RANDOM FIELDS AND THEIR GEOMETRY

Corrections and Commentary

November 15, 2011

Chapter 1

9+15: The upper bound in both integrals, as well as the one three lines below, should (obviously) be $+\infty$ rather than $-\infty$.

10+19: AX should be XA. Ditto for the second last line of Footnote 9.

10-2: (1.2.7) should read

$$m_{i|j} = m^i + (x^j - m^j)C_{jj}^{-1}C_{ji}.$$

10-3: In Footnote 9, the displayed equation should be

$$A \;=\; \begin{pmatrix} I_n & -C_{12}C_{11}^{-1} \\ 0 & I_{d-n} \end{pmatrix}$$

17+8: Replace the second sentence with: Applying (1.3.13) for $u \geq 1$ and checking numerically for $u \in [0,1]$, we have that, for all $u \geq 0$,

18+4: The correct inequality is $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}$.

19-14: Equation should be

$$d(\pi_{j_o}(t), \pi_{j_o}(s)) \;\leq\; d(s,t) + 2r^{-j_o} \;\leq\; 3r^{-j_o}$$

21+1: The upper bound in the integral should be $p^{-1}(\delta)$. The same is true for all integrals on this page.

21-5: $f(t) = f(h^{1/2}(t)$ should be $f(t) = f(W^{1/2}(t))$.

22-2: The notation here, and on the next page, is poor. One needs the interpretation that

$$\sum_{i=1}^{k} h_i t_i' \triangleq \sum_{i=1}^{k} h_i\left(0, \ldots, t_i', \ldots, 0\right),$$

with t_i' appearing in the i-th position. This is an element of $\otimes^k \mathbb{R}^N$, as required.

23+5: The $\widehat{h}_i$ under the limit and at the end of the sentence should be $\widehat{h}_1$.

23-6: The left hand side of (1.4.10) should be

$$\mathbb{E}\left\{\left[\eta_1^{-k}F(t,\eta_1 t') - \eta_2^{-k}F(s,\eta_2 s')\right]^2\right\}.$$

24+3: In the $\eta \neq 0$ part of the definition of $\widehat{F}$, $F(t,\eta t')$ needs to be multiplied by η^{-k}.

24+8: continuity of f → continuity of $\widehat{F}$

27-9: $2 \geq a > b$ should be $2 \geq a \geq b$.

33-7: A_δ should be $A^{(n)}$.

48+1: The proof here is incomplete. Theorem 1.4.1 only establishes that if the *entropy* (not 'energy' as written) integral $\int_\delta^\infty p(e^{-u^2/2})\,du$ is finite, then the entropy integral (1.5.16) converges. In fact, the converse is false. (i.e. There exist processes for which the entropy integral converges, but for which $\int_\delta^\infty p(e^{-u^2/2})\,du$ diverges.) Thus the proof only works in one direction.

For the other direction, argue as follows: Assuming that the left hand inequality in (1.5.18) holds for some $\alpha_1 > 0$, show that $M(\varepsilon)$ (defined in the middle of page 47) satisfies

$$M(\varepsilon) = O\left(e^{N\varepsilon^{-2/(1+\alpha_1)}}\right).$$

Then use the relationship between M and N (also in the middle of the page) to show that the entropy integral diverges, and apply Theorem 1.5.4 to complete the proof.

Chapter 2

51+3: Change $\mathbb{E}\{\|X\|\}$ to $\mathbb{E}\{\|f\|\}$.

52-3: Here and in following three displays $\sqrt{2\pi}$ should be $\sqrt{\pi/2}$.

54+12: The minus sign before the integral should be removed here and in the following line.

54-6: The assumption that h is bounded is also necessary.

55-11: Add the assumption that h is bounded.

55-11: f should be h. Also for 55-5 and 55-4.

55-4: Change to: To remove the C^2 and boundedness assumptions, take a sequence of bounded, C^2 approximations to h ....

56+3: There is an implicit assumption in this paragraph that $f_1, \ldots, f_k$ are independent, standard Gaussians.

56+5: ... from *Lemma* 2.1.6.

56+12: Change from “trivially” as follows: ... trivially Lipshitz. To compute the Lipshitz constant, note that

60-11: $(X_1,'' \ldots, X'_k)$ should be $(X'_1, \ldots, X'_k)$.

61-5: $\sigma^Y_{ij} = \mathbb{E}\{\widetilde{X}_i, \widetilde{X}_j\}$ should be $\sigma^Y_{ij} = \mathbb{E}\{\widetilde{Y}_i, \widetilde{Y}_j\}$.

CHAPTER 3

68-14: Delete Θ and the corresponding brackets from the first term.

72-7: Equals sign missing in $\lambda\psi(t) = \int_0^1 \min(s,t)\psi(s)\, ds = \ldots$

CHAPTER 4

83-12: A strange typo has appeared here. Throughout the proof, replace $\mathcal{P}_{\updownarrow}$ by $\mathcal{P}_\ell$ and $\mathcal{P}_{\updownarrow+\infty}$ by $\mathcal{P}_{\ell+1}$.

95+14: There is a factor of T^N missing on the RHS of (4.4.2).

96+6: The conditioning event should be $f(t_k) = x$ (not $= u$).

98-5: The last line of (4.6.4) is missing the term $\mathbb{P}\{M_u^E(M^\circ) \geq 1\}$, with which the iteration begins.

CHAPTER 5

110+13: $\|f\| \in L^2(\nu)$ should be $f \in L^2(\nu)$.

111+14: Replace (5.4.9) and the remainder of the sentence by $W(A) = \Theta^{-1} 1_A$, where A is a bounded Borel subset of $\mathbb{R}^N$.

113+6: The right hand side needs to be divided by $\prod_{i=1}^k h_i$.

114+1: “$\beta = \gamma = \delta = 0,\ \alpha = 1$” should be “$\alpha = \gamma = \delta = 0,\ \beta = 1$”.

114+7: $f_{jk}(t)$ should be $f_{kl}(t)$ and i, j, k in the following line should be i, l, k.

117+7: Add “up to a multiplicative constant” after “to yield”, and after the display replace “absorbing s_{N-2}” by “absorbing all constants”.

CHAPTER 6

146+20: The claim about the non-negativity of the c_j is only true if ψ is monotone *increasing.*

CHAPTER 7

162+13: “linear ... in Y” should be “linear ... in X”.

CHAPTER 8

187-1: $(-1)^{N-j}$ should be $(-1)^j$.

CHAPTER 9

211-9: (9.3.3) should be (9.3.4).

211-6: Display should read

$$\mu_i(J) \;\overset{\Delta}{=}\; \#\left\{t \in J : \nabla f_{|J}(t) = 0, \iota_{-f,J}(t) = i, f(t) \geq u, \nabla f(t) \in N_t I^N\right\}.$$

211-2: Delete “working with the definition of the C_i”.

CHAPTER 10

222+1: $\rho \leq$ should be $\rho <$.

222+3: $t \in M$ needs to be added to the definition of the region.

238+1: There is nothing wrong here, but the discussion could have been clearer. In particular, despite the last paragraph on p237, (10.5.4) is still in the most general setting $M \subset \widetilde{M} \subset \mathbb{R}^\ell$. On the other hand, (10.5.5) relates to the situation $M \subset \widetilde{M} \equiv \mathbb{R}^\ell$, and so $\dim \widetilde{M} \overset{\Delta}{=} N \equiv \ell$, which is why N disappears in (10.5.5). (It would have been more natural to write (10.5.5) with ℓ replaced by N, and it should probably be read that way.)

246-13: $l = k - j = 0$ should be $m = j - i = 0$.

249-7: $-\widetilde{R} + \kappa I^2/2$ should be $-(\widetilde{R} + \kappa I^2/2)$.

Chapter 11

265-2: whether λ_2 is finite, *or not...*

267-12: $D = N(N+1)/2 + K$ should be replaced by $D = N^2 + K$. Similarly, the integration over $\mathbb{R}^{N(N+1)/2}$ in (11.2.11) should be replaced by integration over $\mathbb{R}^{N^2}$. The same in true in (11.2.13), the three related integrals on page 272, and the one on page 275.

278+14: Condition (g) is not needed for Lemma 11.2.11. The proof requires no real changes. At 278-10, replace "it follows from (11.2.2)" by "it follows from the continuity of the f_j^i". At 279-9, replace "By assumption (g)" by "By assumption (c)".

This condition, and others like it, are needed only for proving the expectation meta-theorem, Theorem 11.2.1.

279-2: Condition (g) is not needed for Lemma 11.2.12.

280-6: The condition that all second derivatives have finite variance is not required. It appears nowhere in the proofs.

This condition, and others like it, are needed only for proving the expectation meta-theorem, Theorem 11.2.1.

281+1: Condition (d) is not required.

281+11: Add "for some $\alpha > 0$" immediately after (11.3.1).

281+18: Theorem 11.3.3 should read "Under the conditions of *Theorem 11.3.1.*

282+22: In (a) the condition $\mathbb{E}\{(XYf)^2\} < \infty$ is not required.

282-15: Condition (d) is not required.

290-7: Q must be a positive definite square root of Λ^{-1}.

299+1: Equation (11.8.1) holds in wide generality, once all the terms are properly defined. However, in the notation of Chapter 11, it only makes sense for isotropic fields with variance and second spectral moment both equal to one.

Chapter 13

333-11: The power of λ in (13.2.1) should be $(j + l)/2$.

Chapter 14

352+17: In Condition (iii) delete “For all $t \in M$”.

357+8: The term $P_L \nabla h(t)$ in the numerator should be $P_L^{\perp} \nabla h(t)$

376+5: Four (not two) applications of l’Hôpital’s rule are needed.

383-10: On the other hand, if $\pi \leq T \leq 2\pi$,...

385-19: M_N has not been defined. It is $\sup_{t \in T} f(t)$, where f is the cosine field on $\mathbb{R}^N$.

Chapter 15

394+1: The $\widetilde{y}$ that appears here is the expansion of y over an ambient manifold $\widetilde{M}$, needed for Morse theory to be applied.

398-1: All four appearences of $n/2 - 1$ should be $(n-1)/2$.

404-3: $G_{n,n^{-1}}$ should be $G_{n,\sqrt{n}}$ and $\nu_{n,n^{-1}}$ should be $\nu_{n,\sqrt{n}}$.

418+10: Delete the term $g_{\mathbb{R}^k}$.

418-13: The expression $\mathcal{L}_{n-1-i}^{1/n}(\pi_{\sqrt{n},n,k}^{-1} D,\, \widetilde{D}_{n-k-1+j})$ is a little misleading. Technically, for this to be consistent with previous notation, such as at (10.7.1), it would be necessary for $\widetilde{D}_{n-k-1+j}$ to be a subset of $\pi_{\sqrt{n},n,k}^{-1} D$ which, in view of the definition of $\widetilde{D}_{n-k-1+j}$ five lines above, is only ‘morally’ true, in terms of isometric embeddings. The notation should therefore be thought of in terms of the warped metric This should be kept in mind in all further uses of this curvature.

418-2: The second expression should read $\widetilde{\nabla}_{F_1}^{\sigma} F_2 = \nabla_{F_1}^2 F_2 - \sigma^2 g_2\,(F_1, F_2)\, \nabla \sigma^2$.

420+12: Display should read $\nu_{k-j} = \sum_{r=1}^{n-k-1} F_r$.

421+12: $\widetilde{D}_{n-k-1+j} \cap S(\mathbb{R}^k) = \emptyset$ should be $D_j \cap S_{\sqrt{n}}(\mathbb{R}^k) = \emptyset$ here, and at 423+7 and 423+9. The condition is written correctly in the statement of Lemma 15.9.2.

424+3: In order that the $\mathcal{M}_i^{\gamma}$ are actually coefficients in the tube formula, we also need the assumptions of Corollary 10.9.6 to hold. Without these, they are as implicitly defined in the last display on the page.

424-6: $(2\pi)^{-i\hat{A}\frac{1}{2}}$ should be $(2\pi)^{-i/2}$.

424-2: The integrability condition (15.9.3) needs to be added to the conditions of Theorem 15.9.4.

426-1: $\gamma_{\mathbb{R}^l}$ should be just γ (i.e. $\gamma_{\mathbb{R}}$) throughout (15.10.6)

427+5: $\gamma_{\mathbb{R}^k}$ and $\gamma_{\mathbb{R}^l}$ should be simply γ here and throughout the proof.

429-9: $e^{-x^2/2}$ should be $e^{-u^2/2}$.