2) \times [2D_4 \sinh(sl/2) + A_4(0)\exp(sl/2)],$$

where $p = (I_1 - I_2)/(I_1 + I_2)$, $s = (\gamma^2 p^2 - \Delta k^2 - 2i\Delta k \gamma)^{1/2}$, $D_2 = [2A_2(0)(i\Delta k - \gamma + s) - \gamma A_1 A_3 A_4(0)]/s$, $D_4 = (D_3/2)I_0(i\Delta k - \gamma - s)/\gamma A_1 A_2$, and $l$ is the interaction length. We note that this problem is formally equivalent to forward four-wave mixing and has an exact solution with no phase mismatch. Figure 3(a) shows the gain of the two scattered waves versus the phase mismatch $\Delta k l$ for various ratios of pumps intensities. In this figure the coupling constant was taken to be real (valid for $\text{BaTiO}_3$ without an external electric field). We can see that when the Bragg condition is fulfilled and the two waves have similar intensities there is a large drop in the gain. This is a result of opposite phase and cancellation of the two gratings induced by the two input beams with their scattered light. This is the case in Fig. 2(a). When the input intensities of the pump waves become significantly different, the feedback decreases and hence the dark ring disappears. Similar behavior was seen in the experiments. When an electric field is applied to the crystal, the coupling constant becomes complex. Figure 3(b) shows the gain of the two scattered waves as a function of the phase mismatch $\Delta k l$. We can see that when the imaginary part of the coupling constant is positive, a bright ring appears outside the phase-matching region as shown experimentally in Figs. 2(b) and 2(c). It is interesting to note that ring scattering patterns were observed also in Kerr media. There, however, two mutually coherent beams were used, and also the phase mismatch was ignored.

The observation of the symmetrical minifanning in the $\pm c$-axis directions around the two input beams is another interesting feature. This scattering occurs when electrical field is applied, as seen in Figs. 2(b) and 2(c) [it is absent in Fig. 2(a)]. This effect is a result of gratings with long wavelengths that can be considered as thin (the Raman–Nath zone). Then the grating of each pair (the input and its scattering) can be read by both inputs. This explains the difference with respect to the regular fanning. The spread to both sides and the violation in the gain direction are possible in thin gratings. The need of an external electric field is due to enhancement of the coupling constant for gratings with long wavelengths. The rings can be obtained for two input beams with different colors. We observed this phenomenon experimentally with two different lines of the argon laser. In this case the apex angle of the ring is changed. The angular deflection $\beta$ with respect to the degenerate case can be calculated in a similar way to Ref. 7 with the double-color-pumped oscillator. The result for small wavelength difference is $\beta = \tan(\alpha/2)\beta_0/\lambda$, where $\alpha$ is the angle between the two input beams.

In conclusion, we have demonstrated experimentally and theoretically a new type of scattering of two mutually incoherent beams in a photorefractive crystal. This effect can be used to transfer information between two mutually incoherent laser beams (which may be of different colors) and to control (by an external field) and limit photorefractive noises.

References