Joint Spectrum Sensing and Direction of Arrival Recovery from sub-Nyquist Samples

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Contributions
- Description of a sub-Nyquist L-shape uniform linear array (ULA) system based on the modulated wideband converter (MWC) [1]
- Formulation of the joint spectrum sensing and direction of arrival (DOA) estimation from sub-Nyquist samples problem
- Derivation of sufficient conditions for perfect recovery of the carrier frequencies, DOAs and input signals
- Derivation of an ESPRIT-based joint frequency and DOA recovery algorithm that achieves the minimal derived sampling rate

Cognitive Radio (CR)
- Address the conflict between spectrum saturation and underutilization
- Grant opportunistic access to spectrum "holes" to unlicensed users
- Perform spectrum sensing task efficiently, in real-time and reliably

Sparse Multiband Signal Model
- Multiband model: M signals with max. bandwidth B and max. frequency \( f_{Nyquist} \)
- Each transmission \( s_i(t) \) is characterized by an angle of arrival (AOA) \( \theta_i \) and carrier frequency \( f_i \)

System Description
- L-shape ULA with \( N \) sensors in x axis and \( N + 1 \) sensors in z axis:
  - Phase accumulation in x axis:
    \[ \Delta \phi_{x_i}(f_i, \theta_i) = \frac{2\pi f_i \cdot \sin(\theta_i)}{c} \]
  - Phase accumulation in z axis:
    \[ \Delta \phi_{z_i}(f_i, \theta_i) = \frac{2\pi f_i \cdot \sin(\theta_i)}{c} \]
- Received signal at \( n \)th sensor in x axis:
  \[ U_n(f) = \sum_{i=1}^{N} S_i(f - f_n) e^{i\Delta \phi_{x_i}(f_n, \theta_i)} \]

Sampling Scheme
- Analog front-end: modified MWC sampling chain [1]
- In each sensor, a unique channel aliases the spectrum so that each band appears in baseband using the same mixing function:

Joint Frequency – Angle Estimation (Joint SVD ESPRIT)
- To overcome the pairing problem:
  Compute cross correlation matrices between ULAs
- Perform joint SVD on the cross-correlations to create same permutations for \( \Phi \) and \( \Psi \)
- Compute \( \theta_i \) and \( f_i \) from the paired eigenvalues:
  Reconstruct \( A(f, \theta) \) and compute \( w = A^T x \)

Simulation Results
- Compared methods:
  - PARAFAC: iterative algorithm (based on alternating least squares) [2]
  - Compressed sensing (CS) exploiting the spectrum sparsity
  - Joint SVD ESPRIT (SVD): analytic solution (as presented)

References