Sub-Nyquist Collocated MIMO Radar in Time and Space

David Cohen, Deborah Cohen and Yonina C. Eldar

Contributions
- Application of the sub-Nyquist framework to collocated MIMO radar in time and space while preserving the range-azimuth resolution
- Low rate sampling and digital processing
- Reduced number of antennas
- Recovery algorithm scaled with problem size by adapting the OMP algorithm to matrix form

Collocated MIMO Radar
- MIMO combines multiple transmit and receiver antenna elements
- Each transmitting element radiates orthogonal waveforms
- Core idea: achieving high spatial resolution by separation and coherent processing of the receivers’ channels
- All space is uniformly lit - beamforming is done at the receiver
- Conventional processing for range-azimuth recovery:
  - Xampling

Resolution in Time and Space
- Classic approach adopts a virtual ULA structure

Resolution in the space domain
- Azimuth resolution: determined by virtual ULA’s aperture
- Problem: High resolution in the azimuth requires large number of elements

Resolution in the time domain
- Range resolution: determined by transmitted signal’s bandwidth
- Problem: High resolution in the range requires high sampling rate
- Goal: break link between number of elements and spatial resolution and sampling rate and time resolution

Proposed Array Structure and Signal Model
- Frequency division approach
- Random array
- Sub-Nyquist in space
  - Preserving azimuth resolution of T transmitters and R receivers while using M-T transmitters and Q-R receivers
  - Elements are randomly located with uniform distribution across the virtual ULA’s aperture of T transmitters and R receivers
- Sub-Nyquist in time
  - Preserving range resolution of signal with bandwidth TBh while the effective sampling rate is lower

The transmissions are performed over total bandwidth TBh.
For each transmission, sub-Nyquist sampling scheme is applied

Xampling in Time and Space
- Received signal at the q-th antenna after demodulation:
  \[ x_q(t) = \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mq}(t) \delta_\tau(t) e^{j2\pi f_m t} \]
- Goal: estimate the targets range and azimuth \( \tau, \varphi \)
- Fourier coefficients of the channel between m-th transmitter and q-th receiver:
  \[ y_{mq}[k] = \sum_{t=0}^{T} h_{mq}(t) e^{j2\pi f_m t} \]
- \( \beta_{mq} \) is governed by the elements location while \( f_m \) by the carriers frequencies
- Xampling: obtain set of Fourier coefficients from low rate samples
- Fourier coefficients for the m-th transmission in matrix form:
  \[ Y = A^* X B^* \]
- \( A \), \( B \): range-azimuth dictionaries
- \( X \): sparse matrix whose elements are located at the targets’ range-azimuth

Joint Range-Azimuth Detection
- Frequency diversity: channels are azimuth-dependent and range dependent
- Range-azimuth coupling resolved by using random array and joint range-azimuth detection
- By processing all channels together: achieve range resolution according to total bandwidth

Recovery algorithm via OMP Matrix Approach

Simulation results

Graph 1: Resolution in range and Azimuth
7 targets including couple of targets with close range and a couple with close azimuth

<table>
<thead>
<tr>
<th>Spatial Reduction</th>
<th>T=20</th>
<th>M=10</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Reduction</td>
<td>N=500</td>
<td>K=250</td>
<td>50%</td>
</tr>
<tr>
<td>Total samples reduction: 12.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph 2: Hit rate vs SNR
10 targets range-azimuth recovery performance

<table>
<thead>
<tr>
<th>Spatial Reduction</th>
<th>T=20</th>
<th>M=10</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Reduction</td>
<td>N=500</td>
<td>K=125</td>
<td>25%</td>
</tr>
<tr>
<td>K=75</td>
<td>15%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total samples reduction: 3.75%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References