Talk Outline

1. Introduction
2. Location Fingerprinting Techniques
3. Single-site Localization via Multipath Fingerprinting
4. Conclusion
1 Introduction
Motivation

- People spend most of their active time indoors.
  - GPS signal is unavailable
- There is a growing interest in indoor position location.
  - Location Based Services (LBS).
- No good indoor position location technology is available yet.
  - High-accuracy, easily deployable, low-cost.

People spend 80-90% of their time indoors.
(Source: Strategy Analytics)
Location Based Services (LBS)

- Pedestrian navigation
- Location based advertising / marketing
- Asset tracking
- Location based analytics
- Public safety / emergency response (E911)
- Location based social networking / check-ins
- Location based actuation / notification
- Location based search / content
- Friends / people finding
- Geofencing/Geotaging (photos, etc)
Current Status

- The classical position location technique, like DOA, TOA, DTOA, are not valid in indoor, where LOS conditions do not exist.

- Current alternative GPS technologies do not meet the requirements of the industry, i.e., being
  - low-cost, high-accuracy, easily deployable, and without additional hardware in user equipment.

- Location Fingerprinting techniques have been recently developed to overcome these problems.
Location Fingerprinting Techniques
Location Fingerprinting (LF)

LF is based on the premise that there is a one-to-one relation between the characteristics of the received signals and the emitter location.

Signal characteristics, known as “fingerprints”, used by LF techniques are:

- Received Signal Strength (RSS),
- Channel Impulse Response (Power Delay Profile)
- Direction of Arrival (DOA),
- Time Difference of Arrival (TDOA)
- etc.
Location Fingerprinting Diagram

### Off-Line Phase: Training
- Location 1: Fingerprint 1
- Location 2: Fingerprint 2
- ... (omitted)
- Location $K-1$: Fingerprint $K-1$
- Location $K$: Fingerprint $K$

### On-Line Phase: Locating
- Multi-channel Receiver or/and Multiple Receivers
  - Baseband signals $p$
- Fingerprint Extraction
- Fingerprint Matching
- Estimated Location

- Fingerprint Database
LF Classification Techniques

LF can be considered as a pattern recognition problem and numerous machine learning techniques can be applied to this problem:

- Probabilistic Methods: Bayesian, MMSE, MDM classifiers.
- Nearest Neighbor Classifier.
- Support Vector Machine.
- Artificial Neural Networks.
Limitations of existing LF techniques

- Fingerprint instability caused by multipath and irrelevant environmental parameters.

- Fingerprint ambiguity caused by ambiguity inherent in the physical environment.

- For an acceptable accuracy, multiple overlapping base stations (BS) are required at each database point.

- The achieved accuracy, usually 5-10m, is still insufficient for LBS.
3 Single-Site Emitter Localization via Multipath Fingerprinting
Current Multipath Fingerprinting Techniques

- **Spatial fingerprint**
  - The directions-of-arrival (DOAs) of the multipath rays.
  - Requires a **wide antenna array** for good resolution.
  - Captured by the array covariance matrix.

- **Temporal fingerprint**
  - The delays and relative powers of the multipath rays.
  - Requires **wide bandwidth** for good resolution.
  - Captured by the impulse response/power delay profile.

*Developed by Wax et al., US Patents issued in 2000-2001*
Our Contributions

- A novel and powerful fingerprint based on the spatial-temporal covariance matrix was proposed
  - Exploits both the DOA and the differential-delays of the multipath signals.
  - Exploits only dominant reflections.
  - Computationally efficient.
  - Powerful similarity-profile matching criterion.

- Necessary and sufficient conditions that guarantee unique localization were presented.

- Frequency domain approach to the multipath fingerprinting using signal subspace was developed.

- Applicability of this method to localization using the array channel impulse response (CIR) or frequency response (CFR) was shown.

- The technique was tested with simulated and real data in indoor environments.

The manuscript based on this thesis was published in the IEEE Transactions on Signal Processing, January 2013.
An arbitrary array composed of \( p \) sensors receives a wideband signal \( s(t) \) impinging on the array through \( q \) reflections with delays \( \tau_1, \ldots, \tau_q \) and corresponding directions \( \theta_1, \ldots, \theta_q \).
Problem Formulation: sampled signal

The outputs of the antenna array are sampled simultaneously at $N$ times ("taps"), with an interval of $D=1/BW$ seconds:

$$x_i(t) = \sum_{k=1}^{q} \gamma_k(t) a_i(\theta_k) s(t+\ell D - \tau_k) e^{-j\omega c \Delta \tau_i(\theta_k)} + n_i(t+\ell D)$$
Problem Formulation: the data structure

The $pN \times 1$ "snapshot" vector $x(t)$, combined from $x_i(t)$ vectors $i = 1, \ldots, p$, can be expressed as

$$x(t) = Ay(t) + n(t)$$

where

$$A = \begin{bmatrix} a(\theta_1) \otimes s(t - \tau_1), \ldots, a(\theta_q) \otimes s(t - \tau_q) \end{bmatrix}$$

$\text{Span}\{A\} = \text{Spatial-temporal Signal Subspace} = \text{Fingerprint}$
Problem Formulation

The snapshot sampled at time $t$ is given by

$$x(t) = A\gamma(t) + n(t)$$

where

$$A = \left[ a(\theta_1) \otimes s(t-\tau_1), \ldots, a(\theta_q) \otimes s(t-\tau_q) \right]$$

$\text{Span}\{A\} =$ Spatial-temporal Signal Subspace = Fingerprint

and

$$a(\theta_k) = \left[ a_1(\theta_k) e^{-j\omega_c \Delta \tau_1(\theta_k)}, \ldots, a_p(\theta_k) e^{-j\omega_c \Delta \tau_p(\theta_k)} \right]^T$$

$$s(t-\tau_i) = \left[ s(t-\tau_i), \ldots, s(t+(N-1)D-\tau_i) \right]^T$$

$$\gamma(t) = \left[ \gamma_1(t), \ldots, \gamma_q(t) \right]^T$$
The Problem:
- DOA estimation of the multipath signals is computationally intensive since the multipath signals are coherent.
  - Requires multi-dimensional non-linear maximization (Ziskind-Wax (1988)).
  - The simpler one-dimensional maximization MUSIC algorithm is applicable only to uniform linear and circular arrays (Shan-Wax-Kailath (1985), Wax-Sheinvald (1994)).
- In typical scenarios the number of multipath rays may be larger than the array dimension, making all the parametric methods void.

The solution:
- Use a computationally simpler entity – the signal subspace - as the basis for the fingerprint.
The Signal Subspace Example

For a 3-antenna array, 2-ray multipath: \((\theta_1, \tau_1, \gamma_1), (\theta_2, \tau_2, \gamma_2)\)
and a single tap (space only), we have

\[
\mathbf{x}(t) = a(\theta_1) s(t - \tau_1) \gamma_1(t) + a(\theta_2) s(t - \tau_2) \gamma_2(t) + n(t)
\]

where \(a(\theta_1)\) and \(a(\theta_2)\) are the “steering vectors” of the array.

Note that snapshots \(\{\mathbf{x}(t_i)\}\)
stay in the 2-dimensional subspace spanned by
\(a(\theta_1)\) and \(a(\theta_2)\).
Assumptions

- The antenna array is sampled $M$-times at $\{t_m\}, \ m=1,\ldots, M$, forming $M$ snapshots.
- The signal is identical for all snapshots.
- The directions-of-arrival and the differential-delays of the multipath reflections are identical for all the $M$ snapshots.
- Propagation coefficients of reflections $\gamma_k$ are fixed during the snapshot, but may vary from snapshot to snapshot.
- All noise samples are assumed to be i.i.d. Gaussian random variables with $\mu=0$ and unknown variance $\sigma^2$. 
The Similarity Metric

Maximum Likelihood Estimator of the emitter location \(i\)

\[
\hat{i} = \arg \max_{A_i, i \in [1,K]} \left\{ \prod_{m=1}^{M} \frac{1}{\pi^{PN}} \frac{1}{\det[\sigma^2 I]} \cdot \exp\left( -\frac{1}{\sigma^2} \|x(t_m) - A_i \gamma(t_m)\|^2 \right) \right\}
\]

Maximization with respect to \(\Gamma = [\gamma(t_1), \ldots, \gamma(t_M)]\) and \(\sigma^2\), yields

\[
\hat{i} = \arg \max_{A_i, i \in [1,K]} \sum_{m=1}^{M} \|PA_i x(t_m)\|^2 = \arg \max_{A_i, i \in [1,K]} Tr\left\{ PA_i \hat{R} \right\}
\]

where

\[
P_A = A \left( A^H A \right)^{-1} A^H
\]

Projection Matrix

\[
\hat{R} = \frac{1}{M} \sum_{m=1}^{M} x(t_m) x^H(t_m)
\]

Sample-covariance Matrix
Estimating the Signal Subspace

The expected value of $\hat{R}$ is given by

$$R = E\left[ x(t)x^H(t) \right] = A\Sigma A^H + \sigma^2 I$$

where

$$\Sigma = E\left[ \gamma(t)\gamma^H(t) \right]$$

Assuming that the $q \times q$ matrix $\Sigma$ has full rank it can be easily verified that

$$\text{Span}\{A\} = \text{Span}\{V_q\} \quad \Rightarrow \quad \hat{P}_A = V_q \left(V_q^H V_q\right)^{-1} V_q^H$$

where $V_q = \{v_1, \ldots, v_q\}$ are eigenvectors of $R$, corresponding to the first $q$ dominant eigenvalues.
Estimating the Signal Subspace

Estimation of the projection matrix $P_i$ of the $i$-th location is carried out as follows

1. Calculate the sample-covariance matrix $\hat{R}_i$ of location $i$ by
   \[
   \hat{R} = \frac{1}{L} \sum_{l=1}^{L} x(t_l) x^H(t_l)
   \]

2. Perform an eigenvalue decomposition of $\hat{R}_i$.

3. Estimate the signal subspace dimension $\hat{q}$.

4. Select the first $\hat{q}$ eigenvectors of $\hat{R}_i$ corresponding to the signal subspace: $V_{\hat{q}} = \{v_1, \ldots, v_{\hat{q}}\}$.

5. Estimate the projection matrix by $\hat{P}_i = V_{\hat{q}} \left(V_{\hat{q}}^H V_{\hat{q}}\right)^{-1} V_{\hat{q}}^H$. 
The dimension of the signal subspace is equal to the number of impinging multipath signals \( q \).

We want to capture only dominant reflections.

**The solution**: select \( q \) as the number of large eigenvalues that contain, say, \( \alpha = 90\% \) of the signal’s energy.

\[
\hat{q} = \min_Q \text{ s.t. } \sum_{i=1}^{Q} \frac{\lambda_i}{pN} \geq \alpha \quad \text{subject to } Q = 1, 2, \ldots, pN
\]

A typical eigenvalue profile of the signal covariance matrix
**Conditions for Unique Localization**

- The M snapshots of the vector $x(t)$ taken at $t_1, \ldots, t_M$ can be expressed as

$$X = A(\Theta, T) \Gamma$$

where $X = [x(t_1), \ldots, x(t_M)]$ and $\Gamma = [\gamma(t_1), \ldots, \gamma(t_M)]$ and $\Theta = \{\theta_1, \ldots, \theta_q\}, T = \{\tau_1, \ldots, \tau_q\}$.

- An array can uniquely localize sources having $q < pN$ reflections if

$$q < \frac{pN + \text{rank}(\Gamma)}{2}$$

i.e. $X = A(\Theta, T) \Gamma \neq A(\Theta', T') \Gamma'$ for every $(\Theta', T') \neq (\Theta, T)$ and any set of $\Gamma'$.
Simulation environment

\((x_i, y_i) = (34, 66) \text{ m}\)

Antenna Array

Tx Locations

(0,0)
Similarity-Profile (SP)

Real Location $\hat{R}$

Potentially Ambiguous Location $Trace\{P_j\hat{R}\}$
Similarity-Profile Matching Criterion

- The SP of the \(i\)-th location in the database is defined by

\[
f_i = \left[ Tr\left\{ P_1 \hat{R}_i \right\}, \ldots, Tr\left\{ P_K \hat{R}_i \right\} \right]
\]

- The query SP obtained from the received signals

\[
f = \left[ Tr\left\{ P_1 \hat{R} \right\}, \ldots, Tr\left\{ P_K \hat{R} \right\} \right]
\]

- The localization is carried out by

\[
\hat{i} = \arg \min_{i \in [1,K]} \left\| f_i - f \right\|_2^2
\]
Algorithm Optimization

\[
\| f_i - f \|^2 = \| P (r_i - r) \|^2 = (r_i - r)^H P^H P (r_i - r)
\]

where

\[
(r_i - r) = \left( \text{vec} \left( \hat{R}_i \right) - \text{vec} \left( \hat{R} \right) \right)
\]

\[
P = \begin{bmatrix}
\text{vec} \left( P_1^T \right), \ldots, \text{vec} \left( P_K^T \right)
\end{bmatrix}^T
\]

Now, using the Cholesky Decomposition:

we get

\[
\hat{i} = \arg \min_{i \in [0, K]} \| G^H (r_i - r) \|^2
\]

For \( K \ll (pN)^2 \) the last formulation provides significant computational and storage savings!
Simulation Results: SP vs ML

802.11g (Wi-Fi)
8 taps (N=8)
BW=20MHz
SNR:0-60dB
Grid Step:1m
ML ≈ Maximum Likelihood
Simulation Results: subspace dimension

- 6 antennas
- BW=20MHz

![Graph showing signal subspace dimension distribution with different tap counts.](image)
Simulation Results: tap influence

6 antennas
BW=80MHz

CDF of position location error

Percent of locations (%)
Error (m)

- 8 taps
- 4 taps
- 2 taps
- 1 tap
Real Data Experiment

- Office floor $33 \times 33 \times 5 \text{m}$.
- 802.11g Wi-Fi access point with 6-antenna uniform circular array.
- $\text{BW}=20\text{MHz}$.
- Database grid step: $0.5\text{m}$
- Emitter: laptop.
Simulation Results: Real Data

6 antennas
8 taps
BW=20MHz

CDF of position location error

Percent of locations (%) vs. Error (m)

Experiment
Simulation
4 Conclusion
Summary

- We have presented a new localization method that
  - Enables single-site localization of wireless emitters in a rich multipath environments.
  - Achieves good localization accuracy of about 1m in typical indoor environments.
  - Applicable to modern communication technologies, like WLAN and 3G/4G, supported by most modern smartphones, tablets, laptops and etc.
  - Does not require new hardware in user equipment.

- The presented technique is a promising candidate for providing high quality, cost-effective and ubiquitous localization in indoor environments.
Thank You!

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It can be shown that baseband noise
\[ n(t) = n_{c,LPF}(t) + jn_{s,LPF}(t) \]
is Decoupled Circular Gaussian Process. Hence, because of independence of noise in different antennas of the array, for each time instance \[ n(t_i) = \{n_1(t_i),...,n_p(t_i)\} \]is complex jointly-Gaussian circularly symmetric random vector.

The probability density of circularly-symmetric Gaussian vectors is defined as
\[
 f_Z(z) = \frac{1}{\pi^n \det(K_Z)} \exp(-z^H K_Z^{-1} z)
\]
where
\[ Z = (Z_1,Z_2,...,Z_n)^T \]is a circularly-symmetric jointly-Gaussian complex random vector completely determined by its covariance matrix
\[ K_Z = E\left[ ZZ^H \right], \text{ where } Z^H = Z^{T*} \]

For more details, see “R. G. Gallager. Circularly-symmetric Gaussian random vectors. January 1, 2008”.