

State Estimation in Hybrid Systems with a Bounded Number of Mode Transitions in the Presence of Spurious Measurements

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Abstract—We consider the problem of tracking the state of a hybrid system capable of performing a bounded number of mode transitions in the presence of spurious, or cluttered measurements. The system is assumed to follow, at each time, one of a predefined dynamical models. Two types of uncertainties make the problem challenging. The first is the data uncertainty that follows from the fact that the true measurement of the state is indistinguishable from the clutter measurements that do not carry useful information. The second problem is the intrinsic model uncertainty. Both reasons prevent the computation of the optimal estimator. On the other hand, the mode transitions are not Markov, thus ruling out the direct use of standard approaches for state estimation in cluttered environment. We derive an efficient estimation scheme for systems in cluttered environments capable of performing a bounded number of mode transitions. At the heart of this scheme is a transformation of the non-Markov model set to an equivalent Markovian one and a subsequent utilization of standard approaches matched to the new mode set. The algorithm’s performance is evaluated via a simulation study, and shown to outperform the standard popular approaches.

I. INTRODUCTION

Hybrid systems are characterized by a continuously varying state vector and a discretely varying (switching) parameter vector, that takes values in some finite set [1]. The switching parameter vector is usually referred to as the system mode. The hybrid system framework is frequently used in multi-sensor fault-prone systems. Typical examples are navigation systems using the signals of global navigation satellite systems (that are prone to jamming and spoofing) and inertial sensors that are, frequently, of low grade. Another typical application of the hybrid system framework is in maneuvering target tracking, where it is assumed that the target can maneuver, at any time, in one of a finite set of maneuvers (modes).

Since many hybrid systems are characterized by stochastic state and/or parameter vector, much work has been devoted to the simultaneous estimation of both state and parameters. As is well known, the mean squared optimal filtering algorithm for hybrid systems requires computation resources that grow exponentially in time [2]. Therefore, a variety of suboptimal estimation techniques was proposed [3]–[7]. In all of these cases, the number of mode switches permitted by the system’s model was assumed to be unbounded, and the switching

parameter vector was assumed to obey a Markovian law.

When the number of mode switches permitted by the system is bounded (assuming some knowledge about this bound), the Markovian assumption no longer holds. Typical problems featuring a bounded number of mode switches are encountered in maneuvering target tracking, particularly during the end-game phase of the interception scenario. Thus, [8] analyzes a stochastic ballistic missile interception scenario where the incoming ballistic missile executes a random bang-bang maneuver consisting of a maximal maneuver in one direction, followed by a randomly-timed single switch to a maximal maneuver in the opposite direction. Further, in [9], optimal differential games-based strategies are derived, showing that, for both adversaries, these are of the bang-bang type, with possibly a single direction change if the target has non-minimum phase dynamics. In [10], two possible target strategies are assumed: a constant maximum maneuver, and a square wave maneuver with a small number of direction changes.

In [11], an efficient algorithm for tracking the state of a hybrid system capable of performing a (hard) bounded number of mode transitions was proposed. The algorithm was shown to outperform the state-of-the-art Interacting Multiple Model (IMM) filter when the latter is applied in a straightforward manner to the problem, at the expense of an increased computational burden in terms of the number of primitive Kalman filters involved. A similar problem was considered in [12], where the the bound on the number of transitions “soft”, that is, random, but finite with probability 1. In both cases, the system under track was assumed to evolve in a clutter-free environment such that the obtained measurements were noisy observations of the state of interest.

In the present paper we consider the problem of estimating the state of a system capable of performing a bounded number of mode transitions in a cluttered environment. Specifically, at each time a random number of measurements are acquired. Among these, at most one measurement is the true observation of the state, and the rest do not carry useful information. However, these are indistinguishable from the true measurement in the sense that all observations are unlabeled and assumed to carry only partial (and noisy) information about

the state and the corresponding time tag. In classical scenarios, when the mode transitions are Markov, a smart combination of the IMM and the Probabilistic Data Association (PDA) [13] techniques, termed IMM-PDA [14] is commonly used. Since in the bounded case (in either of its variants) the mode transitions do not possess Markovian dynamics, a naive utilization of the above technique is not desirable.

In [15] it is shown that both soft- and hard-bounded problems can be tackled by redefining the system's *natural* mode to an equivalent, augmented one possessing the Markov property. The natural mode, that comprises the system matrices, is augmented with the information on the number of transitions the system has performed. The resulting process may be shown to be Markov. This observation is also used in the present work. Instead of the direct, ad-hoc, utilization of the IMM-PDA approach on the natural mode sequence, the mode set is first augmented by accompanying the actual dynamical model with the number of the elapsed mode transitions. The resulting augmented Markov mode sequence is then used in the standard IMM-PDA routine matched to the augmented mode sequence. The computational cost of the resulting algorithm, termed Bounded IMM-PDA (BIMM-PDA), is higher than that of the standard IMM-PDA due to the increased number of the primitive PDA modules involved in the computation, and some possible savings are discussed in the sequel.

The remainder of the paper is organized as follows. In Section II we provide a mathematical formulation of the problem. In Section III we derive the proposed method and its reduced-complexity version. Several numerical examples are given in Section IV. Concluding remarks are made in Section V.

II. PROBLEM FORMULATION

Consider the following state-space representation of a stochastic dynamical system

$$x_{k+1} = A_k x_k + C_k w_k, \quad (1)$$

where $\{w_k\}$ is a zero-mean, unit-variance white Gaussian process noise sequence independent of the initial state x_0 , that is assumed to be Gaussian with mean \bar{x}_0 and covariance P_0 .

The system's state is observed via the following linear measurement equation

$$y_k = H_k x_k + G_k v_k. \quad (2)$$

We assume that the observing sensor has a known detection probability P_D such that (2) is valid only at the time steps when the system is detected. The sequence $\{v_k\}$ is assumed to be white Gaussian with zero-mean and unit-variance such that $\{w_k\}$, $\{v_k\}$, and x_0 are independent.

The system (1)–(2) is specified by four matrix sequences $\{A_k\}$, $\{H_k\}$, $\{C_k\}$, and $\{G_k\}$. At each time k the set $\mathcal{M}_k \triangleq \{A_k, H_k, C_k, G_k\}$ comprises the *natural mode* of the system. Different values of the mode correspond to, for example, different flight regimes (e.g., maneuvering/non-maneuvering) of an aircraft, or different sensor conditions (e.g., nominal/faulty).

We consider the case where at time k the mode \mathcal{M}_k may assume one of r possible values m_1, \dots, m_r . In addition, the evolution of the sequence $\{\mathcal{M}_k\}$ does not occur in a deterministic manner. Suppose that the total number of mode transitions is upper bounded by a known deterministic constant L_{\max} . We define $\{N_k\}$ to be a sequence of random variables assuming the values $0, \dots, L_{\max}$, such that $N_k = \ell$ if ℓ mode transitions have occurred by time k . The considered switching law is given in 3 (appearing at the next page), where $\{p_{ij}, i, j \in \{1, \dots, r\}\}$ are known probabilities and δ_{ij} is Kronecker's delta.

Equations (1)–(3) define a hybrid stochastic system, as the continuous uncertainty associated with the state vector x_k is accompanied by a discretely-varying uncertainty associated with the (discrete) random mode transition law.

In addition to the actual measurement defined in (2), at each time, a number of clutter measurements are obtained. These originate from false (or ghost) targets and do not carry any information about the target of interest. They are, however, indistinguishable from true detections, but are not affected by the sensor's detection probability. At each time, the clutter measurements are assumed to be independent of each other, of the clutter measurements at other times, and of the true state and observation. The prior probability distribution of the number of clutter measurements is known. In addition, these measurements are uniformly distributed in space. The set of measurements obtained at time k (that comprises some number of clutter returns and, possibly, a single true target detection) is denoted by $Y_k \triangleq \{y_k^1, \dots, y_k^{n_k}\}$, where n_k is the total number of measurements acquired at time k .

The goal of the paper is to obtain an efficient algorithm for the estimation of the state x_k using the available data $\mathcal{Y}_k \triangleq \{Y_0, \dots, Y_k\}$. Two reasons make the described problem challenging. The first has to do with the model ambiguity, that stems from the fact that the system mode is random. The second has to do with the data ambiguity due to the noninformative clutter measurements. Both reasons preclude the computation of the minimum mean-squared error (MMSE) filter, because this filter requires unbounded computational resources that grow exponentially in time [15], [16]. Thus, resorting to suboptimal approaches is inevitable. An efficient suboptimal method is developed in the next section.

III. FILTER DERIVATION

In this section we describe the proposed method for state estimation in hybrid systems in cluttered environment with modes evolving according to (3). For cases where $\{\mathcal{M}_k\}$ is a Markov process, the problem may be solved efficiently, in a suboptimal manner, by the IMM-PDA approach [14], which is a combination of the IMM algorithm, originally designed to cope with model uncertainty, with the PDA filter proposed to deal with state estimation in clutter [13]. However, as shown in [15], the natural mode sequence $\{\mathcal{M}_k\}$, with transition dynamics obeying (3), is not Markov, which precludes the direct utilization of IMM-PDA. We overcome this obstacle by defining an augmented system mode, that possesses the

$$\begin{aligned} & \Pr \{ \mathcal{M}_{k+1} = m_j \mid \mathcal{M}_k = m_i, \mathcal{M}_{k-1} = m_{i_1}, \dots, N_k = \ell, N_{k-1} = \ell_1, \dots \} \\ & = \Pr \{ \mathcal{M}_{k+1} = m_j \mid \mathcal{M}_k = m_i, N_k = \ell \} = \begin{cases} p_{ij}, & \ell < L_{\max}, \quad i, j \in \{1, \dots, r\}, \quad k = 0, 1, \dots \\ \delta_{ij}, & \ell = L_{\max} \end{cases} \end{aligned} \quad (3)$$

Markov property, thus revealing a ‘‘hidden Markovian nature’’ in the hybrid system. This permits a straightforward application of a standard IMM-PDA algorithm to the equivalent Markovian mode set. Thus, the exposition of the new method essentially reduces to deriving the mode set transformation. For completeness, nevertheless, we outline the principles of the IMM-PDA filtering algorithm.

A. Background on IMM-PDA

Roughly speaking, the IMM-PDA is obtained from a standard IMM by replacing the bank of (primitive) Kalman filters with PDA modules. The algorithm assumes that the mode sequence $\{\mathcal{M}_k\}$ constitutes a Markov chain on a finite state space $\{m_1, \dots, m_r\}$ with known transition probability matrix (TPM) $P_{r \times r} = (p_{ij})$. It maintains a bank of (primitive) PDA filters, each matched to a different model in the given model set. At step k of the algorithm, the j -th filter produces a local estimate $\hat{x}_j(k)$ with an associated error covariance $P_j(k)$, using its initial estimate $\hat{x}_j^0(k-1)$ and the associated covariance $P_j^0(k-1)$, which are generated externally, and the current measurement set Y_k . This measurement set, that might have been validated by a standard windowing test, gets processed by all PDAs in the bank. In addition, each filter produces a current value of its own (model-matched) likelihood function $\Lambda_j(k)$. Inherited from the standard IMM, the key element of the IMM-PDA scheme is the interaction block that generates, using all local estimates, covariances, and likelihoods from the previous cycle, individual initial conditions for each of the primitive filters in the bank. For completeness, we summarize the steps of the algorithm as they appear in [17]. To avoid cumbersome notation, we drop the time index from the matrices comprising the system mode and denote by $A(j)$, $C(j)$, $H(j)$, and $G(j)$ the realizations of, respectively, A_k , C_k , H_k , and G_k corresponding to the m_j -th value of \mathcal{M}_k .

a) *Mixing Probabilities:* For $i, j = 1, \dots, r$ compute

$$\begin{aligned} \mu_{i|j}(k-1) & \triangleq \Pr \{ \mathcal{M}_{k-1} = m_i \mid \mathcal{M}_k = m_j, \mathcal{Y}_{k-1} \} \\ & = \frac{1}{c_j} p_{ij} \mu_i(k-1), \end{aligned} \quad (4)$$

where c_j is a normalizing constant and $\mu_i(k) \triangleq \Pr \{ \mathcal{M}_k = m_i \mid \mathcal{Y}_k \}$.

b) *Mixing Step:* For $j = 1, \dots, r$ compute the initial state estimate for the filter matched to m_j

$$\hat{x}_j^0(k-1) = \sum_{i=1}^r \hat{x}_i(k-1) \mu_{i|j}(k-1) \quad (5)$$

and the corresponding covariance $P_j^0(k-1)$.

c) *Predicted States and Measurements:* For $j = 1, \dots, r$ compute the mode-matched predicted measurements and innovation covariances

$$\hat{y}_j(k \mid k-1) = H(j)A(j)\hat{x}_j^0(k-1) \quad (6)$$

$$\begin{aligned} S_k^j & = H(j)(A(j)P_j^0(k-1)A^T(j) \\ & \quad + C(j)C^T(j))H^T(j) + G(j)G^T(j). \end{aligned} \quad (7)$$

d) *Measurement Validation:* Set a validation window centered at the predicted measurement of the mode corresponding to the largest mode-conditioned innovation covariance with size determined by the latter. It is possible, however, to introduce modifications. For example, one may consider a single validation window similarly to the single predicted measurement, take a union of the mode-conditioned windows, or propose an adaptive sizing of the window.

e) *Mode-Matched Filtering:* For $j = 1, \dots, r$, using (5) and the corresponding covariance compute the mode-matched estimate $\hat{x}_j(k)$ and $P_j(k)$ as well as the likelihood $\Lambda_j(k)$, which is computed in a standard PDA manner

$$\begin{aligned} \Lambda_j(k) & = p(y_k^1, \dots, y_k^{n_k} \mid \mathcal{Y}_{k-1}) \\ & = (V(k))^{-m} \gamma_0(n_k) \\ & \quad + (V(k))^{-m+1} \sum_{n=1}^{n_k} P_G^{-1} \mathcal{N}(y_k^n; \hat{y}_j(k \mid k-1), S_k^j) \gamma_j(n_k), \end{aligned} \quad (8)$$

where $V(k)$ is the volume of the validation window, n_k is the total number of the validated measurements at time k , P_G is the probability that the true measurement is inside the validation window, $\mathcal{N}(y; \mu, \Sigma)$ is a Gaussian density with mean μ and covariance Σ , evaluated at y , and

$$\gamma_j(n_k) = \begin{cases} \frac{1}{N_k} P_D P_G, & j = 1, \dots, n_k \\ 1 - P_D P_G, & j = 0, \end{cases} \quad (9)$$

with P_D being the detection probability of the sensor.

f) *Mode Probability Update:* For $j = 1, \dots, r$

$$\mu_j(k) = \frac{1}{c} \Lambda_j(k) \sum_{i=1}^r p_{ij} \mu_i(k-1), \quad (10)$$

where c is a normalization factor.

g) *Output Computation:* The algorithm output at time k is obtained as a fused version of the local estimates:

$$\hat{x}(k) = \sum_{j=1}^r \hat{x}_j(k) \mu_j(k). \quad (11)$$

The associated covariance is computed in a similar manner.

Note that the likelihood $\Lambda_j(k)$ incorporates the contribution of the validated measurements according to their distance

from the predicted measurement. This is the “soft validation” window of the PDA algorithm. Namely, unlikely, distant measurements contribute very little due to their small likelihood. Hence, one may, in principle, apply the scheme without pre-validating the measurements, allowing the “soft validation” to effectively discard the unlikely ones.

B. Revealing the System’s Hidden Markovian Nature

The general case algorithm rests on the following lemma, proved in [15], that allows the transformation of the mode sequence to a Markov sequence.

Lemma 1. *The joint sequence $\{\mathcal{M}_k, N_k\}$, with $\{N_k\}$ defined in Section II, is Markov with transition probability matrix having the following entries*

$$\Pr\{\mathcal{M}_{k+1} = m_j, N_{k+1} = \ell_j \mid \mathcal{M}_k = m_i, N_k = \ell_i\} = \begin{cases} \delta_{ij}, & \ell_i = L_{\max} \\ p_{ij}\delta_{ij}, & \ell_i < L_{\max}, i = j \\ p_{ij}\delta_{\ell_j, \ell_i+1}, & \ell_i < L_{\max}, i \neq j. \end{cases} \quad (12)$$

Equipped with Lemma 1, an efficient algorithm for the hard-bounded case may be obtained by defining a *supermode* (instead of the natural mode) sequence $\{\mathcal{M}_k, N_k\}$, which constitutes a Markov process, and invoking a standard IMM/PDA algorithm matched to this supermode sequence equipped with the TPM (12).

Clearly, the number of primitive PDA filters required for the BIMMPDA is equal to the number of different values of the supermode, namely $(L_{\max} + 1)r$. Although this number of primitive PDAs constitutes a significant reduction relative to the optimal filter, which requires a number of PDAs that is exponential in L_{\max} , it still requires a considerable computational effort. Therefore, we present a reduced-complexity version of the algorithm in the next section.

C. Complexity Reduction

Clearly, to reduce the computational load of the algorithm one needs to reduce the number of different values of the supermode $\{\mathcal{M}_k, N_k\}$. We can do this by applying the following additional assumptions to the general problem formulation.

A1: The number of different dynamical models is taken to be $r = 2$. The model m_1 is the nominal one. In target tracking applications it represents the typical, non-maneuvering motion regime of a target. In FDI applications, it may represent the fault-free condition of the sensor. The second possible mode, m_2 , represents the abnormal situation, e.g., a maneuvering target, or a faulty sensor. It is possible to describe various maneuver regimes using, e.g., Singer’s exponentially correlated acceleration (ECA) model [18], [19], thus relaxing the limitation caused by reducing the number of possible dynamical models.

A2: The initial model is known. Without loss of generality, it is taken to be the nominal one. Under assumptions A1 and A2, the number of mode transitions, $\{N_k\}$, constitutes

a Markov process defined over the state space $\{0, \dots, L_{\max}\}$ with the following TPM

$$\begin{pmatrix} p_{11} & p_{12} & 0 & \cdots & 0 \\ 0 & p_{22} & p_{21} & 0 & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & & 0 & 1 \end{pmatrix}. \quad (13)$$

Note that, in this case, N_k uniquely determines the current dynamical model. Hence, the supermode space defined in Lemma 1 contains only $L_{\max} + 1$ possible modes each matched to either the nominal dynamical model m_1 or the abnormal one m_2 .

D. Related Problems

- The standard IMM/PDA procedure may easily be applied to multisensor scenarios by performing additional measurement validation and filtering steps using the measurements of the next sensor after the conditional filtering step described above [17]. Applying the BIMMPDA to multisensor problems is thus also straightforward.
- We note that Lemma 1 remains valid irrespective of the actual dependence of the LHS of (3) on ℓ . Hence, a broader class of problems may be cast into the proposed framework. For example, one may consider the case where the probability to perform a transition from, say, m_i to m_j , decreases as the number of past such transitions increases. This may be done by replacing the probabilities p_{ij} on the RHS of (3) with p_{ij}^ℓ for $i \neq j$, with a corresponding normalization. The only difference would be the corresponding correction to the TPM of (12).
- A related problem of state estimation with a soft bound on the number of mode transitions has recently been considered in [12]. In this case, the number of mode transitions was considered random, but finite with probability 1. The solution approach was based on transforming the mode sequence into a Markovian one and applying a standard IMM matched to the modified mode space. Hence, the soft-bounded case may easily be extended to handle cluttered environments using the same reasoning as used in the hard-bounded case.

IV. NUMERICAL STUDY

In this section we test the performance of the proposed BIMMPDA algorithm and compare it with that of the standard IMM/PDA filter, adapted in an ad-hoc manner to the problem at hand. We first describe the experimental setup, consisting of the models m_1 and m_2 , and the measurement generation model. We then present several examples, differing in their mode evolution mechanisms.

A. Simulation Setup

We consider the state equation (1) describing the dynamics of a maneuvering target, where the state vector $x_k = [p_k \ v_k \ a_k]^T$ comprises the target’s position, velocity, and

acceleration. At time k , the pair $\{A_k, C_k\}$ may take one of two values, $m_1 = \{A^1, C^1\}$ or $m_2 = \{A^2, C^2\}$. In the (nominal) regime m_1 , the system obeys the dynamics of the discrete white noise acceleration (DWNA) model [20], specified by:

$$A^1 = \begin{pmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, C^1 = \begin{pmatrix} T^2/2 \\ T \\ 0 \end{pmatrix} \sigma_1, \quad (14)$$

where T is the sampling period corresponding to a single time step of the system (1), σ_1 is some nominal process noise intensity. In the (abnormal) regime, m_2 , the corresponding model is chosen to be the discrete Wiener process acceleration (DWPA) model [20], specified by the following matrices:

$$A^2 = \begin{pmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}, C^2 = \begin{pmatrix} T^2/2 \\ T \\ 1 \end{pmatrix} \sigma_2, \quad (15)$$

where σ_2 is the abnormal process noise intensity.

The target measurements are generated according to (2) with $P_D = 1$, where, irrespective of the current motion regime, $H_k = [1 \ 0 \ 0]$, and $G_k = \sigma_v$. In addition, at each sampling time, a number of clutter measurements is generated. These are uniformly distributed in the surveillance region. For simplicity, we do not set a validation window, but utilize all the measurements from a given time resting on the ‘‘soft’’ validation of the PDA. Hence, we set $P_G = 1$ as the true detection is always present in the current measurement set.

In all examples below we used the following parameters: $\sigma_1 = 0.3 \text{ m/s}^2$, $\sigma_2 = 2.5 \text{ m/s}^2$, $\sigma_v = 25 \text{ m}$, $T = 5 \text{ s}$. These parameters correspond to a maneuvering index $\lambda = \sigma T^2 / \sigma_v$ of 0.3 for $\sigma = \sigma_1$, and about 2.5 for $\sigma = \sigma_2$, meaning that the problem cannot be solved to a satisfactory level of accuracy using a single, non-adaptive filter (see, e.g., [21]) and the use of adaptive filters of the multiple model variety is inevitable. In the first three examples in the sequel the average number of clutter measurements in an interval of one standard deviation of the measurement noise σ_v is kept constant at ≈ 0.25 . In the final experiment, this number varied between 0.05 and 0.3. In addition, without loss of generality, the initial state is $x_0 = [0 \ 0 \ 0]$ and P_0 is an all-zero matrix, and scenarios of 30 time units are considered. Each filter is initialized in a perfect manner and the initial model is set to be the nominal one.

B. Performance Evaluation

In all cases we compare the performance of the BIMMPDA filter and a standard IMMPPDA (designed for the natural mode sequence) adapted in an ad-hoc (yet reasonable) manner to the problem at hand. In addition, we generate a ‘‘Genie’’ PDA filter that possesses perfect information on the natural mode value at each time. This non-realistic filter serves to provide an overall performance bound. Note, that despite knowing the system mode, the genie PDA is still a suboptimal algorithm, since it implements a standard (suboptimal) PDA routine for target tracking in clutter. Finally, we also present the performance of a naive, non-adaptive PDA, which is matched to the model m_1 with process noise standard deviation which is taken to be

the average of σ_1 and σ_2 . This naive choice aims to make a compromise between the two motion regimes.

All the presented results are averaged over 1000 independent Monte Carlo runs. In the first three examples we compute the Root Mean-Squared (RMS) position and velocity errors for all filters for those MC runs in which the target was not lost. The target is declared lost if, before the end of the simulation interval, the distance between the true and estimated target positions has deviated by more than $5\sigma_v$ for three consecutive times. In the last experiment we record the percentage of the lost tracks vs. clutter density.

1) *Deterministic Case:* First, we consider the following scenario, lasting 30 time units.

- From $k = 1$ to $k = 5$ the target moves according to m_1 ,
- At $k = 6$ the target switches to m_2 and complies with this model for additional 5 time units,
- At $k = 11$, the target switches back to m_1 and persists at this model until the end of the scenario.

In this (deterministic) scenario, mode switches occur after 5 and 10 time units. To comply with the assumed model we set the probabilities for a mode switch in the BIMMPDA to $p = \frac{1}{6}$ as well as $L_{\max} = 2$. The IMMPPDA filter is designed for the natural mode sequence. Since for $L_{\max} > 1$ this mode sequence is not Markov, the transition probability matrix of the IMM needs to be adjusted in some ad hoc manner. According to our modeling, a reasonable choice would be setting the probability for a transition from m_1 to m_2 and vice versa to $p = \frac{1}{6}$, such that

$$\text{TPM}_{\text{IMMPPDA}} = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}. \quad (16)$$

The Monte Carlo RMS position and velocity errors are presented in Fig. 1. It is readily seen that before the second mode transition the IMMPPDA and BIMMPDA attain similar performance. Both are inferior to the genie filter and both outperform the naive PDA. The more interesting phenomenon, however, occurs after the second (and last) switch, where the BIMMPDA clearly dominates the IMMPPDA, approaching the genie filter’s performance. The superiority of the proposed method at the final interval of the scenario, after the maximum number of model switches has been exhausted, is best understood via observing Fig. 2 that shows the mode probabilities computed by the BIMMPDA and IMMPPDA. Notice that after the maximum number of switches, L_{\max} , has been exhausted, the BIMMPDA knows precisely the model being currently in effect. Therefore, it nullifies all probabilities μ_ℓ for $\ell < L_{\max}$ and assigns $\mu_{L_{\max}} = 1$. This means that the scheme’s state estimate is based on the true model, which yields superior performance. The IMMPPDA filter, on the other hand, cannot but assume that the mode transitions obey a Markovian structure, which is not true in our case. Therefore, unless $L_{\max} = 1$, it cannot nullify any of the modal probabilities in order to keep both models ‘alive’ (so as to be prepared for a possible future switch). This introduces an additional mismatched term to the output estimate, thus increasing the estimation error.

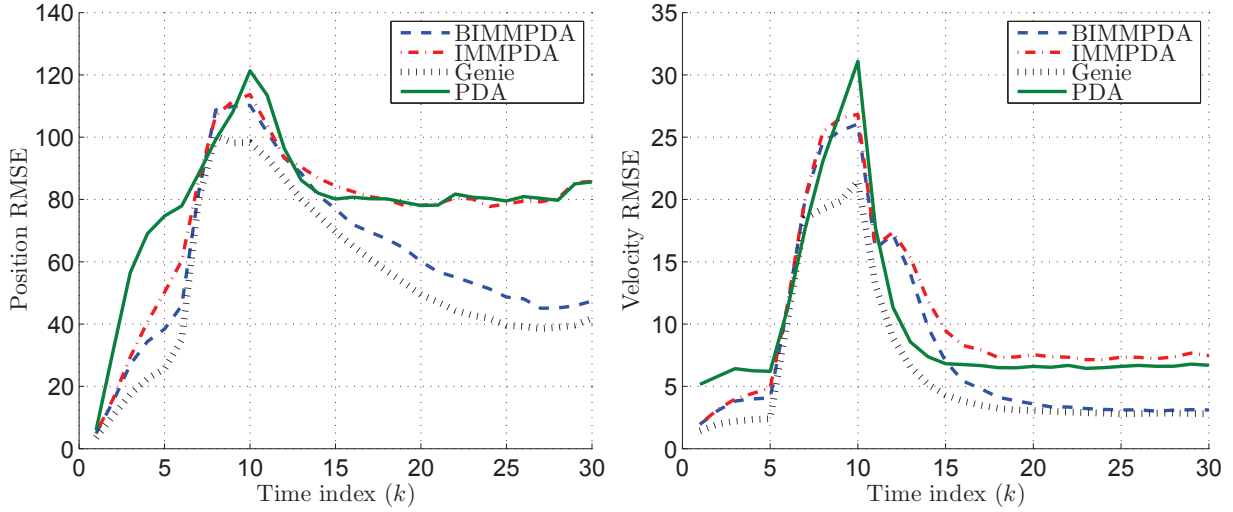


Fig. 1. RMS position (left) and velocity (right) estimation errors for the deterministic scenario.

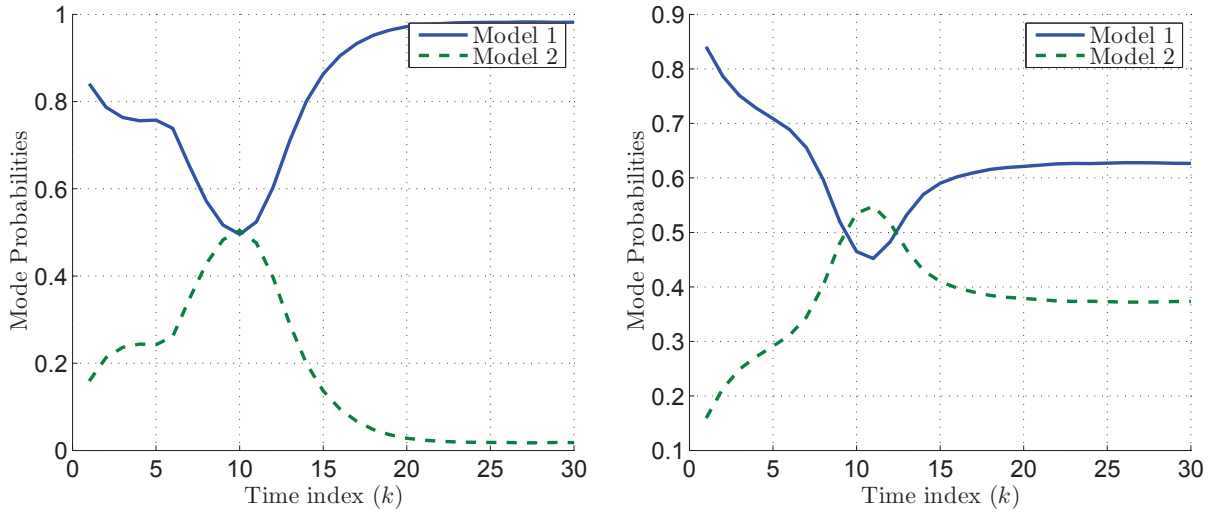


Fig. 2. Model probabilities of BIMMPDA (left) and IMMPDA (right) estimation errors for the deterministic scenario.

2) *Bounded Case*: In this experiment the transitions between m_1 and m_2 are generated according to the modeling mechanism underlying the BIMMPDA algorithm, according to (12), such that the system may perform exactly 2 mode transitions with probability $\frac{1}{6}$ for a switch between models. Note the difference between this example and the deterministic one, where the times of the two transition have been kept constant in every Monte Carlo run. The TPM of the proposed filter, as well as that of the IMMPDA filter, remain unchanged. The corresponding RMS position and velocity errors of all algorithms are presented in Fig. 3. Operating under nominal conditions, the BIMMPDA algorithm attains superior performance, approaching the genie PDA errors at the end of the simulation interval, after all 2 transitions have been exhausted. Similarly to the first (deterministic) example, the IMMPDA filter operates with acceptable accuracy (relatively to BIMM-

PDA) as long as the system alternates between m_1 and m_2 , since these alternations resemble then Markovian transitions, but the performance gap, relatively to BIMMPDA, increases after the second transition is made and the system follows a constant model (approximately after 11 time steps, which is the mean time for performing 2 mode transitions). The poorer performance of the IMMPDA filter is explained by the fact that the filter is not aware of the fact that, after the hard bound on mode transitions has been achieved, no more transitions can occur; therefore, it maintains a mismatched primitive PDA, keeping the overall MSE higher than BIMMPDA.

It may seem at first that the naive nonadaptive PDA performs relatively well, outperforming, at some parts of the interval, the more sophisticated algorithms. This is not necessarily true since, as will be demonstrated in the sequel, its track loss rate is significantly higher.

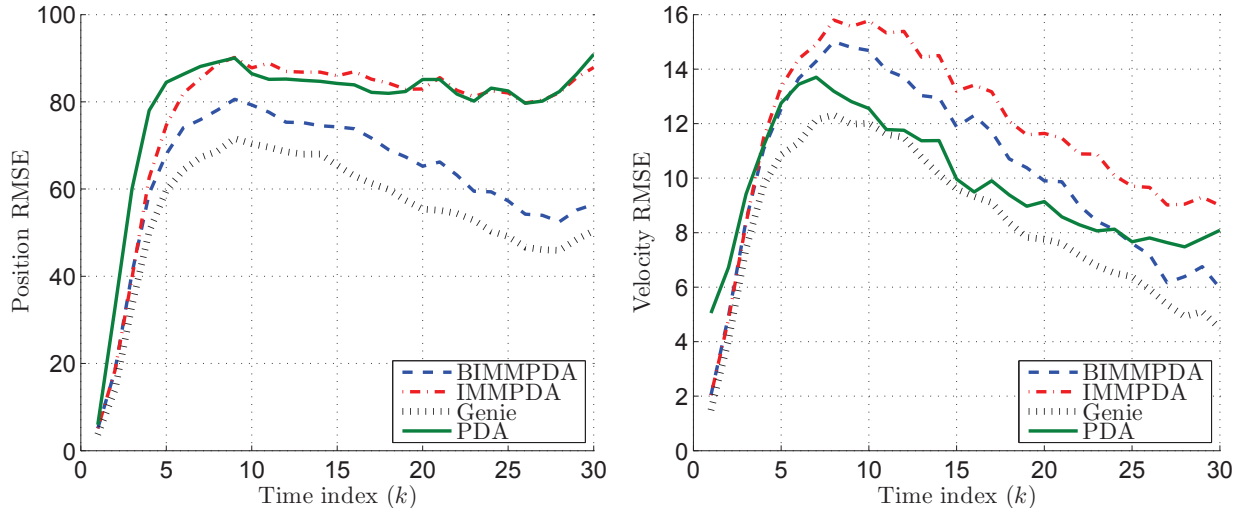


Fig. 3. RMS position (left) and velocity (right) estimation errors for the bounded scenario.

Note that in the present example, since L_{\max} is even, the final target dynamics is dictated by the nonmaneuvering model. This explains the smaller steady-state errors obtained by all filters in comparison to the following example.

3) *Markovian Case*: In this case, the states are generated according to Markovian dynamics without bounding the number of transitions. All other parameters are kept unchanged. The BIMMPDA operates under mismatched conditions, under- or over-estimating the number of switches. However, as shown in Fig. 4, relatively to IMMPDA, the performance degradation is graceful and the filter does not diverge.

4) *Track Loss Rate*: In this final experiment we record the percentage of the lost tracks, according to the criterion described in the beginning of this section, versus varying clutter density, namely the average number of clutter measurements in an interval of length σ_v . We consider Markov and bounded cases and the results are presented in Fig. 5.

As expected, the nonadaptive PDA has the highest track loss rate, rendering it useless in tracking a *maneuvering* target in clutter. The BIMMPDA has a visible advantage over IMMPDA for the bounded case. In the Markov case, on the other hand, the difference between the two is not significant.

V. CONCLUSION

We considered the problem of tracking the state of a hybrid system capable of performing a bounded number of mode transitions in the presence of spurious, or cluttered measurements. Due to the bound on the number of transitions, the popular IMMPDA approach cannot be invoked in a direct manner. Similarly to several related contributions, we showed that a simple modification of the system's mode results in Markov switching dynamics thus allowing the utilization of standard approaches. We showed in simulation that the proposed scheme is, indeed, preferable over a direct use of IMM-based methods in the sense that it attains lower estimation errors and track loss rates. Moreover, the scheme is robust

to modeling assumptions and introduces only moderate gaps relatively to IMMPDA when applied to a Markov scenario.

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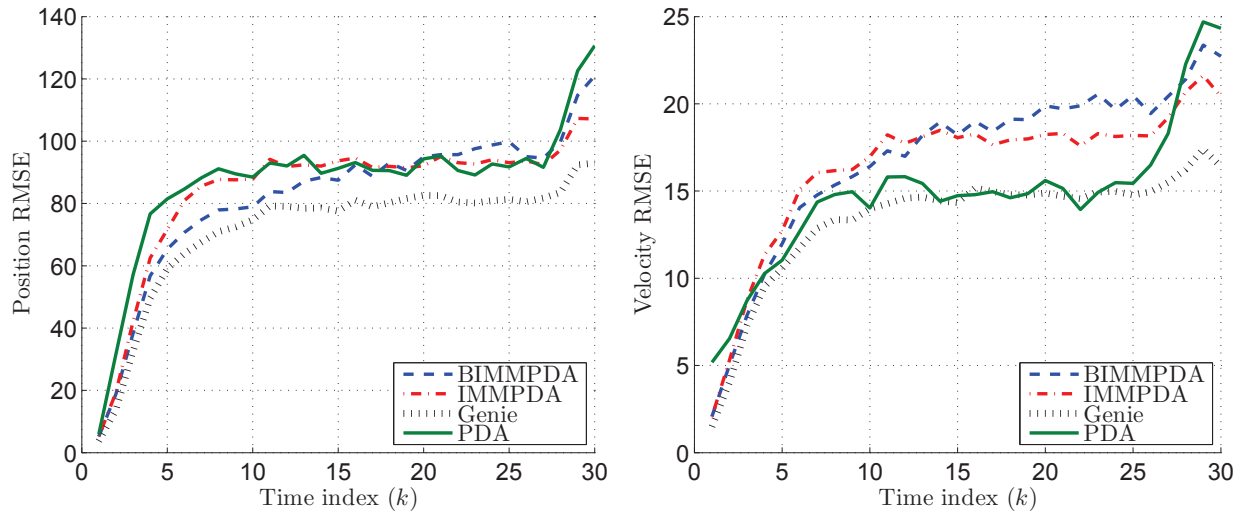


Fig. 4. RMS position (left) and velocity (right) estimation errors for the Markovian scenario.

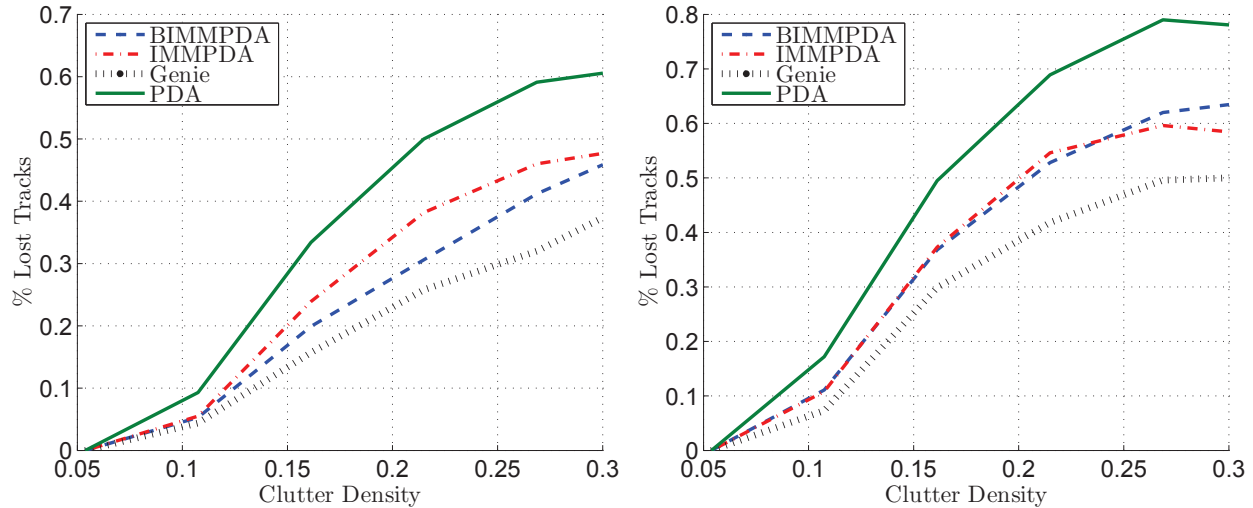


Fig. 5. Average track loss rates for the bounded (left) and Markov (right) scenarios.

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