

# The Multi-session Multi-layer Broadcast Approach for Two Cooperating Receivers

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**Abstract**—We study the problem of a single source transmitting, over a non-fading Gaussian channel, to two cooperating users exhibiting different signal-to-noise ratios (SNR). Both users are required to decode the source message. This model is similar to the broadcast channel with a common message, however the users cannot reliably decode the message independently, and require some form of conversation in order to decode. The cooperation is performed over a noiseless link with a total capacity limit  $C_{coop}$ , which the users may utilize for conversation. In order to maximize the achievable throughput, it is suggested to employ the multi-layer broadcast approach combined with multi-session cooperation, such that every session another layer can be decoded. Since the cooperation link is limited, an efficient cooperation scheme is required. The relaying user performs compression accounting for the side information of the decoder in the Wyner-Ziv (WZ) spirit. In multi-session cooperation the compression is in the form of successive-refinement WZ coding. After the first user decodes a layer, the minimal required information is sent back to the second user to allow both of them to decode the message. The minimal required information is achievable with random coding using a binning strategy. While it is expected that multi-session cooperation will potentially be more efficient than a single-session cooperation we show that the maximal throughput is achieved in a single session.

## I. INTRODUCTION

In recent years, interest in communication networks has increased, and various applications of it, such as sensor networks energy sensitive networks and Ad-hoc networks, have gained popularity due to the promising capacity benefits. Here, we consider one transmitter that sends the same information to two cooperating users, where both users are interested in the same message. The conference cooperation model was introduced by Willems in [1], with orthogonal links available between two transmitting users over a multiple-access channel (MAC), the capacity region here was obtained. Receiver cooperation in general appears to be less understood than transmitter cooperation; for example, the capacity of the broadcast channel with cooperating users is found for the degraded broadcast channel in [2], and remains an open problem in the general case. Multi-session decoding of a single message by two terminals was considered in [3], where the efficiency of multi-session cooperation is shown.

Communication between a single transmitter and a destined user, with a helping colocated user, over a block Rayleigh-fading channel is studied in [4]. For a fading channel, with receiver only channel state information (CSI), it is beneficial to use the broadcast transmission strategy [5]. The broadcast

approach is useful in case of transmitter CSI uncertainty, since a multi-layer coding approach can allow decoding as many layers as possible per block depending on the channel realization. The better the channel the more layers are decoded. Broadcasting to two colocated users over a fading channel was also considered in [6].

Another related work is [7], where the problem of communication from a transmitter to destination via a relay node is considered, where relay and destination cooperate. Unlike [7], in our setting both receivers are required to reliably decode the common message. In addition, cooperation with multiple conferencing sessions was actually not considered in [7], since the original message was not a layered message.

The main contribution of this work is in showing that the highest achievable (common) rate, over an AWGN broadcast channel with two cooperating users, is obtained with a single session. That is, the design of optimal multi-layered coding such that every session another layer is decoded ends up with allocating all transmit power to one layer, which reduces the scheme to a single session, single layer coded scheme.

The rest of the paper is organized as follows. In Section II, the channel model is defined, and the cooperation schemes are defined. The single session cooperation for the non-symmetric broadcast channel is considered in Section III. its extensions to multi-session are considered in Section IV. Finally, concluding remarks in Section V conclude this work.

## II. SYSTEM MODEL AND PRELIMINARIES

We describe here the channel model, which is illustrated in Figure 1, and the basic assumptions. Consider the following non-fading channel model, with two colocated users,

$$y_1 = \sqrt{g_1}x + n_1 \quad (1)$$

$$y_2 = \sqrt{g_2}x + n_2 \quad (2)$$

where  $x$  is the transmitted signal, and  $y_1, y_2$  are the received signals of user-1 and user-2 respectively. the power gains  $g_1$  and  $g_2$  represent the channel gains for user-1 and user-2 respectively. the additive white Gaussian noise for each channel is denoted  $n_1, n_2$  for the first and second user, respectively,  $n_1, n_2 \sim \mathcal{CN}(0, 1)$ . The transmitter power constraint is an average power constraint  $P_s$ , i.e.  $E[|x|^2] \leq P_s$ . Note that  $x, y_1, y_2, n_1, n_2 \in \mathbb{C}$ , and  $g_1, g_2 \in \mathbb{R}_+$ . The channel is assumed to be static. That is, the channel gains  $g_1, g_2$  remain fixed throughout the communications, and over

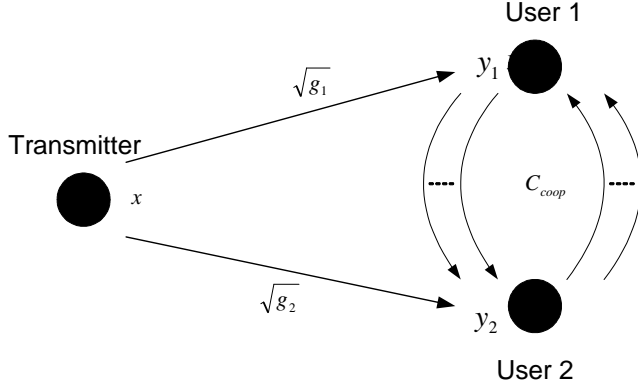


Fig. 1. A schematic description of the broadcast channel model with two cooperating users. The total capacity on the noiseless cooperation links is denoted  $C_{coop}$ .

multiple blocks. In addition, every node has perfect CSI, i.e. both nodes have  $g_1, g_2$ . There is no loss of generality in restricting the channel gains to be real, as the receivers can zero-phase the observed signals.

The users cooperate via finite noiseless orthogonal links of a total capacity  $C_{coop}$ . The sum-rate capacity on the cooperation link is the only constraint, and there is no limitation on the data rate to be exchanged every session. This is formalized by defining the cooperation messages:  $m_{i,k} \in \mathcal{M}_{i,k}$ , where  $m_{i,k}$  is the message sent by the  $i^{th}$  user on the  $k^{th}$  session; and  $\mathcal{M}_{i,k}$  represents the code alphabet for the  $i^{th}$  user on the  $k^{th}$  session. This gives a cooperation link capacity limit, for a block length  $L$ ,

$$\frac{1}{L} \sum_{k=1}^K \log |\mathcal{M}_{1,k}| + \log |\mathcal{M}_{2,k}| \leq C_{coop} \quad (3)$$

where  $K$  is the maximal number of sessions, and the  $i^{th}$  user alphabet cardinality on the  $k^{th}$  session is  $|\mathcal{M}_{i,k}|$ .

The binning strategy for cooperation for efficient decode-and-forward (DF) relaying was suggested in [8]. We briefly review the binning strategy. Consider a single level coded message transmitted at rate  $R_1$ . The transmitted signal is denoted  $x$ , and the corresponding message is denoted  $m$ . The Compress and binning strategy includes a compress and forward (CF) step from user 2 to user 1. This allows user 1 to correctly decode the message  $m$ . At this stage user 1 needs to efficiently send  $m$  to user 2, just enough information to allow user 2 correct decoding of  $m$ . User 1 bins the  $2^{LR_1}$  codewords into  $2^{LR_{bin}} = 2^{L(R_1 - I(x;y_2) + 2\epsilon)}$  bins and transmits to user 2 the index of the bin containing the codeword  $m$ . User 2, intersects the contents of this bin with the list of codewords jointly typical with its observation  $y_2$ . Generally speaking, within a bin there are  $2^{LI(x;y_2)}$  codewords, and hence since these are random, the normalized information  $I(x;y_2)$  is adequate to resolve those. In the case of multi-layer and multi-session cooperation, the message rate per layer is defined by  $R_k$ . The transmitted signal  $x$  is composed of  $K$  layers, and the  $k^{th}$  layer is denoted by  $x_k$ . The binning rate

on the  $k^{th}$  session is  $R_{bin}^{(k)} = R_k - I(x_k; y_2 | x_1, \dots, x_{k-1})$ .

### III. SINGLE SESSION COMPRESS AND BINNING COOPERATION

In this section, the single session achievable rates are specified, and their optimality will be shown in the next section, where multi-session cooperation is considered.

In a single session with CF and binning, one user performs WZ compression, and then the other user decodes the original message, and bins back to the second user the necessary information so this user can also decode the original message. In what follows, user 2 performs WZ, and then user 1 bins the message back. In this case the total achievable rate is

$$R_1 = \log \left( 1 + P_s \left[ g_1 + \frac{g_2}{1 + D_1} \right] \right) \quad (4)$$

with distortion (quantization noise variance),

$$D_1 = \frac{1 + (g_1 + g_2)P_s}{(1 + g_1P_s)(e^{R_{wz1}} - 1)}. \quad (5)$$

The distortion  $D_1$  manifests itself as additional noise, on the channel, thus the equivalent channel gain after cooperation is  $s_{eq} = g_1 + \frac{g_2}{1 + D_1}$ . Detailed derivation of (4)-(5) may be found in [4]. The total cooperation rate constraint, denoted  $C_{coop}$ , is

$$C_{coop} = R_{wz1} + R_1 - \log(1 + g_2P_s). \quad (6)$$

Figure 2 demonstrates the achievable rate  $R_1$  in (4) as function of  $C_{coop}$ , for two pairs of non-symmetric channels  $(g_1, g_2) = (1, 2)$  and  $(g_1, g_2) = (2, 1)$ . As may be noticed clearly, it is more beneficial to have the user with a weaker channel perform WZ compression, and let the stronger user decode first, and bin back the message to the weaker user. It may also be noticed that for  $(g_1, g_2) = (1.5, 1.5)$ , which is a symmetric broadcast channel, the attainable throughput for a given  $C_{coop}$  is smaller than that associated with  $(g_1, g_2) = (2, 1)$ . The upper bound in Figure 2 is identical for all considered cases, since  $I(x; y_1, y_2) = \log(1 + (g_1 + g_2)P_s)$ .

### IV. MULTI SESSION COOPERATION APPROACHES

Several generic multi-session schemes are considered, including K-sessions compress and binning, and its continuous layering extension. In all suggested schemes, it is shown that the optimal power allocation reduces into the single session compress and binning.

#### A. Two-Sessions - Non-Symmetric Compress and Binning

In this two session scheme, the following protocol is studied. Define a 'strong' user as the user with the highest channel gain, and the other is a 'weak' user. Consider a two-level coding scheme, where the strong user first decodes the first layer independently (without any help from the other user), then bins just enough information to the weak user who decodes and subtracts the first layer from the original signal. Then starts the process of compress and binning single session; the weak user performs WZ compression on the residual signal, allowing the stronger user to decode the second layer.

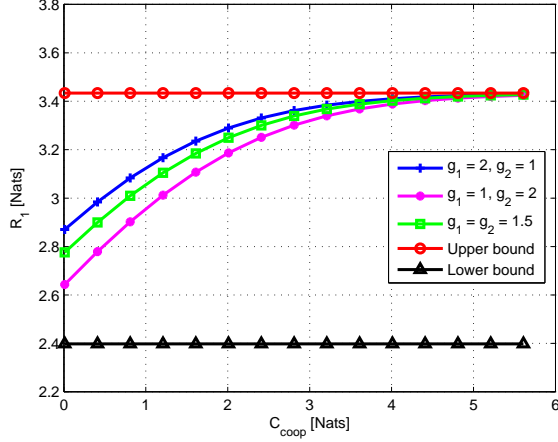


Fig. 2. Achievable Rates with compress and binning single session cooperation; A comparison of attainable rates for three exemplary broadcast channels characterized by  $(g_1, g_2)$ ;  $P_s = 10$  dB.

After decoding the second layer this user bins back to the weak user just enough information to decode the second layer.

The layering rates at the source are obtained by allocating  $\alpha P_s$  for the first layer, and  $(1 - \alpha)P_s \equiv \bar{\alpha}P_s$  for the second layer. The achievable rates are given by

$$R_1 = \log \left( \frac{1 + g_1 P_s}{1 + \bar{\alpha} g_1 P_s} \right) \quad (7)$$

$$R_2 = \log \left( 1 + \left( g_1 + \frac{g_2}{1 + D_1} \right) \bar{\alpha} P_s \right), \quad (8)$$

where it is implicitly assumed that  $g_1 \geq g_2$ . The WZ quantization distortion  $D_1$  is given by

$$D_1 = \frac{1 + (g_1 + g_2) \bar{\alpha} P_s}{(1 + g_1 \bar{\alpha} P_s)(e^{R_{wz2}} - 1)}. \quad (9)$$

The cooperation rate constraint here is

$$C_{coop} = R_{wz2} + R_1 + R_2 - \log(1 + g_2 P_s) \quad (10)$$

where  $R_1, R_2$  are specified in (7),(8), respectively. The optimization problem and its Lagrangian are specified by

$$\begin{aligned} & \max_{\bar{\alpha} \in [0, 1]} R_1 + R_2 \quad (11) \\ & \text{s.t. } C_{coop} = R_{wz2} + R_1 + R_2 - \log(1 + g_2 P_s) \end{aligned}$$

The next proposition determines the optimal strategy here.

*Proposition 4.1:* In a non-symmetric non-fading AWGN broadcast channel, a single session cooperation is more beneficial than a two session cooperation, as specified in (7)-(10).

*Proof:* In order to prove the proposition, it is required to show that  $\bar{\alpha} = 1$  is the optimal solution for the optimization problem in (11). This is a convex optimization problem, so a Lagrange multiplier  $\lambda$  is added to incorporate the cooperation link constraint. It may be shown that the gradient of the Lagrangian in the direction of  $\bar{\alpha}$  is always positive. More details are available in [10]. ■

## B. Two-Session Compress and Binning

Following the result of the non-symmetric compress and binning with one time WZ compression and two binning sessions, it is suggested to perform two compress and binning 'symmetric' sessions, and on the first session begin with a WZ CF, add binning, and repeat all over on the residual signal, after removing the first layer on both receivers. The following equations specify the achievable rates

$$R_1 = \log \left( 1 + \frac{\left( g_1 + \frac{g_2}{1 + D_1} \right) \alpha P_s}{1 + \left( g_1 + \frac{g_2}{1 + D_1} \right) \bar{\alpha} P_s} \right) \quad (12)$$

$$R_2 = \log \left( 1 + \left( g_1 + \frac{g_2}{1 + D_2} \right) \bar{\alpha} P_s \right). \quad (13)$$

The associated successive refinement (SR) WZ rates are

$$R_{wz1} = \log \left( 1 + \frac{1 + (g_1 + g_2) P_s}{D_1 (1 + g_1 P_s)} \right) \quad (14)$$

$$R_{wz2} = \log \left( \frac{D_1}{D_2} \frac{1 + (g_1 + g_2) \bar{\alpha} P_s + D_2 (1 + g_1 \bar{\alpha} P_s)}{1 + (g_1 + g_2) \bar{\alpha} P_s + D_1 (1 + g_1 \bar{\alpha} P_s)} \right) \quad (15)$$

The cooperation link rate constraint is given by

$$C_{coop} = R_{wz1} + R_{wz2} + R_1 + R_2 - \log(1 + g_2 P_s) \quad (16)$$

It is shown here that the optimal power allocation is a single layer power allocation, which means that a two-session approach is sub-optimal here. The optimization problem is

$$\begin{aligned} & \max_{\bar{\alpha} \in [0, 1], D_1 \geq D_2 \geq 0} R_1 + R_2 \quad (17) \\ & \text{s.t. } C_{coop} = R_{wz1} + R_{wz2} \\ & \quad \quad \quad + R_1 + R_2 - \log(1 + g_2 P_s) \end{aligned}$$

*Proposition 4.2:* In a non-symmetric non-fading AWGN broadcast channel, a single session cooperation is more beneficial than a two session compression and binning cooperation, as specified in (12)-(16).

The proof is omitted due to space limitations, see [10].

## C. K-Session Compress and Binning

The two session case is extended here to a general setting with  $K$  coded layers and  $K$  sessions, where each session includes a SR-WZ compression, add binning back to allow both users to decode one layer per session. Consider a  $K$ -layer code, where the power allocated to the  $k^{\text{th}}$  layer is denoted  $\Delta_k$ . The accumulated residual power is denoted  $P_k$ , where

$$P_k = \sum_{n=k}^K \Delta_n. \quad (18)$$

The rate associated with the  $k^{\text{th}}$  layer is

$$R_k = \log \left( \frac{1 + (g_1 + g_2) P_k + D_k (1 + g_1 P_k)}{1 + (g_1 + g_2) P_{k+1} + D_k (1 + g_1 P_{k+1})} \right) \quad (19)$$

where  $P_{K+1} = 0$ , and  $P_1 = P_s$ . The associated SR-WZ compression rates are given by

$$R_{wz,1} = \log \left( 1 + \frac{1 + (g_1 + g_2)P_s}{D_1(1 + g_1P_s)} \right) \quad (20)$$

$$R_{wz,k} = \log \left( \frac{D_{k-1}}{D_k} \frac{1 + (g_1 + g_2)P_k + D_k(1 + g_1P_k)}{1 + (g_1 + g_2)P_k + D_{k-1}(1 + g_1P_k)} \right) \quad (21)$$

where  $k = 2, \dots, K$  in  $R_{wz,k}$  (21). The cooperation link rate constraint is given by

$$C_{coop} = -\log(1 + g_2P_s) + \sum_{k=1}^K R_{wz,k} + R_k. \quad (22)$$

It is shown here that the optimal power allocation is a single layer power allocation, which means that for any  $K > 1$ , a sub-optimal cooperation is approach, and the maximal attainable throughput is achievable within a single session.

The optimization problem is specified by

$$\begin{aligned} & \max_{\substack{0 \leq P_K \leq P_{K-1} \leq \dots \leq P_2 \leq P_s, \\ 0 \leq D_K \leq D_{K-1} \leq \dots \leq D_1,}} \sum_{k=1}^K R_k \quad (23) \\ \text{s.t. } & C_{coop} = -\log(1 + g_2P_s) + \sum_{k=1}^K R_{wz,k} + R_k \end{aligned}$$

*Proposition 4.3:* In a non-symmetric non-fading AWGN broadcast channel, a single session cooperation is more beneficial than a  $K > 1$  compression and binning multi-session cooperation, for any integer  $K > 1$ . The multi-session cooperation model is specified in (18)-(22).

*Proof:* A detailed proof is available in [10]. In order to prove that the multi-session scheme is sub-optimal, we adhere to the following steps. Take an arbitrary set of  $K$  distortions  $\{D_k\}_{k=1}^K$ , which satisfy the boundary constraint  $D_K < D_{K-1} < \dots < D_1$ . The strict inequality is required to guarantee all layers are effective. It will be shown that for every  $k$ , the gradient of the Lagrangian in the direction of  $P_k$  is an increasing function, which means that if the power allocation starts with the  $K^{th}$  layer  $P_K = P_s$  is optimal.

This is a convex optimization problem, so a Lagrange multiplier  $\lambda$  is added to incorporate the cooperation link constraint. Let  $J(P_1, \dots, P_K, D_1, \dots, D_K)$  be the Lagrangian, given by

$$\begin{aligned} J(P_1, \dots, P_K, D_1, \dots, D_K) &= \sum_{k=1}^K R_k \\ &+ \lambda \left( \sum_{k=1}^K (R_{wz,k} + R_k) - \log(1 + g_2P_s) - C_{coop} \right). \quad (24) \end{aligned}$$

For optimal selection of  $P_K$  which maximizes  $J(P_1, \dots, P_K, D_1, \dots, D_K)$  above, we take the gradient of  $J(P_1, \dots, P_K, D_1, \dots, D_K)$  in the direction of  $P_K$ , for a given  $D_K < D_{K-1} < \dots < D_1$ . This is given by the partial

derivative of  $J(P_1, \dots, P_K, D_1, \dots, D_K)$  w.r.t.  $D_K$ .

$$\begin{aligned} & \frac{\partial J(P_1, \dots, P_K, D_1, \dots, D_K)}{\partial P_K} \\ &= \frac{(1 + 2\lambda)(g_1 + g_2 + g_1D_K)}{1 + (g_1 + g_2)P_K + D_k(1 + g_1P_K)} \\ & \quad - \frac{(1 + 2\lambda)(g_1 + g_2 + g_1D_{K-1})}{1 + (g_1 + g_2)P_K + D_{K-1}(1 + g_1P_K)} \quad (25) \end{aligned}$$

which can be simplified into

$$\frac{\partial J(P_1, \dots, P_K, D_1, \dots, D_K)}{\partial P_K} = \frac{(1 + 2\lambda)g_2(D_{K-1} - D_K)}{\beta} \quad (26)$$

where  $\beta > 0$  is a positive scalar, and is a function of  $(g_1, g_2, P_K, D_{K-1}, D_K)$ . From the above result in (26), it is clear that for  $D_{K-1} > D_K$ ,  $\frac{\partial J}{\partial P_K} > 0$ . This means that  $P_K = P_s$  is optimal, which gives  $\Delta_1 = \Delta_2 = \dots = \Delta_{K-1} = 0$ . This leaves a single effective layer - the  $K^{th}$  layer. ■

#### D. Continuous broadcasting with Compress and Binning

Let us now consider the case of multi-session cooperation for the non-symmetric AWGN broadcast channel, with a non-limited number of sessions. The session index is taken to the limit, and a fractional cooperation bandwidth is allocated for each session. This is clearly not a practical setting, but provides the information theoretic upper bound for the achievable throughput for applications which are not delay sensitive.

Starting from the discrete case, of  $K$  layered coding with multiple SR-WZ and binning sessions, the distortion after the  $k^{th}$  session is obtained by extracting  $D_{k+1}$  from Eq. (21),

$$D_{k+1} = \frac{D_k \cdot (1 + (g_1 + g_2)I(s_k))}{(1 + g_1I(s_k))(1 + \delta_k(1 + D_k)) + g_2(1 + \delta_k)I(s_k)} \quad (27)$$

where  $\delta_k(R_{wz,k}) = \exp(R_{wz,k}) - 1$ , and  $I(s_k)$  is the residual interference associated with the  $k^{th}$  layer. In general,  $I(s_k)$  is the residual accumulated power defined in (18), not including the current layer, that is  $I(s_k) = P_k - \Delta_k$ . The equivalent channel gain is  $s_k = g_1 + \frac{g_2}{1 + D_k}$ . The throughput is the sum-rate over the fractional rates of the  $K$  layers, which is

$$R_K = \sum_{k=1}^K \log \left( 1 + \frac{s_k \Delta_k}{1 + s_k I(s_k)} \right) \quad (28)$$

where  $\Delta_k$  is the power allocated to the  $k^{th}$  layer, and  $I(s_k)$  is the corresponding residual interference.

In order to form the continuous broadcasting  $K$  is taken to the limit of  $K \rightarrow \infty$ . The power allocation per layer is denoted by  $\lim_{K \rightarrow \infty} \Delta_k = \rho(s)ds$ , where  $\rho(s) = -I'(s)$ . From the definition of  $\delta_k$ , it may be noticed that  $\delta(s)ds = R_{wz}(s)ds$ . The distortion, specified in Eq. (27), in the continuous, using the limit  $ds \rightarrow 0$ , is given by

$$\frac{dD(s)}{D(s)} = \frac{-[1 + D(s)(1 + g_1I(s)) + (g_1 + g_2)I(s)]\delta(s)ds}{1 + (g_1 + g_2)I(s)} \quad (29)$$

The relation in (27) is used in the continuous case to express  $D(s)$  and  $D'(s)$  as function of  $s, g_1, g_2$ ,

$$D(s) = \frac{g_1 + g_2 - s}{s - g_1}, \quad \frac{dD(s)}{ds} = \frac{-g_2}{(s - g_1)^2}. \quad (30)$$

The above  $D(s)$  and  $D'(s)$  in (30) are substituted in Eq. (29), and yield the following expression for  $\delta(s)$ ,

$$\delta(s) = \frac{1 + (g_1 + g_2)I(s)}{(g_1 + g_2 - s)(1 + sI(s))}. \quad (31)$$

The equivalent SNR gain lies in  $g_1 \leq s \leq g_1 + g_2$ . We assume at this point a single non-zero continuous interval of power allocation  $[s_0, s_1] \subseteq [g_1, g_1 + g_2]$ , hence the bandwidth limitation on the WZ cooperation is

$$R_{wz} = \int_{s_0}^{s_1} \delta(s) ds = \int_{s_0}^{s_1} \frac{1 + (g_1 + g_2)I(s)}{(g_1 + g_2 - s)(1 + sI(s))} ds. \quad (32)$$

The multi-session cooperation here follows the same guidelines of the  $K$  discrete layering multi-session with the SR-WZ and binning protocol, which was analyzed in the previous subsection. Let one user perform SR-WZ compression. The other user after decoding the next layer simply performs binning to send minimal required information to let the first user decode that layer too. Both users locally subtract the last decoded layer from their received signal, and repeat the same protocol for the next layer. Under the SR-WZ and binning protocol, the overall cooperation link capacity is given by

$$\begin{aligned} C_{coop} &= R_{wz} + R_{bin} \\ &= \int_{s_0}^{s_1} [\delta(s) + dR(s)] ds - \log(1 + g_2 P_s) \\ &= \int_{s_0}^{s_1} \left( \frac{1 + (g_1 + g_2)I(s)}{(g_1 + g_2 - s)(1 + sI(s))} + \frac{-sI'(s)}{1 + sI(s)} - \frac{\log(1 + g_2 P_s)}{s_1 - s_0} \right) ds \\ &\triangleq \int_{s_0}^{s_1} G(s) ds \end{aligned} \quad (33)$$

We can state the optimization problem:

$$\begin{aligned} &\max_{I(s),} \int_{s_0}^{s_1} \frac{-sI'(s)}{1 + sI(s)} ds \\ \text{s.t. } &I(s_0) = P_s, I(s_1) = 0, \\ &\int_{s_0}^{s_1} G(s) ds \leq C_{coop} \end{aligned} \quad (34)$$

where  $\int_{s_0}^{s_1} I'(s) ds = -P_s$ , where  $I(s_0) = P_s$ ,  $I(s_1) = 0$ ,  $g_1 \leq s_0 \leq s \leq s_1 \leq g_1 + g_2$ , and (33).

**Proposition 4.4:** In a non-symmetric non-fading AWGN channel, continuous layering is sub-optimal, following the setup specified in (33)-(34).

*Proof:* Denote the integrand in (34) by  $J(s, I, I') \triangleq \frac{-sI'(s)}{1 + sI(s)}$ . The condition for extremum in its general form accounting for the subsidiary constraints is given by

$$J_I + \lambda G_I - \frac{d}{ds}(J_{I'} + \lambda G_{I'}) = 0. \quad (35)$$

Then

$$J_I - \frac{d}{ds} J_{I'} = \frac{1}{(1 + sI(s))^2} \quad (36)$$

$$\delta_I = \frac{1}{(1 + sI(s))^2} \quad (37)$$

$$G_I - \frac{d}{ds} G_{I'} = \delta_I + J_I - \frac{d}{ds} J_{I'} \quad (38)$$

and  $\delta_{I'} = 0$ . This yields the following extremum condition

$$\frac{1 + 2\lambda}{(1 + sI(s))^2} = 0. \quad (39)$$

This condition cannot be met for any  $0 \leq I(s) < P_s$ , and therefore it may be concluded that a continuous layering is a sub-optimal approach for the multi-session non-symmetric AWGN channel. ■

## V. CONCLUSION

In this paper we introduce the problem of transmission from a single source over a non-fading channel with two cooperating users. Both users are required to decode the source message. The cooperation is performed over a noiseless link with a total capacity limit  $C_{coop}$ , which the users may utilize for conversation. Since the cooperation link is limited, it is required to efficiently use this link. Thus every session the relaying user performs compression accounting for the side information of the decoder in the WZ spirit. In multi-session cooperation the compression is in the form of SR-WZ coding. After the first user decodes a layer, a random coding with a binning strategy is used in order to send the minimal required information back to the second user. It is shown in [10] that such cooperation is more efficient than other known alternatives such as compress-and-forward by both users. We also show here that with this strategy a single session cooperation is more beneficial than multi-session cooperation.

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