LDPC Coded MIMO Multiple Access Communications

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***Abstract*— An efficient scheme for the multiple-access MIMO channel is proposed, which operates well also in the single user regime, as well as in a DS-CDMA regime. The design features scalability and is of limited complexity. The system employs optimized LDPC codes and an efficient iterative (belief propagation) detection which combines Linear Minimum Mean Square Error (LMMSE) and iterative interference cancellation. Asymptotic density evolution is used to optimize the degree polynomials of the underlining LDPC code, and thresholds as close as 0.77 dB to the channel capacity is evident for a system load of 2. Replacing the LMMSE with the complex individually optimal multiuser detector (IO-MUD) further improves the performance up to 0.14 dB from the capacity. Comparing the thresholds of good single-user LDPC code to the multiuser optimized LDPC code both on the above multiuser channel reveals surprising 8 dB difference. The asymptotic analysis of the proposed scheme is verified by simulations of finite systems, which reveal meaningful differences between performances of MIMO systems with single and multiple users and demonstrate similar performance to previously reported techniques, but with higher system loads, and significantly lower receiver complexity.**

I. Introduction

Many schemes are designed to approach the overwhelming MIMO capacity [1], in particular, the case of CSI known at the receiver but not at the transmitter is assumed. (a nice overview of various communication systems is given in [2]). On the other end, iterative receivers, in which the detector and the decoder exchange extrinsic information, are known to perform well in a variety of communication systems, specifically in multiple access systems such as CDMA ([3],[4]). Similar multiuser detection techniques achieve very good performance also in MIMO systems [5]. Caire et al. in [3] present density evolution (DE) analysis for such iterative CDMA multiuser receiver, when the transmitters use convolutional codes. This density evolution analysis is used in [6] also for regular LDPC codes. The use of LDPC-BP decoder with LMMSE multiuser detector enables elegant density evolution analysis of the decoder for the asymptotic realm. In this paper we utilize these techniques to construct a multiuser MIMO communication system employing LDPC codes and LMMSE multiuser receiver. Furthermore, we optimize the degree distribution pairs (DDP) [7] of the LDPC code when iteratively decoded with the LMMSE detector. Asymptotic techniques [8], [3] are used for the code optimization and for the capacity evaluation. Codes are optimized for both linear MMSE detectors and individually optimal multiuser detector (IO-MUD). Comparing the thresholds to the capacity reveals excellent performance, with gap to capacity of 0.7 dB for the LMMSE and 0.14 dB for the IO-MUD, which is realized with prohibitive complexity. The LDPC codes are optimized with a global search algorithm (DE-Differential Evolution) that finds good LDPC polynomials that achieve low bit error rate. Simulations of finite systems verify the asymptotic expectations on both CDMA and MIMO for both single and multiple user systems with block lengths of 1.5×10^5 and 10^4 respectively.

II. System Description

In this paper we deal with the multi access communication channel where each user transmits independent information by a single antenna over independent Rayleigh fading channel to a receiver equipped with M antennas. We assume full synchronization, perfect power control scheme and a single class of K users. Generalization of this work to systems with several classes and/or received powers follows the same lines. The channel load α is defined by $\frac{K}{M}$ in parallel to an equivalent DS-CDMA setting [3]. The receiver, in the proposed scheme, has full knowledge about the channel state information (CSI), while the transmitters have no CSI available. The MIMO channel coefficients $\{h_{k,t}\}_{k=1}^{K}$ are assumed to remain constant along entire transmitted block and are then randomly and independently chosen again for the next transmitted block, so we drop the time index t. Such statistics approximates quasi-static block fading MIMO channel. The model also suits more generalized multi access MIMO system of $K_u \neq K$ users, where each transmitter can use number of antennas $\frac{K}{K_u}$, not necessarily a single one. We can also consider the case of single user MIMO system, as a special case of the generalized system above, where there is one user $K_u = 1$ transmitting out of $\frac{K}{1} = K$ antennas. The designed scheme also suits non-orthogonal CDMA or any other multi access system that uses some randomly generated vector as the common channel and when this vector is generated by some i.i.d. elements. For example, in multiuser DS-CDMA systems M stands for the processing gain, and the random channel attenuations coefficients stands for the K-users signatures sequences.

A. Transmitters and the channel

The channel (1) is defined along the lines of [3]. The received signal vector y_t of length M, at time t, consists of linear superposition of the K transmitted symbols $\{x_{k,t}\}_{k=1}^{K}$

multiplied by the channel coefficients vectors $\{\boldsymbol{h}_k\}_{k=1}^K$, by scalar random phases $\{e^{j\theta_{k,t}}\}_{k=1}^K$ and by received amplitude $\sqrt{\frac{\gamma}{M}}$. It also suffers additive Gaussian complex noise $\boldsymbol{\nu}_t \sim \mathcal{N}_{\mathbb{C}}(0,\boldsymbol{I})$[1].

$$\boldsymbol{y}_t = \sqrt{\frac{\gamma}{M}} \sum_{k=1}^{K} \boldsymbol{h}_k e^{j\theta_{k,t}} x_{k,t} + \boldsymbol{\nu}_t. \tag{1}$$

Here, $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(0,\boldsymbol{I})$. We use QPSK signals $x \in \{\pm\frac{1}{\sqrt{2}}, \pm\frac{j}{\sqrt{2}}\}$. $e^{j\theta_{k,t}}$ is randomly, independently and uniformly generated from $\{\pm\frac{1}{\sqrt{2}}, \pm\frac{j}{\sqrt{2}}\}$ to render the multiple access interference independent of the transmitted codewords [3]. The receiver has full knowledge of the received powers $\frac{\gamma}{M}$, the channel coefficients $\boldsymbol{h}_k$ and the random phases $e^{j\theta_{k,t}}$. Notice that we normalized the received power with M so that each transmitting antenna is received with total received power of $\frac{E_s}{N_0} = \gamma$, regardless of the number of receiving antennas. All the users encode their information bits with the same Low density Parity Check (LDPC) code and then apply independent random bit interleavering before modulating QPSK symbols. We use LDPC block code with length N and rate R and therefore each transmitted block consists of $t = 1, \ldots, \frac{N}{2}$ QPSK symbols and $\frac{E_b}{N_0} = \frac{E_s}{N_0}\frac{1}{\log_2(4)R} = \frac{\gamma}{2R}$.

B. The Receiver

The received signal is detected by a multiuser detector (MUD) which produces soft information about the individually coded bits of the K users. These are passed to the K (or k_u) LDPC decoders. The soft outputs of the single user decoders are fed back to the MUD, such that the receiver corrects the erroneous bits along the iterations.

1) The multi user detector: Recall from equation (1) that the inputs to the receiver are $\boldsymbol{y}_t, \{\boldsymbol{h}_k\}_{k=1}^K, \{\theta_{k,t}\}_{k=1}^K, \gamma$ and the inputs to the multiuser detector in the iterative scheme are $\boldsymbol{y}_t, \{\boldsymbol{h}_k\}_{k=1}^K, \{\theta_{k,t}\}_{k=1}^K, \gamma, \hat{\boldsymbol{x}}$, where $\hat{\boldsymbol{x}}$ stands for the soft estimations of the transmitted symbols as obtained from the SISO decoders at the previous iteration. The LLR for the n-th bit of the k-th user codeword can be calculated by the non-linear and highly complex individually optimal multi user detector (IO-MUD) ([8] and [9]), that ignores any code structure. It can be approximated, however, by linear filters combined with interference cancellation (IC), which ignore any constellations constraints, are widely used and can be implemented with polynomial complexity with the number of users. Such linear multiuser detector is the linear minimum mean square error (LMMSE). For LMMSE filter in the iterative interference cancellation scheme (LMMSE-IC) we define ξ_j to be the estimated power of the cancelled j user:

$$\xi_j = \mathrm{E}|x_j - \hat{x}_j|^2 \tag{2}$$

where $\mathrm{E}(x_j - \hat{x}_j) = 0$[2] and $\mathrm{E}(x_k - \hat{x}_k)^*(x_j - \hat{x}_j) = 0$, $\forall\, j \neq k$, since the users are uncoordinated and as long as the cycle-free assumption holds. Conditional LMMSE filter considers the power ξ_j conditioned on $\hat{x}_{j,t}$, while unconditional LMMSE estimates it for the entire block of the user $\frac{1}{N}\sum_{t=1}^{\frac{N}{2}} \hat{x}_{j,t}$, which results in significantly lower complexity. If we define $\boldsymbol{\Sigma}_k = \boldsymbol{I} + \gamma \sum_{i\neq k} \xi_i \boldsymbol{h}_i \boldsymbol{h}_i^H$ (the covariance matrix of the multiple access interference (MAI) plus the noise) and $\hat{\boldsymbol{y}}_k = \sqrt{\frac{\gamma}{M}} \sum_{i\neq k} \xi_i \boldsymbol{h}_i e^{j\theta_i} \hat{x}_i$, the estimation of the symbol of the k-user from the LMMSE-IC MUD is:

$$z_k = \frac{\sqrt{\gamma}\boldsymbol{h}_k^H \boldsymbol{\Sigma}_k^{-1}(\boldsymbol{y} - \hat{\boldsymbol{y}}_k)}{\gamma \boldsymbol{h}_k^H \boldsymbol{\Sigma}_k^{-1} \boldsymbol{h}_k}. \tag{3}$$

Note that this symbol estimation still needs to be converted to LLRs of the code bits.

2) The Decoder: to decode the LDPC codes, the receiver utilizes the well known BP decoder. Such decoders are often described using a Tanner graph, which is composed of variable nodes and check nodes and of interconnecting edges. The variable nodes for the single user channel are initialized by the channel outputs (which remain constant along the iterations). The multiuser receiver graph, however, is composed of three classes of nodes: the $N \times K$ LDPC variable nodes and $N \times (1 - R) \times K$ check nodes that represent the individual codes' parity check equations, including any interleaving and the $\frac{N}{2}$ multiuser detector nodes that iteratively improve the LLRs of the codewords' bits. This is illustrated in figure 1, where two users with regular (2,4) LDPC user code of length 8, are parallel wise decoded and detected using four multiuser detectors nodes for two users with QPSK modulation, and two LDPC tanner graphs.

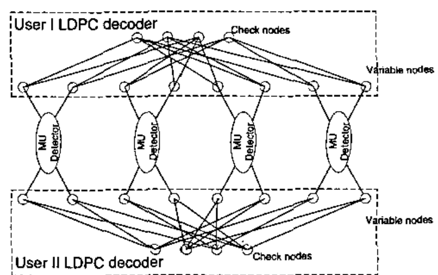

Fig. 1. LDPC BP decoder with multiuser detector scheme

3) Iterative detection and decoding: At first iteration, when receiving the signal, the multi user detector estimates $K \times \frac{N}{2}$ symbols without any a priori information from the SISO decoders, which means that it acts as detector in separate detection-decoding scheme. All the likelihood ratios, calculated from these symbol estimations are simultaneously forwarded to all the respective variable nodes of all the users' decoders. These variable nodes, then send messages to the check nodes, which replay back. The messages from variable to check and from check to variable nodes are denoted as VC and CV respectively. The replied CV messages are added at each variable node, to provide the multiuser detector with independent estimations of the symbols for the next iteration. These CV messages represent the present extrinsic decoder estimations of the corresponding bits. The proposed receiver

[1] $\mathcal{N}_{\mathbb{C}}(0, I)$ represents the vector complex Gaussian density with covariance equals to identity matrix I and zero mean

[2] E is the expectation operator

uses parallel scheduling scheme, so that all the users are simultaneously detected, decoded and then subtracted in every iteration. It is different than e.g. BLAST techniques which sequentially detect, decode and subtract user after user. We search for good codes, so that the LDPC decoder would iteratively improve the multiuser detector's a priori inputs ($\hat{x}$) and reach low bit error rate.

III. Asymptotic Analysis

We follow [3] and [8] and use asymptotic systems analysis, where the code length, number of users and number of antennas is taken to infinity, while the channel load remains fixed. Such analysis, which considers the statistics of the channel, gives rise to a cycle free decoder graph and also to elegant analytical expressions [3]. It is noticed that although the time variant LDPC code is not trellis code as in [3], the cycle free assumption still holds, by the arguments presented in [7]. It is known from [8] that in such realm, the outputs of both the linear filters and the IO-MUDs converge to Gaussian random variables. So that the multiuser efficiency η is sufficient to describe the density. Where η is defined as the output SINR from the multiuser detector divided by the SNR of the user, without the interferers.

A. Analysis of the LDPC SISO decoder

The limiting density of the asymptotic LDPC decoder's output can be determined by the density evolution procedure [7]. To obtain numerical results, we assume without any loss of generality that the transmitted codeword is the all zero codeword and use the technique that was suggested by [10] to calculate densities that are defined over discrete and finite space. In this manner, the calculation of the densities of the CV and VC messages, along the iterations, can be efficiently performed by FFT for the VC and by using some lookup tables for the CV densities. The calculated densities are, in fact, the limiting densities of the messages, while $N \to \infty$ and while the cycle free assumption holds. Less complex and less accurate analysis assumes only Gaussian densities [11], and calculates only a single parameter instead of complete densities. We modified the Gaussian approximation technique of [11] to approximate the multi access receiver, where the log likelihood ratios (LLRs) originate from the MUD, instead of from the channel and therefore change along the iterations.

B. Optimal multiuser detector analysis

The asymptotic multiuser efficiency η for the IO-MUD in the iterative scheme over real channel, is given by Caire et al. in [9], which extended previous results on separated scheme [8]. It is the solution of the implicit equation (4):

$$\frac{1}{\eta} = 1+\alpha\gamma \mathrm{E}_t[(1-t^2)\int \frac{1-\tanh(z\sqrt{\eta\gamma}+\eta\gamma)}{1-t^2\tanh^2(z\sqrt{\eta\gamma}+\eta\gamma)}Dz], \quad (4)$$

where $t = \tanh(\frac{\mathrm{LLR}}{2})$ is the **LLR** corresponding to $\hat{x}$, and $Dz = \frac{\exp(-\frac{z^2}{2})}{\sqrt{2\pi}}dz$. Similar analysis applies for complex channels.

C. Linear multiuser detector analysis

For the linear MMSE in IC schemes, we can use the results of Caire *et al.* in [3] which include a closed form expressions for the filter's output descriptive statistics, as a functional of the density of the estimated symbol. These elegant and useful results rely on the fact [12] that when dealing with random matrices in the asymptotic regime, the resulting SIR converges to a deterministic limit. If the conditions defined in [12] are fulfilled, the resulting asymptotic multiuser efficiencies η of LMMSE filter converges (weakly) to a deterministic value, as $K \to \infty$. This value is the solution of the following fixed point equation:

$$\eta = \frac{1}{1+\alpha\int_0^\infty \frac{\xi}{1+\xi\eta}dF_\xi(\xi)}, \quad (5)$$

where $F_\xi(\xi)$ is the limiting cumulative probability function of mean power of the users: $F_\xi(\xi) = \lim_{K\to\infty}\frac{1}{K}\sum_{j=1}^{K}\mathrm{u}(\xi-\xi_k)^3$. This asymptotic result stems from the asymptotic distribution of the eigenvalues of the channel matrices (specifically, the eigenvalues of $\sum_{k=1}^{K} \boldsymbol{h}_{k,t}^H \boldsymbol{h}_{k,t}$), which is robust to the specific distribution of the matrix elements, and remains true for complex valued H.

Notice that from the asymptotic analysis point of view, the only difference between the LMMSE-IC and the IO-MUD is in the resulting η. Where for the LMMSE, it is the solution of (5), and for the optimal detector, it is the solution of (4). For the conditional LMMSE $F_\xi(\xi)$ is directly calculated from the output of the SISO decoder, that changes for every iteration, every symbol and every user and for the unconditional LMMSE, $F_\xi(\xi) = \mathbf{u}(\xi-\bar{\xi})$, where $\bar{\xi}$ is the block-wise average of the residues and therefore the unconditional filter remains constant for all the symbols of the same user at the same iteration. Since the integral over ξ reduces to a simple expression, the fixed point of (5) for the unconditional LMMSE is explicitly written in equation (6):

$$\eta = \frac{2}{1+(\alpha-1)\bar{\xi}+\sqrt{4\bar{\xi}+(1+(\alpha-1)\bar{\xi})^2}}. \quad (6)$$

It is noticed that conditional LMMSE's performance will never be worse than performance of unconditional LMMSE, because of the convexity of $\frac{\xi}{1+\xi\eta}$ in equation (5).

IV. Search Results

In this section we will review the search results for good LDPC degree distribution pairs (DDPs). The problem of finding good codes is a global optimization problem, that maximizes the rate of the code R under the constraint of asymptotic error free decoding. The search for globally good codes is performed by a stochastic genetic algorithm known as differential evolution. This algorithm improves initial random population, by inflicting random mutations on it, and keeping only the better mutants. The mutations are done while maintaining the constraints originating in the asymptotic analysis.

[3] $\mathbf{u}(x)$ is the indicator function, that equals one for $x \geq 0$ and 0 otherwise

TABLE I

DISTANCE TO CAPACITY IN [dB] OF SEARCH RESULTS FOR UNCONDITIONAL LMMSE, OBTAINED WITH THE DIFFERENTIAL EVOLUTION WITH CONSTRAINT UTILIZING GAUSSIAN APPROXIMATION

E_s/N_0	1 dB	2 dB	3 dB	4 dB
$\alpha = 0.2$	0.33	0.31	0.38	0.40
$\alpha = 0.5$	0.44	0.44	0.55	0.63
$\alpha = 1$	0.66	0.69	0.8	1
$\alpha = 1.5$	0.77	1.06	1.09	1.26
$\alpha = 2$	0.77	1.16	1.35	1.69

TABLE II

DISTANCES TO CAPACITY OF THE CODE DESIGNED FOR MULTI ACCESS AND FOR A CODE OPTIMIZED FOR THE AWGN ON BOTH SCENARIOS

	distance to capacity on AWGN [dB]	distance to capacity on multi access [dB], α =2
multi access code R=0.352	2	0.77
AWGN code R=0.5	0.2	9

Good pairs are found for five channel loads: α =0.2,0.5,1,1.5 and α =2, each with four threshold constraints: E_s/N_0 =1,2,3 and 4 dB. The DDPs were obtained while limiting the search space by constraining the check and variable degrees. The use of QPSK signalling is quite sufficient for the lower powered schemes. Notice that the number of antennas at the receiver and the MUD's complexity are reduced by the usage of high channel loads. Table I presents the results of the search process as the gap between the achieved asymptotic thresholds and the asymptotic capacities. These complex channel capacities are conjectured, and lean on the technique in [8] which is used there for the real valued channel. The loss due to the QPSK restriction can be appreciated by comparing this capacity to the optimal MIMO capacity, for CSI known at the receiver and without feedback channel. Telatar in [1] calculated this optimal capacity, and we noticed a gap of 0.05 dB for load $\alpha = 2$ and $\gamma = 4$ dB. Notice that for α =0.2 the suggested scheme reached up to 0.3 dB from the channel capacity, and for α =2 as close as 0.77 dB away from the channel capacity. We compared the code that was designed for channel load of 2 to LDPC code that was designed for AWGN channel (from [7]). The differences between the thresholds of these codes and the channel capacity are shown in table II for both multi access and single user channels. Notice that although the AWGN-code performs well on the AWGN, it's performance on the multi access channel is poor (even when considering the fact that it has higher rate). In contrast, the code, that was designed for the multi access channel, performed reasonably well on the AWGN channel (difference of 2 dB). This emphasizes the importance of including the multiuser analysis in the search for good codes.

The gaps to the channel capacity for high channel loads, as appear in table I, are mainly due to the limitations of the linear MUD. To confirm this conclusion, we searched for good DDPs for iterative schemes which include the conditional LMMSE-IC and the IO-MUD. The distances between the thresholds of these systems to the capacity, with channel load α=2 are presented in table III. We see insignificant improvement of 0.1 dB for the conditional LMMSE over the unconditional LMMSE and much more significant 0.63 dB improvement for the IO-MUD. This exemplifies the linear detectors limitations, especially for high channel loads.

TABLE III

DISTANCE TO CAPACITY, AS ACHIEVED WITH IO-MUD, CONDITIONAL LMMSE AND UNCONDITIONAL LMMSE, FOR α=2 AND THRESHOLD OF $\gamma = 1$ dB.

	unconditional LMMSE	conditional LMMSE	IO-MUD (MMSE)
distance [dB]	0.77	0.67	0.14

V. SIMULATION RESULTS

The system was simulated to verify the analysis and to asses performance with finite number of users. All the simulations were performed with channel load of $\alpha = 2$. The LDPC code was constructed according to the degree distribution pair that was found by the asymptotic analysis for threshold of $E_s/N_0 = 1$ dB and for channel load of α =2. The resultant code is of $R = 0.352$.

1) Comparison of the asymptotic analysis to the simulations of finite systems: The asymptotic analysis expectation and simulation results are drawn in figure 2 as function of the iterations. These simulation were done for multiuser MIMO system with K =200 users ($\alpha = 2$) and $\gamma = E_s/N_0$ =1.45 dB (E_b/N_0 =2.98 dB). It can be seen that the density evolution predicts the performance of the system well, until about the 20-th iteration. The simulations will probably agree with the asymptotic density evolution prediction along more iterations if the codeword length N will be increased.

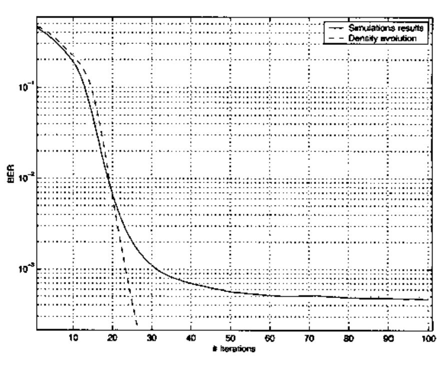

Fig. 2. Simulation results versus density evolution of the check to variable messages, of MIMO system with K =200 users, that use LDPC code with $R = 0.352$, and with a channel load α =2, E_s/N_0 =1.45 dB (E_b/N_0 =2.98 dB), as function of iterations

2) Single user versus Multiple user MIMO block fading: The multiple access MIMO system simulation approximated the asymptotic analysis prediction with well with $K = 200$ users (although sufficient performance was achieved with only $K = 50$ users). That means using $M = 100$ antennas in the receiver. Significantly less users, such as $K = 15$ with $\alpha = 1.875$ results in severe degradation in the performance. Single user MIMO system, however, can utilize single LDPC code with multiplexer to transmit from all the antennas. The corresponding receiver uses single decoder that will benefit from much more diverse channel (factor of K) than the multiple access decoders, so that the probability of all the entries of the channel matrix being very weak is very low. This can be seen in figure 3, where we simulated a single user and multiple user MIMO systems with code lengths of $N = 15 \times 10^4$ and $N = 10^4$, respectively and with $K = 15$ and $K = 200$ transmitting antennas. It is clear that the single user systems outperform the respective multiuser systems. Single user MIMO system with $K = 200$ reaches BER of 10^{-4} within 0.2 dB from the predicted density evolution threshold, and within 1 dB from the channel capacity. It is comparable to the excellent performance reported in [5] for system which includes sphere decoder, except that our proposed system operates at higher channel load, require less complex receiver but uses more antennas. It is evident that in our setting more users are required to achieve the asymptotic performance than the number required to achieve the ergodic single user capacity [1]. This is a consequence of our block fading model which prevents coding over differently faded symbols, thus causing erroneous blocks for small number of users.

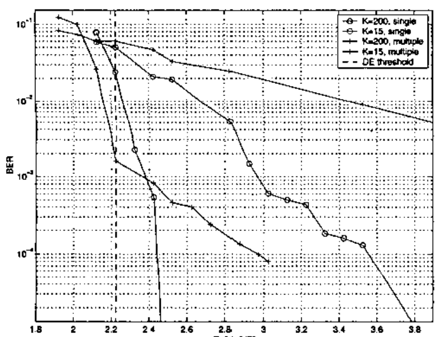

Fig. 3. Simulation results of multiuser vs. single user MIMO systems, with $K = 200$ and $K = 15$ users with channel loads $\alpha = 2$ and $\alpha = 1.875$, where the transmitters use LDPC code with $R = 0.352$ and lengths of $N = 10^4$ and $N = 1510^4$ respectively. The BER presented was measured after sufficient number of iterations, as function of E_b/N_0. DE threshold is also presented for comparison.

VI. Conclusions

In this paper we described an efficient MIMO communication scheme for both single and multiple users. The users' LDPC code was optimized with asymptotic analysis for the entire iterative receiver, which is a variation of what was presented by Caire et al. in [3] for trellis codes. The resulting low complexity scheme performs well and for channel load of $\alpha = 2$, the asymptotic threshold of the system is only 0.77 dB away from the corresponding capacity (evaluated with the replica method). It is also compared to both conditional LMMSE and IO-MUD systems, for the assessment of the cost of using linear filter, which turns out to be less than 0.6 dB. Comparing the performance of the optimized code to that of LDPC code which was optimized for AWGN reveals substantial merit of the modified search. The asymptotic analysis is verified by simulations of finite systems which indicate differences between single and multiple users systems. It is seen that single user system does well enough with only 15 antennas whereas the multiuser system requires considerably more. These differences stem from the non-ergodic nature of the block fading channel and can be reduced by using more than one transmitting antenna for each user, or by adopting faster fading model, so that each decoder experiences several attenuations in single block.

Acknowledgments

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