Motivation and Goals

- High resolution radar requires high bandwidth signals
- Wideband signals need a complex analog front end receiver design which consumes high power
- Digital processing of wideband signals requires large memory and large computational power
- We present a sub-Nyquist sampling and recovery method implemented in hardware which reduces the rate by 30 fold
- This approach provides both simple recovery and robustness to noise by performing beamforming on the low rate samples
- Clutter rejection is also performed on the sub-Nyquist samples by adapting standard methods to our setting

**Sub-Nyquist Radar Algorithm**

\[ x(t) \xrightarrow{\text{Xampling}} \text{Clutter Filtering} \xrightarrow{\text{Doppler Focusing}} \text{Parameter Estimation} \xrightarrow{\text{Time Delay Doppler Shift Amplitude}} \]

**Xampling** - A process of sampling a signal at a low rate in such a way that preserves the required information

**Clutter Filtering** - Adaptation of standard clutter algorithms to fit our low rate samples

**Doppler Focusing** - A method of digitally beamforming the low rate samples which is both numerically efficient and robust to noise

**Estimation** - A modified OMP, matched to our samples, produces target locations and Doppler frequencies

**Input Signal Model**

- \( L \) targets, each defined by 3 degrees of freedom: amplitude \( \alpha_x \), delay \( \tau_x \), and Doppler frequency \( \nu_x \)
- After transmitting \( P \) equispaced high-bandwidth pulses \( h(t) \), the received signal:
  \[
  x(t) = \sum_{i=0}^{P-1} \sum_{k=1}^{L} \alpha_h(t - \tau_{p} - \nu_{p} \tau) e^{-j\nu_{p} \tau} \]

- This is an FRI model as \( x(t) \) is completely defined by \( 3L \) parameters

**Xampling Scheme – Acquiring Fourier Coefficients**

- The signal’s parameters are embodied in its Fourier coefficients
- Multichannel analog processing and low rate sampling scheme are used to extract spectral information for specific frequency bands:
  - Analog signal
  - BPF4
  - BPF3
  - BPF2
  - Band-pass Filter 1
  - Low rate ADC
  - Baseband down-converter

- Calculating Fourier coefficients is performed digitally after sampling

**Clutter Filtering**

- The target signal is contaminated with clutter + thermal noise:
  \[
  y(t) = x(t) + c(t) + n(t)
  \]
- Assume the clutter interference is modelled as “colored” noise - a WSS random process whose spectrum is Gaussian:
  \[
  S_c(f) = \frac{P_c}{2\pi\sigma_c^2} \exp \left[ \frac{(f - f_c)^2}{2\sigma_c^2} \right]
  \]
- Clutter + Thermal Noise autocorrelation matrix:
  \[
  M(m,n) = \left( \frac{P_c}{P_s} e^{-2\pi\sigma_c^2} \delta_{m,n} \right) + \delta_{mn}
  \]
- Filtering is performed by using the whitening matrix \( M^{-1} \) to whiten the interference and proceeding with Doppler focusing.

**Simulation Results**

- Measure performance by “hits” and RMS error
- A “hit” is a Delay-Doppler estimate in the interior of an ellipse around the true target one tenth the Nyquist Rate and at -25dB SNR, Doppler focusing achieves performance equivalent to matched filter processing sampling at the Nyquist rate
- When we concentrate the signal’s energy contents in the sampled frequencies, Doppler focusing outperforms matched filtering at Nyquist rate
- Under SNR of -16dB and 100 pulses used:
  - Without clutter filtering, only 3 out of 5 targets are detected
  - Using clutter filtering algorithm, all 5 targets are detected
- The performance of sub-Nyquist algorithm is equivalent to classic Nyquist rate processing.