

Communication via Decentralized Processing

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Abstract—The common problem of a nomadic terminal sending information to a remote destination via agents with lossless connections is investigated. Such a setting suits, e.g. access points of a wireless network, where each access point is equipped with a different connection bandwidth. The case where these agents do not have any decoding ability is fully characterized for the Gaussian channel, when the transmitter uses "typical" codewords. For general discrete memoryless channels, lower and upper bounds are derived. An achievable rate with unrestricted agents, which are capable of decoding, is also given and then demonstrated by a numerical example for the Gaussian channel.

I. INTRODUCTION

The information theory of networks, and especially wireless networks, is in the focus of an extensive research activity. This interest is partly due to many recent results about the multiple antenna channel, which demonstrate significant improvements, especially for fading channels.

Many papers propose and analyze, in information theoretic terms, ad-hoc wireless networks. The relaying technique, or as it is sometimes called, multi-hopping, makes use of several intermediate wireless nodes to help the communication between two distant nodes. An information theoretic view of the relay channel was given by Cover and El Gamal in [1] for a single relay node and extended by [2] for several relaying nodes. Relaying can be coarsely divided into compress-and-forward (amplify-and-forward is viewed as a special case) and decode-and-forward, depending on whether the relays decode the transmitted message or just forward the received signal to the destination. An upper bound derived by [3] suggests that as the number of users in an ad-hoc network increases, the total rate of each user tends to zero. This bound motivates the use of networks that are not solely ad-hoc, but are composed of base stations or access points as well.

The problems of conveying a source that is observed by remote agents to a single destination are built around similar settings, where the source is an i.i.d. random variable. Many problems are analyzed within an information theoretic setting, such as distributed source coding, CEO [4] and sensor networks. A connection between these rate-distortion problems to network scenarios is reported in [5],[6],[7]. The CEO setting is also addressed by tools of estimation theory (where it is commonly referred to as fusion center), such as [8],[9] and [10]. The dissertation of Schein [11] focuses on the problem of communication via two agents (parallel relays), which are connected via lossless links to the final destination, and several achievable rates are demonstrated there.

Here we consider the problem of communication through either non-decoding or decoding agents. This means that the agents use a noisy version (via their respective channels) of the transmitted message, and are able to transmit a predetermined number of bits to the destination without any errors. The destination views the source only via the agents' transmissions as in [11].

The rest of the paper is organized as follows: in section II the setting of the problem is given. An achievable rate and an upper bound are presented in sections III and IV respectively. An achievable rate for the case where the agents can decode is the focus of section V. Section VI deals with the Gaussian channel and includes a complete characterization for the case where the agents are unaware of the code used.

II. PROBLEM SETTINGS

We use capital letters for random variables, capital letters with subscripts X_i for the i -th element in a random vector, and a capital letter with a superscript X^n for the vector (X_1, \dots, X_n) . The notation X_k^m refers to the vector (X_k, \dots, X_m) . Calligraphic letters \mathcal{X} or \mathcal{T} denote sets. Lower-case letters will usually be used for indices or realizations of random variables.

We consider the problem of a single transmission through T agents, playing the role of decentralized processors, as is seen in Fig. 1. Our model consists of a nomadic transmitter S , which uses random codebooks, where the agents do not know the codebook used. Such random coding is also used in [12] for a mis-match scenario, while the advantages of random codebooks were demonstrated in [13] for unknown channels.

The following properties and definitions hold, unless stated otherwise:

- 1) The output of the transmitter S is $X \in \mathcal{X}$ for every channel use.
- 2) The T agents A_1, \dots, A_T receive the outputs of a memoryless broadcast channel, defined by

$$P_{Y_1^k, \dots, Y_T^k | X^k}(Y_1^k, \dots, Y_T^k | X^k) = \prod_{i=1}^k P_{Y_1, \dots, Y_T | X}(Y_{1,i}, \dots, Y_{T,i} | X_i), \quad (1)$$

for any $k = [1, 2, \dots]$ and where $Y_t \in \mathcal{Y}_t$. Denote $\mathcal{T} \triangleq \{1, \dots, T\}$.

- 3) The bandwidth C_t , in bits per channel use, characterizes the lossless link that connects the agent A_t to the final destination D .

- 4) The ensemble of all $N_C = |\mathcal{X}|^{k2^{kR}}$ codebooks with rate R , codeword length k and input channel alphabet \mathcal{X} is denoted as \mathcal{C} .
- 5) The key F is an index mapping into a code from \mathcal{C} , so $1 \leq F \leq N_C$.
- 6) The transmitter sends $X^k = \phi_{S,F}(M)$, where

$$\phi_{S,F} : [1, \dots, 2^{kR}] \rightarrow \mathcal{X}^k, \quad (2)$$

in k channel uses. The function $\phi_{S,F}$ represents the coding with code F , and M is the message to be sent, $M \in [1, \dots, 2^{kR}]$.

- 7) The transmitter S and the final destination D randomly select the key F from N_C , in an initialization stage, according to the probability

$$P_F(F) = \prod_{M=1}^{2^{kR}} P_{X^k}(\phi_{S,F}(M)), \quad (3)$$

where $P_{X^k}(X^k) = \prod_{i=1}^k P_X(X_i)$, for some single letter probability $P_X(X)$. The agents are not informed about the selected key F .

- 8) The agents encode every $n \leq k$ channel outputs (where $m = k/n$ is an integer) with T encoding functions:

$$0 < t \leq T, \phi_{At} : \mathcal{Y}_t^n \rightarrow [1, \dots, 2^{nC_t}] \quad (4)$$

so that

$$0 < j \leq m : V_{t,j} = \phi_{At} \left(Y_{t,(j-1)n}^{jn} \right). \quad (5)$$

- 9) The final destination D decodes the message M from the set $V_T^m \triangleq (V_{T,1}, \dots, V_{T,m})$, which consists of T vectors, each of length $m = k/n$, which are sent to the destination from the T agents:

$$\hat{M} = \phi_{D,F}(V_T^m), \quad (6)$$

where $\phi_{D,F} : [1, \dots, 2^{m \sum_{t=1}^T nC_t}] \rightarrow [1, \dots, 2^{kR}]$.

Notice that with the knowledge of F , X^k is uniformly distributed over 2^{kR} codewords, while without the key, X^k is distributed according to $\prod_{i=1}^k P_X(X_i)$. We use the following simple lemma in the sequel:

Lemma 1: Without the key F , the received vector X^k is distributed according to $P_{X^k}(X^k) = \prod_{i=1}^k P_X(X_i)$, and therefore Y_t^k are also distributed as

$$P_{Y_t^k}(Y_t^k) = \prod_{i=1}^k \sum_X P_{Y_t|X}(Y_{t,i}|X) P_X(X). \quad (7)$$

This lemma is proved in [14].

It is easy to see from the probability laws and the fact that all possible codebooks are considered, that for a given F , the code is a random code chosen according to the probability law $P_{X^k}(X^k) = \prod_{i=1}^k P_X(X_i)$.

The above setting models the problem where the final destination decodes the message from the transmitter via simple agents, which are not able to decode the transmitted message and use compression of the received signals.

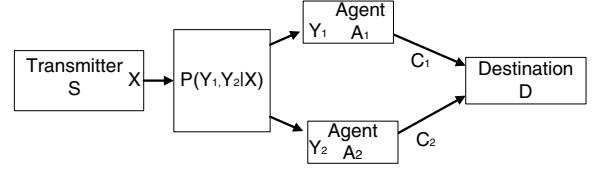


Fig. 1. A system with two agents between the transmitter and the destination.

III. AN ACHIEVABLE RATE

We denote the model which is described by the setting of section II as nomadic transmitter. The following theorem is a special case of theorem 3 (proved in [14], see also [2]).

Theorem 1: Define a positive R such that (8) is fulfilled

$$R < \max I(X; U_T) \quad (8)$$

with the constraints (9) on U_T

$$\forall \mathcal{S} \subseteq \mathcal{T} : \sum_{t \in \mathcal{S}} C_t > I(U_S; Y_S | U_{\mathcal{S}^c}) \quad (9)$$

and the joint distribution (10)

$$P_{X, U_T, Y_T}(X, U_T, Y_T) = P_X(X) P_{Y_T|X}(Y_T|X) \prod_{t=1}^T P_{U_t|Y_t}(U_t|Y_t). \quad (10)$$

Then a rate R is achievable for the case of nomadic transmitter.

The probability space (10) results with the Markov relations:

$$U_t - Y_t - \{X, U_{T \setminus t}, Y_{T \setminus t}\}. \quad (11)$$

Since this achievable rate is attained in theorem 3 through the use of compression which is independent of the message index and the codebook used by the transmitter, theorem 1 is indeed a special case of theorem 3.

IV. AN OUTER BOUND

We start by stating the maximum rate R so error free decoding is possible at the destination, when the agents' encoding functions are given. Using Fano's inequality, an error free decoding at the destination is possible only if:

$$H(M|V_T^m, F) \leq k\epsilon_k, \quad (12)$$

where $k\epsilon_k \rightarrow 0$ as $k \rightarrow \infty$.

Defining $\mathbf{X}_j \triangleq (X_{(j-1)n+1}, \dots, X_{jn})$, we now have:

$$kR = H(M) = I(V_T^m, F; M) + H(M|V_T^m, F) \quad (13)$$

$$\leq I(V_T^m; M|F) + k\epsilon_k \quad (14)$$

$$= I(V_T^m; \mathbf{X}^k(M, F)|F) + k\epsilon_k \quad (15)$$

$$\leq \sum_{j=1}^m [H(V_{T,j}|F) - H(V_{T,j}|\mathbf{X}_j, F) + n\epsilon_k] \quad (16)$$

$$\leq m \max_{P_{X^n}(X^n)} I(V_T; X^n|F) + k\epsilon_k, \quad (17)$$

where (14) follows since F is independent of M so $H(F|M) = H(F)$, and (16) is due to properties II - 2 and II - 8. From (17) we conclude that the transmission rate R is upper bounded by

$$R \leq \max_{P_{X^n}(X^n)} \frac{1}{n} I(V_T; X^n|F). \quad (18)$$

Next we would like to upper bound

$$\frac{1}{n} I(V_T; X^n|F). \quad (19)$$

By defining $U_{t,i} \triangleq (V_t, Y_T^{i-1}, X^{i-1})$, we get to the following theorem, which is proved in [14], using techniques similar to the ones used to show theorem 2 of [15]:

Theorem 2: The maximum reliable communication rate for a nomadic transmitter is upper bounded by

$$R \leq \max I(X; U_T). \quad (20)$$

Here U_T must satisfy the constraints:

$$\forall S \subseteq \mathcal{T} : \sum_{t \in S} C_t \geq I(U_S; Y_T|U_{S^c}). \quad (21)$$

and also fulfill the Markov relations:

$$U_t - Y_t - \{X, Y_T \setminus t\}. \quad (22)$$

V. AGENTS WITH CODE KNOWLEDGE

In this section we diverge from the nomadic model described in section II. Suppose the agents know the codebook so that the agents and the transmitter can be jointly optimized. This enables to transmit a broadcast message that is decoded by the agents and forwarded to the destination, in addition to the compression operation. Denote this model as decoding agents. Such generalized approach might increase the overall transmission rate.

In the following, we will denote all messages that are decoded at the agents as broadcast messages, although they are always intended for the same final destination.

Denote by M_t the message to be decoded at agent t , so that now $M = (M_T, M_{CF})$ (M_{CF} is the message that is decoded only at the final destination). Since the agents have decoding ability, we assume that they also process much longer output sequences, so that now $n = k$, and we can drop the k index altogether.

Theorem 3: For the decoding agents case, any rate R satisfying (23)

$$R < I(X; U_T|W_T) + \sum_{t=1}^T R_{BC,t}, \quad (23)$$

with the constraints (24) on $R_{BC,T}, U_T, W_T$ (W_0 is a constant)

$$\begin{cases} \forall 0 < t \leq T : 0 \leq R_{BC,t} < \min \{I(W_t; Y_t), C_t\} \\ \forall S \subseteq \mathcal{T} : \\ \sum_{t \in S} [C_t - R_{BC,t}] > I(U_S; Y_S|U_{S^c}, W_T) \\ \sum_{t \in S} R_{BC,t} < \sum_{t \in S} [I(W_t; Y_t) - I(W_t; W_{\tilde{T}(S,t)})], \end{cases} \quad (24)$$

when $\tilde{T}(S,t) \triangleq \{i : i \in S \text{ and } i < t\}$ and the joint distribution (25)

$$P_{X,Y_T,W_T,U_T}(X, Y_T, W_T, U_T) = P_{W_T}(W_T) P_{X|W_T}(X|W_T) P_{Y_T|X}(Y_T|X) \cdot \prod_{t=1}^T P_{U_t|Y_t,W_t}(U_t|Y_t, W_t), \quad (25)$$

is achievable.

The agent A_t forwards $nR_{BC,t}$ decoded bits to the destination along with $n(C_t - R_{BC,t})$ bits for the compression. The achievable rate may be further increased to the convex hull of (23) and (24).

Proof: The proof appears in [14] and uses compression in addition to Marton's scheme (not including common messages). ■

Remark 1: The above achievable rate can be increased by sending only partial messages to the final destination. This means in fact doing Slepian Wolf (SW) distributed compression of the messages to the final destination.

Remark 2: Another improvement upon (23) is done by sending common broadcast messages in addition to the individual broadcast messages to the agents. This is achieved by extending theorem 2 in [16] to more than two users and including compression. Notice that such construction contains theorem 3 and theorem 4 (to follow) as special cases.

Remark 3: The scheme described in theorem 1 is obtained as a special case of the above scheme, by taking all W_T to be constants.

For the case where the channels $P_{Y_t|X}(Y_t|X)$ are either stochastically or physically degraded [17], that is Y_t is better than Y_{t-1} (without loss of generality), we can use superposition coding, which is known to achieve capacity for degraded broadcast channels. The received signal Y_2 is physically degraded compared to Y_1 if the following Markov chain

$$X - Y_1 - Y_2 \quad (26)$$

is satisfied. Notice that this relation leaves $I(X; Y_1, Y_2) = I(X; Y_1)$. On the other hand, stochastically degraded Y_2 means that the marginal probability $P_{Y_2|X}(Y_2|X)$ can be calculated from $P_{Y_1|X}(Y_1|X)$ through some $P_{Y_2|Y_1}(Y_2|Y_1)$. Notice that since (26) is not necessarily true for stochastic degradedness, we have that $I(X; Y_1, Y_2) \geq I(X; Y_1)$. So although superposition coding is optimal for the degraded broadcast channel, it is not optimal, in general, for our model.

Theorem 4: For decoding agents with a channel $P_{Y_T|X}(Y_T|X)$ that satisfies stochastic degradedness (27),

$$\forall 0 < t \leq T : P_{Y_{t-1}|X}(Y_{t-1}|X) = \sum_{Y_t} P_{Y_t|X}(Y_t|X) P_{Y_{t-1}|Y_t}(Y_{t-1}|Y_t), \quad (27)$$

any rate R satisfying (23) with the constraints (28) on

$R_{BC,T}, U_T, W_T$ (W_0 is a constant)

$$\begin{cases} \forall 0 < t \leq T : 0 \leq R_{BC,t} \leq C_t \\ \forall 0 < t \leq T : \sum_{i=1}^t R_{BC,i} < \sum_{i=1}^t I(W_i; Y_i | W^{i-1}) \\ \mathcal{S} \subseteq \mathcal{T} : \sum_{t \in \mathcal{S}} C_t - R_{BC,t} > I(U_{\mathcal{S}}; Y_{\mathcal{S}} | U_{\mathcal{S}^c}, W_T), \end{cases} \quad (28)$$

and the joint distribution (29)

$$P_{X|W_T}(X|W_T)P_{Y_T|X}(Y_T|X) \prod_{t=1}^T P_{U_t|Y_t, W^t}(U_t|Y_t, W^t) \cdot \prod_{t=1}^T P_{W_t|W^{t-1}}(W_t|W^{t-1}), \quad (29)$$

is achievable.

Theorem 3 does not seem to include theorem 4 as a special case, as it does not account for the common rate in Marton's region.

VI. THE GAUSSIAN CHANNEL

In this section we investigate the Gaussian channel under two cases, where the agents are ignorant about the code and where they are cognizant of the codebook used by the transmitter. Using the latest results of Tse et. al., for the Gaussian CEO rate-distortion problem [18], a converse for the reliable communication rate is shown for the former case. We use the results of theorem 1 with continuous alphabets, where the extension relies on standard arguments.

A. Non-decoding agents

The Gaussian channel is defined by $Y_t = X + N_t$, where N_t are independent Gaussian random variables with $\mathbb{E}N_t^2 = P_{N_t}$ ¹ and $\mathbb{E}N_t = 0$. Let $P_X(X)$ be zero mean Gaussian with variance $\mathbb{E}X^2 = P_X$.

We prove the following result.

Theorem 5: The capacity of the nomadic transmitter, for the Gaussian channel (described above), is

$$R = \max_{r_t \geq 0} \min_{\mathcal{S} \subseteq \mathcal{T}} \left\{ \frac{1}{2} \log_2 \left(1 + P_X \sum_{t \in \mathcal{S}^c} \frac{1 - 2^{-2r_t}}{P_{N_t}} \right) + \sum_{t \in \mathcal{S}} [C_t - r_t] \right\}. \quad (30)$$

The theorem is proved in [14] using the technique of [18]. In the sequel, the constants r_t are interpreted as rates associated with noise quantization [18]. The capacity is determined by calculating the sufficient bandwidths $\underline{C} \triangleq (C_1, \dots, C_T)$ for communication at rate R , and by showing that it equals the necessary bandwidths $\overline{C} \triangleq (C_1, \dots, C_T)$ for reliable communication with rate R .

Proof: The altered proof is briefly sketched here, for completeness.

First, the sum-bandwidths under the constraints

$$\sum_{t=1}^m C_t \leq a_m, \quad m = 1, \dots, T-1 \quad (31)$$

is shown to be $(\bar{a} \triangleq \{a_1, \dots, a_{T-1}\})$

$$C_{\min}(a_1, \dots, a_{T-1}) = R + \min_{r_T \in \nabla(R, \bar{a})} \sum_{t=1}^T r_t, \quad (32)$$

where the space $\nabla(R, \bar{a})$ is defined as r_T that satisfy the following $2T$ conditions:

$$\begin{cases} \forall 0 < t \leq T : 0 \leq r_t \\ \forall 0 < m \leq T-1 : \\ a_m \geq R - \frac{1}{2} \log_2 \left(1 + P_X \sum_{t=m+1}^T \frac{1 - 2^{-2r_t}}{P_{N_t}} \right) + \sum_{t=1}^m r_t \\ R = \frac{1}{2} \log_2 \left(1 + P_X \sum_{t=1}^T \frac{1 - 2^{-2r_t}}{P_{N_t}} \right). \end{cases} \quad (33)$$

Next, the above sum-bandwidth identity leads to identical bandwidth-regions ($\underline{C}(R) \leq \overline{C}(R)$). This is since both regions are convex [18] and since for all non-negative vectors $(\alpha_1, \dots, \alpha_T)$:

$$\min_{(C_1, \dots, C_T) \in \overline{C}(R)} \sum_{t=1}^T \alpha_t C_t \geq \min_{(C_1, \dots, C_T) \in \underline{C}(R)} \sum_{t=1}^T \alpha_t C_t, \quad (34)$$

where (34) is proved in [18]. ■

B. Example: agents with decoding capabilities

Consider the symmetric case of a Gaussian channel with statistically equivalent agents (both suffering from an additive Gaussian noise with variance P_N). In addition, both agents are connected via lossless links with equal bandwidth C , to the final destination. The combined approach of broadcast and compression for the degraded channel (theorem 4) is employed. The rate R is achievable provided that:

$$R < \sum_{t=1}^2 R_t + \frac{1}{2} \log_2 \left(1 + \alpha \frac{P_X}{P_N} \sum_{t=1}^2 (1 - e^{-2r_t}) \right) \quad (35)$$

where the $\{r_t, R_t, \alpha\}$ satisfy:

$$\begin{cases} 0 \leq \alpha \leq 1 \\ t = 1, 2 : 0 \leq R_t \leq C \\ \sum_{t=1}^2 R_t < \frac{1}{2} \log_2 \left(\frac{P_N + P_X}{P_N + \alpha P_X} \right) \\ \forall \mathcal{S} \subseteq \{1, 2\} : \\ \sum_{\mathcal{S}} [C - R_t] > \sum_{\mathcal{S}} r_t + \\ \quad \frac{1}{2} \log_2 \left(1 + \alpha P_X \sum_{t=1}^2 \frac{1 - 2^{-2r_t}}{P_{N_t}} \right) + \\ \quad - \frac{1}{2} \log_2 \left(1 + \alpha \frac{P_X}{P_N} \sum_{\mathcal{S}^c} (1 - 2^{-2r_t}) \right). \end{cases} \quad (36)$$

The convex hull is found to improve rates for this example. The achievable rate as a function of the bandwidth C , for signal to noise ratio $\frac{P_X}{P_N} = 10$, is presented in Fig. 2. In this figure, the left most dashed line $R = 2C$, and the upper flat dashed line $R = \frac{1}{2} \log_2(1 + 2\frac{P_X}{P_N})$ are the two cut-set bounds [17], and the lower flat dashed line is the rate of a system without compression $R = \frac{1}{2} \log_2(1 + \frac{P_X}{P_N})$. The dotted line represents time-sharing, which is useful here. This figure illustrates that if the sum of capacities of the corresponding broadcast channel, (calculated by the signal to noise ratios

¹E stands for statistical expectation

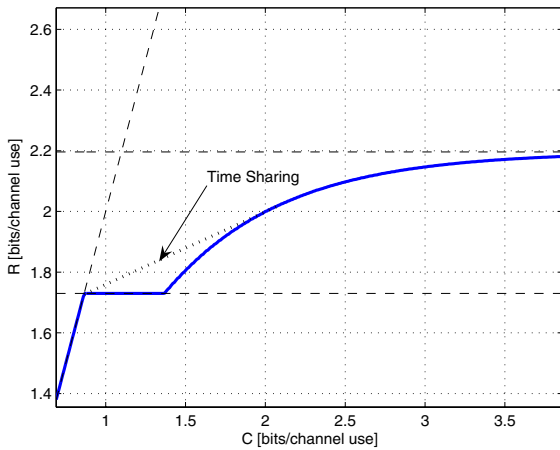


Fig. 2. The achievable rate of a system with two agents, each with link bandwidth of C and signal to noise ratio of 10 dB. The dotted line designates time sharing, and the dashed lines represent the cut-set bounds [17]. The lower flat dashed line is the achievable rate for a system without compression.

at the agents) is smaller than the sum of the bandwidths of the links, a compression scheme can significantly improve the performance. A rate of up to 0.2 bits from the cut-set bound is observed with $C = R = 2$, which means 50% excess bandwidth compared to the achievable rate.

VII. CONCLUSION

Communication via separated agents is considered, focusing on two cases: (1) the agents do not possess any knowledge about the codebook used by the transmitter, and (2) the agents do possess decoding capability. For the first case, a direct coding theorem based on decentralized quantization and an outer bound were presented. Considering the Gaussian channel, a converse was proved by the entropy power inequality invoking the results of [18]. An achievable rate was derived also for the case where the agents are cognizant of the codebook used by the transmitter. These achievability arguments combine Marton's achievable rates for the broadcast channel with decentralized quantization. The utility of time sharing was demonstrated in the Gaussian example.

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