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Time Delay Estimation in Continuous Linear Time-Invariant Systems

Julio Tuch, Arie Feuer, and Zalman J. Palmor

***Abstract*—The use of a time delay in modeling LTI systems is quite common. However, attempts to estimate these time delays in continuous systems typically resorted to methods which increase the number of parameters in the system, in contradiction to the use of time delay in the model to begin with. We present here an attempt to estimate the time delays directly. The algorithm we present is supported both by analysis and simulations with very encouraging results.**

I. Introduction

Recursive parameter identification of linear time-invariant systems has been extensively studied. Many algorithms exist covering a wide range of accuracy v.v. simplicity. These algorithms are supported by analytical studies and results and are well understood. Considerably less is known ,however, about the identification of systems with unknown time delays (also referred to as dead time). The difficulty arises from the way the time delay parameter enters the system model.

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J. Tuch is with the Israel Electric Corporation, P.O. Box 10, Haifa 31000, Israel.

A. Feuer is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Technion City, Haifa 32000, Israel.

Z. J. Palmor is with the Department of Mechanical Engineering, Technion—Israel Institute of Technology, Technion City, Haifa 32000, Israel.

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A number of attempts of dealing with systems with time delays are reported in the literature. In the discrete case, the common approach is based on knowledge of an upper bound on time delay value. The system transfer function numerator is expanded by the given upper bound and, accordingly, the number of parameters to be estimated is increased. The drawback of this approach is clear as an increase in computational effort which may be costly at best. An example to this approach is [1]. Another approach for discrete systems is described in [2] where the algorithm increases the time delay by one sample time at each step. This is based on a parallel computation of three error terms corresponding to +1, 0, and −1. The decision then is made by choosing the smallest error.

In the continuous-time models, typically, various approximations have been used, such as Pade of various orders (see, e.g., [3]–[5]), with limited success. In [6], an attempt was made to define an error function which depends on the time delay only. A polynomial match of this function for a collection of preselected time delay values enables the computation of the best parameter, the one which minimizes the given function. Once the time delay is determined, the model becomes linear for the other parameters. This procedure is very slow and requires a large computational effort. In most cases described above, to be able to estimate the time delay (which is one additional parameter) one is required to estimate a number, sometimes large, of parameters. In addition, no analysis was attempted as to the convergence of the proposed algorithms. The main thrust of our paper is the estimation of the time delay. We propose an algorithm to do this for a continuous system. An analysis is presented which proves the convergence of the algorithm, and robustness issues are discussed as well. The proposed algorithm is then expanded to include the case where, in addition to the time delay, one estimates all other parameters. For this, however, we have no analytical results. To verify the analysis and to demonstrate the feasibility of the algorithm, we present some of the simulation results which were carried out.

II. Identification of Time Delay in a Known LTI System

Consider a continuous-time linear time-invariant system with a single input and a single output. The system is given by the equation

$$A(D)y(t) = B(D)u(t-\tau) \tag{2.1}$$

where

$$A(D) = D^n + a_{n-1}D^{n-1} + \cdots + a_0$$

$$B(D) = b_m D^m + b_{m-1}D^{m-1} + \cdots + b_0, \qquad m < n. \tag{2.2}$$

$D = d/dt$ is the differential operator and τ is the time delay (or the dead-time, as it is sometimes referred to) of the system.

Let us assume now that both the coefficients a_i of $A(D)$ and b_i of $B(D)$ are known, and we would like to estimate the time delay τ. In addition, we would like to perform this estimation on line as more and more data are being collected. Namely, we want to develop an adaptive identification algorithm for τ. To do that, let us assume that $B(s)/A(s)$ is stable and minimum phase and apply the Laplace

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transform on both sides of (2.1) to get

$$Y(s) = -\sum_{i=0}^{n-1} \frac{a_i}{s^{n-i}} Y(s) + e^{-s\tau} \sum_{i=0}^{m} \frac{b_i}{s^{n-i}} U(s) \tag{2.3}$$

where $Y(s)$ and $U(s)$ are the transforms of $y(t)$ and $u(t)$, respectively.

Let us denote the estimate of τ by $\hat{\tau}$; then $\hat{y}(t)$, the estimate of $y(t)$, is defined through its Laplace transform by

$$\hat{Y}(s) = -\sum_{i=0}^{n-1} \frac{a_i}{s^{n-i}} Y(s) + e^{-s\hat{\tau}} \sum_{i=0}^{m} \frac{b_i}{s^{n-i}} U(s) \tag{2.4}$$

and the error $e(t)$ through its transform

$$E(s) = Y(s) - \hat{Y}(s) = (e^{-s\tau} - e^{-s\hat{\tau}}) \sum_{i=0}^{m} \frac{b_i}{s^{n-i}} U(s). \tag{2.5}$$

Denote

$$\psi_i(s) = \frac{1}{s^{n-i}} U(s), \qquad i = 0, 1, \cdots, (m+1) \tag{2.6}$$

and define the sensitivity function

$$\Phi(s) = \frac{\partial \hat{Y}(s)}{\partial \hat{\tau}}. \tag{2.7}$$

Then, by (2.4) and (2.6), we get

$$\Phi(s) = -e^{-s\hat{\tau}} \sum_{i=0}^{m} b_i \psi_{i+1}(s)$$

or

$$\phi(t) = -\sum_{i=0}^{m} b_i \psi_{i+1}(t - \hat{\tau}). \tag{2.8}$$

Motivated by the continuous recursive least square algorithm as presented in the literature (see, e.g., [7]), we define the following algorithm for adaptively estimating τ

$$\left.\begin{aligned} \dot{\hat{\tau}} &= \frac{P(t)\phi(t)e(t)}{1+P(t)\phi(t)^2} \\ \dot{P}(t) &= -\frac{P(t)^2\phi(t)^2}{1+P(t)\phi(t)^2} \end{aligned} \quad P(0) > 0 \right\}. \tag{2.9}$$

Note that the $\psi_i(t)$ are generated by passing $u(t)$ through a bank of integrators, and recall that we assumed $n > m$ so ψ_{m+1} can be at most equal to $u(t)$.

The convergence properties of this algorithm can be summarized in the following theorem.

Theorem 2.1: Let $B(s)/A(s)$ be stable and minimum phase and $u(t) \geq \epsilon > 0$ for all $t > 0$. Then, in the algorithm defined by (2.9)

$$\lim_{t \to \infty} \hat{\tau}(t) = \tau. \tag{2.10}$$

Proof: Define the following function

$$v(t) = \frac{1}{2}(\hat{\tau}(t) - \tau)^2. \tag{2.11}$$

Then, using (2.9), we get

$$\dot{v}(t) = \frac{P(t)\phi(t)e(t)}{1 + P(t)\phi(t)^2}(\hat{\tau}(t) + \tau). \tag{2.12}$$

From (2.6), $\psi_i(t)$ are clearly positive and monotonically increasing. Hence,

$$\begin{aligned} &[\psi_i(t-\tau) - \psi_i(t-\hat{\tau})][\hat{\tau} - \tau] \\ &\quad = [\psi_i(t-\tau) - \psi_i(t-\hat{\tau})][(t-\tau) - (t-\hat{\tau})] > 0. \end{aligned} \tag{2.13}$$

Since $B(s)/A(s)$ is minimum phase, $b_i > 0$; hence, from (2.8),

$$\phi(t) < 0 \tag{2.14}$$

and from (2.5) and (2.13)

$$e(t)[\hat{\tau}(t) - \tau] = \sum_{i=0}^{m} b_i[\psi_i(t-\tau) - \psi_i(t-\hat{\tau}(t)][\hat{\tau}(t) - \tau] > 0. \tag{2.15}$$

From (2.9), clearly, $P(t) > 0$ so, combining (2.14) and (2.15) into (2.12), we conclude that

$$\dot{v}(t) < 0 \qquad \forall t.$$

This implies that $v(t)$ is bounded for all t and so is $\hat{\tau}(t)$. It can readily be shown that

$$|\hat{\tau}(t)| \leq \overline{\tau} \tag{2.16}$$

where $\overline{\tau} = \tau + |\hat{\tau}(0) - \tau|$ and $\hat{\tau}(0)$ is the initial value for $\hat{\tau}(t)$. Now, from (2.6),

$$\psi_i(t) = \int_0^t \psi_{i+1}(\sigma)\, d\sigma$$

and since $u(t) \geq \epsilon$

$$\psi_i(t) \geq \frac{\epsilon}{(n-i)!} t^{n-i} \tag{2.17}$$

so

$$\begin{aligned} &[(t-\tau) - (t-\hat{\tau})][\psi_i(t-\tau) - \psi_i(t-\hat{\tau})] \\ &= [(t-\tau) - (t-\hat{\tau})] \int_{t-\hat{\tau}}^{t-\tau} \psi_{i+1}(\sigma)\, d(\sigma) \\ &\geq \frac{\epsilon}{(n-i-1)!}[(t-\tau) - (t-\hat{\tau})] \int_{t-\hat{\tau}}^{t-\tau} \sigma^{n-i-1}\, d\sigma \\ &\geq \frac{\epsilon}{(n-i)!}[(t-\tau) - (t-\hat{\tau})][(t-\tau)^{n-i} - (t-\hat{\tau})^{n-i}] \\ &\geq \frac{\epsilon}{(n-i)!}\Big[(t-\tau)^{n-i-1} + (t-\hat{\tau})^{n-i-2}(t-\hat{\tau}) + \cdots \\ &\quad + (t-\hat{\tau})^{n-i-1}\Big][\hat{\tau} - \tau]^2 \\ &\geq \frac{\epsilon\tau^{n-i-1}}{(n-i)!}[\hat{\tau} - \tau]^2, \qquad \text{for } t > 2\overline{\tau}. \end{aligned} \tag{2.18}$$

Also, from (2.8) and (2.17), we have

$$\begin{aligned} |\phi(t)| &= \sum_{i=0}^{m} b_i \psi_{i+1}(t-\hat{\tau}) \geq \epsilon \sum_{i=0}^{m} \frac{(t-\hat{\tau})^{n-i-1}}{(n-i-1)!} \\ &\geq \epsilon \sum_{i=0}^{m} \frac{\tau^{n-i-1}}{(n-i-1)!}, \qquad \text{for } t > 2\overline{\tau}. \end{aligned} \tag{2.19}$$

Combining (2.12), (2.18), and (2.19), we get

$$\begin{aligned} \dot{v}(t) = \frac{P(t)\phi(t)^2}{1 + P(t)\phi(t)^2} \cdot \frac{e(t)}{\phi(t)}(\hat{\tau} - \tau) &\leq -c[\hat{\tau}(t) - \tau]^2 \\ &\leq -cv(t) \end{aligned} \tag{2.20}$$

where

$$c = \frac{\sum_{i=0}^{m} b_i \frac{\tau^{n-i-1}}{(n-i)!}}{\sum_{i=0}^{m} b_i \frac{\tau^{n-i-1}}{(n-i-1)!}} > 0.$$

Thus,

$$v(t) \leq v(o)e^{-ct}$$

and clearly, $v(t) \to 0$ as $t \to \infty$ so that

$$\lim_{t \to \infty} \hat{\tau}(t) = \tau.$$

□

Remark 2.1: In case $u(t) \leq -\epsilon < 0$ for all t, Theorem 2.1 holds as well (the proof is almost identical).

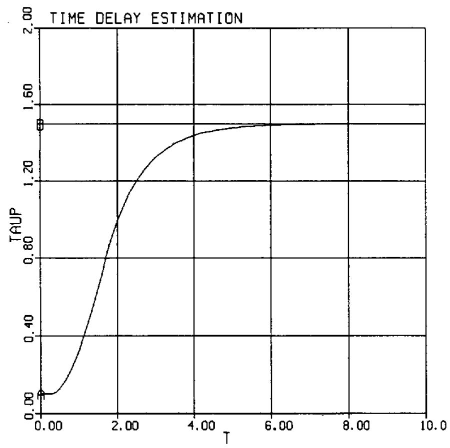


Fig. 1. No modifications with unit step input.

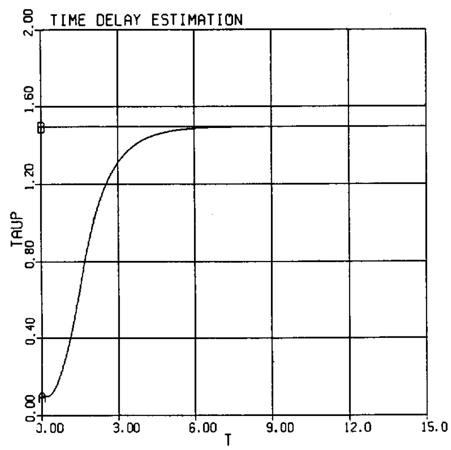


Fig. 2. Square wave input with period 3.

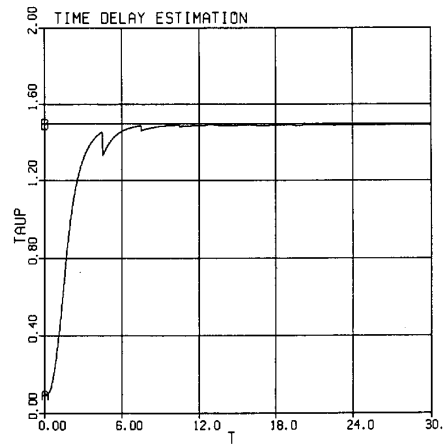


Fig. 3. Step input with integrator resetting.

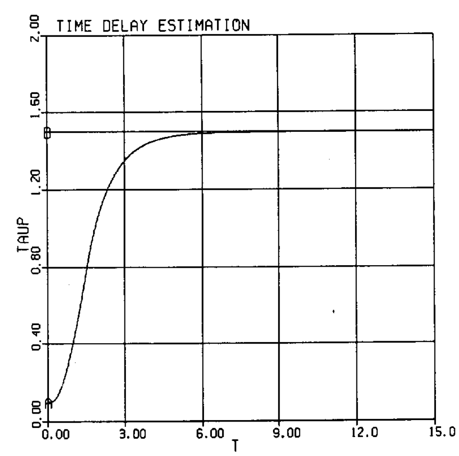


Fig. 4. Step input and first-order delay with time constant= 4 instead of integrator.

Remark 2.2: From the definition of the signals $\psi_i(t)$ and the condition on $u(t)$, clearly, in practical implementations, the algorithm has to be modified so that the $\psi_i(t)$ do not saturate. Several modifications are possible, and all of them were tried successfully in our simulations.

1) In light of Remark 2.1, one could use an input signal which changes signs (e.g., a square wave). As long as the time distance between sign changes is considerably larger than τ and $\hat{\tau}(t)$, convergence will be guaranteed.

2) Periodic resetting of the integrators to zero will also guarantee bounded values for $\psi_i(t)$, and if the resetting period is long enough, again, convergence is guaranteed.

3) Instead of integrators, one could use first-order lags with long time constants. This will also guarantee bounded $\psi_i(t)$ as long as $u(t)$ is bounded.

Remark 2.3: In Theorem 2.1, we required a minimum phase system. Actually, as one can observe from the proof of the theorem, all that is required is that $b_i < 0$.

In our analysis, we have assumed that the system parameters are known. An important question is: How sensitive is the algorithm to this assumption? To discuss this question of robustness, let us consider, for simplicity, a first-order system where

$$\left.\begin{aligned} A(D) &= 1 + aD \\ B(D) &= b. \end{aligned}\right\}. \tag{2.21}$$

Suppose now that the value used in our algorithm is b_1, rather than b. In this case, the error becomes [see (2.15)]

$$e(t) = b\psi(t-\tau) - b_1\psi(t-\hat{\tau}). \tag{2.22}$$

Suppose $u(t)$ is a unit step input, and $\psi(t)$ is a ramp. It is straightforward to show that to optimize the integral of the error squared, the estimated time delay will be

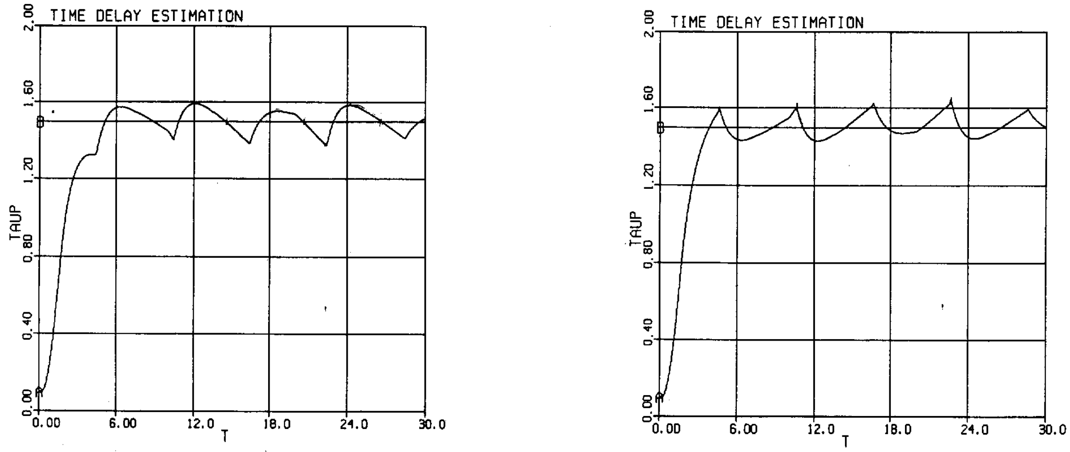
$$\hat{\tau}^*(t) = \frac{b_1 - b}{b_1}t + \frac{b}{b_1}\tau, \qquad t \geq \tau. \tag{2.23}$$

Clearly, no matter how small the difference $b_1 - b$ is, $\hat{\tau}^*(\tau)$ will diverge to infinity. This means that the algorithm is not robust. However, we will show that the modifications suggested in Remark 2.2 do remedy this robustness problem.

Let us consider the first modification, noting that the discussion on the second one will be quite similar with similar conclusions.

With a square wave of period T and "amplitude" 1, we get

$$\psi(t) = \begin{cases} t - kT, & \text{for } k\tau \leq t < kT + T/2 \\ (k+1)T - t, & \text{for } kT + T/2 \leq t < (k+1)T \end{cases}, \quad k = 0, 1, 2, \cdots. \tag{2.24}$$



(a) (b)

Fig. 5. Robustness experiment with square wave input. (a) $b_1 = 1.4$. (b) $b_1 = 1.6$.

Since $\psi(t)$ in (2.24) is a periodic function, let us confine our attention to one period $[kT+\tau,\ (k+1)T+\tau)$ and ask how $\hat{\tau}(t)$ can be chosen to minimize $e(t)$ for every t in this interval. By straightforward observation, we can conclude that the following $\hat{\tau}^*(t)$ will accomplish it

$$\hat{\tau}^*(t) = \begin{cases} \frac{b_1-b}{b_1}(t-kT)+\frac{b}{b_1}\tau, \\ \quad \text{for } kT+\tau \le t < kT+\tau+\frac{b_1}{2b}T \\ t-kT-T/2, \\ \quad \text{for } kT+\tau+\frac{b_1}{2b}T \le t < kT+\tau+\left(1-\frac{b_1}{2b}\right)T \\ \frac{b_1-b}{b_1}[t-(k+1)T]+\frac{b}{b_1}\tau, \\ \quad \text{for } kT+\tau+\left(1-\frac{b_1}{2b}\right)T \le t < (k+1)T. \end{cases} \tag{2.25}$$

Note that for $b_1 > b$, the second interval becomes empty.

The above means that the algorithm will thrive to converge to a periodic function oscillating around τ with amplitude given by $|b_1 - b/b_1|T/2$. Namely, it is linearly dependent on both the error in the gains and the period T. This means that the algorithm, with the above modification, is robust to errors in the parameters.

III. Identification of an LTI System with Time Delay

Identification of a system with a time delay in which all parameters are unknown is a considerably more difficult problem. Modifying the RLS algorithm of the previous section to identify the parameters a_i and b_i as well as the time delay can be done as follows.

Let us denote

$$\theta_0 = [a_0,\ a_1, \cdots, a_{n-1},\ b_0,\ b_1, \cdots, b_m,\ \tau]^T \tag{3.1}$$

$$\left.\begin{aligned} \Psi_i(s) &= -\frac{1}{s^{n-i}}Y(s), \qquad i = 0, 1, \cdots, n-1 \\ \Psi_i(s) &= \frac{1}{s^{n-i}}U(s), \qquad i = n, \cdots, n+m+1 \end{aligned}\right\}. \tag{3.2}$$

Here, since the parameters are all unknown, we define

$$\hat{y}(t) = -\sum_{i=0}^{n-1}\hat{a}_i\psi_i(t) + \sum_{i=0}^{m}\hat{b}_i\psi_{n+i}(t-\hat{\tau}) \tag{3.3}$$

and the regression vector (see [8])

$$\phi(t) = \frac{\partial\hat{y}(t)}{\partial\hat{\theta}} = \Big[\psi_0(t), \cdots, \psi_{n-1}(t),\ \psi_n(t-\hat{\tau}), \cdots, \psi_{n+m}(t-\hat{\tau}), \\ -\sum_{i=0}^{m}\hat{b}_i\psi_{n+i+1}(t-\hat{\tau})\Big]^T \tag{3.4}$$

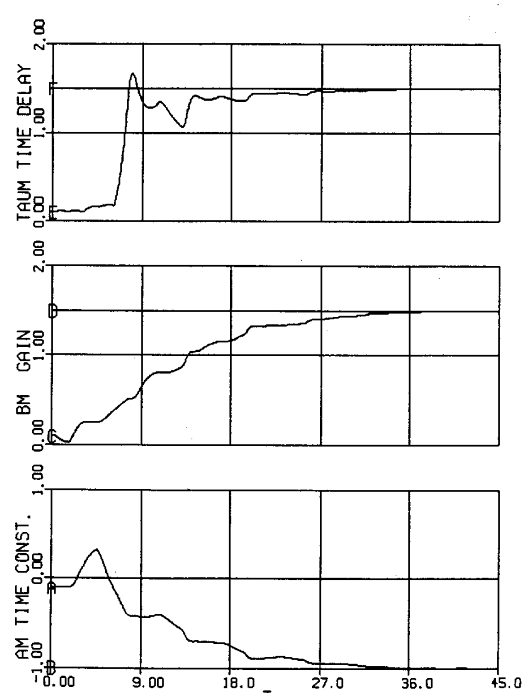


Fig. 6. Identification of all parameters with square wave input.

where $\hat{\theta}$ is the estimate of θ_0, namely,

$$\hat{\theta} = [\hat{a}_0,\ \hat{a}_1, \cdots, \hat{a}_{n-1},\ \hat{b}_0, \cdots, \hat{b}_m,\ \hat{\tau}]^T.$$

The algorithm then is given by

$$\left.\begin{aligned} \dot{\hat{\theta}}(t) &= \frac{P(t)\phi(t)e(t)}{1+\phi(t)^T P(t)\phi(t)} \\ \dot{P}(t) &= -\frac{P(t)\phi(t)\phi(t)^T P(t)}{1+\phi(t)^T P(t)\phi(t)} \end{aligned}\right\} \tag{3.5}$$

where

$$e(t) = y(t) - \hat{y}(t). \tag{3.6}$$

The analysis of this algorithm is very difficult and is beyond the scope of this paper. Extensive simulations conducted show very good results, some of which are presented here.

IV. Simulation Results

Extensive simulations have been conducted, some to verify the analysis of Section II and some to test the algorithm for the problem in Section III. In all our simulations, the following first-order system has been used

$$G(s) = \frac{1.5e^{-1.5s}}{s+1}. \tag{4.1}$$

In Fig. 1, we see the result of using the algorithm with a unit step input and employed as in Theorem 2.1. The convergence is fast and seems to be exponential, as predicted by the proof of Theorem 2.1. In Figs. 2–4, we repeated the experiment, each time with a different modification proposed in Remark 2.2. In each case, the convergence is quite similar to the one in Fig. 1, as predicted in Remark 2.2. To test for robustness, we have tried to use the algorithm without any modification when $b_1 \neq b$, and the algorithm diverged. In Fig. 5, we see the behavior of the algorithm when we use a square wave as the input. Again, the behavior verifies our discussion and (2.25). In Fig. 5(a), we take $b_1 < b$, and in Fig. 5(b), $b_1 > b$, both with similar results.

Finally, we have used the algorithm proposed in (3.5) for the case when a and b are unknown, with a square wave input. The results are very encouraging and are given in Fig. 6. We see that all three parameters converge to the correct values.

V. Conclusion

An algorithm for direct identification of an unknown time delay in an LTI system was presented. It is based on the commonly used RLS algorithm. The convergence of the proposed algorithm for minimal phase and stable systems, where only the time delay is unknown, is analyzed and proven. The robustness of the proposed algorithm to the knowledge of other parameters is also discussed. It is shown that, with an oscillating input such as the square wave or with integrators resetting, the algorithm is robust to inaccuracies in system parameters.

The algorithm is extended to the case where all parameters of the system are unknown. For this, there is no analytical support; the simulations conducted ,however, show very encouraging results.

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Dissipative H_2/H_∞ Controller Synthesis

Wassim M. Haddad, Dennis S. Bernstein, and Y. William Wang

***Abstract*— In certain applications, such as the colocated control of flexible structures, the plant is known to be positive real. Hence, closed-loop stability is unconditionally guaranteed as long as the controller is also positive real. One approach to designing positive real controllers is the LQG-based positive real synthesis technique of Lozano-Leal and Joshi. The contribution of this paper is the extension of this positive real synthesis technique to include an H_∞–norm constraint on closed-loop performance.**

I. Introduction

In certain applications, such as the control of flexible structures, the plant transfer function is known to be positive real. This property arises if the sensor and actuator are colocated and also dual, for example, force actuator and velocity sensor or torque actuator and angular rate sensor. In practice, the prospects for controlling such systems is quite good since, if sensor and actuator dynamics are negligible, stability is unconditionally guaranteed as long as the controller is strictly positive real [1]–[3]. Although there is no general theory yet available for designing positive real controllers, a variety of techniques have been proposed based on H_2 theory [4]–[10] and H_∞ theory [11], [12].

In this paper, we focus on the H_2-based positive real controller synthesis method of Lozano-Leal and Joshi [7]. In [7], it is shown that if the plant is positive real and if the error and disturbance matrices satisfy certain constraints, then the LQG controller is also positive real. This approach is appealing in practice since it requires only standard LQG synthesis techniques. Our goal in this note is to extend the synthesis technique of [7] to include an H_∞-norm bound on the closed-loop transfer function [13]. This extension thus provides the control designer with more flexibility in specifying closed-loop system performance.

II. Preliminaries

In this section, we establish definitions and notation. Let $\mathcal{R}$ and $\mathcal{C}$ denote the real and complex numbers, let $(\ )^T$ and $(\ )^*$ denote transpose and complex conjugate transpose, respectively, and let I_n or I denote the $n \times n$ identity matrix. Furthermore, we write $\|\cdot\|_2$ for the Euclidean norm, $\|\cdot\|_F$ for the Frobenius matrix norm, $\sigma(\cdot)$ for the maximum singular value, tr for the trace operator, and

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W. M. Haddad is with the Department of Mechanical and Aerospace Engineering, Florida Institute of Technology, Melbourne, FL 32901.

D. S. Bernstein and Y. M. Wang are with the Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109.

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