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IDENTIFICATION OF A CLASS OF BLUR FUNCTIONS FROM BLURRED AND NOISY IMAGES

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ABSTRACT

In this work a robust method for the identification of a class of point spread functions (PSF) from blurred and noisy images is presented. It is assumed that the original image is passed through a linear two-dimensional blurring system and that wideband noise is added to the observed image. Two types of blurring are considered: motion blur and out-of-focus blur.

The fact that the spectra of these blurring functions have periodic zeros is the basis of an already known blur identification method. This periodicity is manifested by discrete negative peaks or circles in the cepstral domain. The location of these impulses allows the classification of the parameters of the blur functions of the type considered. However, this method is found to be highly sensitive to noise. We propose therefore the following improvement to the above basic method. First, adding a pre-processing stage for noise reduction, using a modified spectral assumption approach, and second, applying an adaptive comb-like window (stirling) in the cepstral domain to enhance the best parameter identification.

The proposed algorithm is found to provide adequate identification of blur function parameters from noisy blurred images with signal-to-noise ratio down to 0.05 for motion blur and 0.02 for out-of-focus blur, as compared to 0.1 dB for the original method.

1. INTRODUCTION

The problem of estimating noisy images has been a difficult challenge for many years and is addressed in numerous publications. The main trends and methods in restoration are known to be found in any already "Classics" book [1].

The major part of image restoration algorithms require some knowledge about the degradation process and associated parameters. These include the classical Wiener filter restoration techniques [2], the more recent iterative restoration algorithms [3,15], and others [8,17]. The problem is that the information is not always available and that the restoration results would be highly sensitive on the blurring system model used. And on the accuracy by which its parameter are identified from the degraded image. The approaches taken in other image restoration works can be put into one of the following two categories:

a. Identification of the PSF parameters in order to use it later in one of the known restoration algorithms.

b. Interpretation of the identification procedure in the restoration algorithm.

The work of Gorodnitsky [2] who tried to identify the PSF parameters in the spectral domain, the work of Mitre and Fienup [7], for identification in the space and spectral domains, and the work of Cote [6] and Chinn [14] for identification in the log-spectral and cepstral domain fall in the first category. The works of Bradski and Perez [10] and Toet and Kuo [13] can be considered as members of the second category. The principal advantage in [10,11,12] is to assume that the degraded image can be modeled as a product of an NMA system where the pre-processing part represents the image and the linear average part represents the degradation process.

Cowie and Dorrill [8] assume a new model for the degraded image description and propose an iterative process for completed identification and restoration, thus also belonging to the second category.

The main drawback of all the above mentioned methods is the high sensitivity to additive noise. The relatively simple methods of [2] and [7] are restricted to very high signal-to-noise ratios (SNR) of the degraded image under test. In papers [8,10,12], the lower SNR required is 20dB. This was also the result of our experiments of Caunt's method [4].

In this paper we present a robust method for identifying the PSF parameters in motion and out-of-focus blurred images with additive noise, which is partly based on the approach in [5].

2. BLURRING-SYSTEM MODEL

We assume that the image acquisition process is as depicted in Fig. 1 with a linear space-variant imaging system.

Mathematically, it can be described as follows:

\[ g(x, y) = f(x, y) * h(x, y) + \eta(x, y) \]

where \( f(x, y) \) - original image, \( g(x, y) \) - observed image, \( h(x, y) \) - point-spread function of the imaging system, \( \eta(x, y) \) - additive noise.

The Fourier transform domain, equation (1) takes the form:

\[ G(u, v) = F(u, v) * H(u, v) + N(u, v) \]

2.1 Blur Functions

As noted above, two types of blur functions are dealt with in this paper: motion blur, caused by relative motion between the object and the camera along the x axis during exposure time, and out-of-focus blur, caused by modulation of the camera's f/ring having a circular aperture.

For motion blur, \( h(x, y) \) is given by a one dimensional rectangle:

\[ h(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \frac{\text{blur \ length}}{\text{blur \ length}} \\ 0 & \text{otherwise} \end{cases} \]

(3)

Where \( d \) is the "blur-length" and is proportional to the relative velocity between the camera and the object and to the film exposure time. The Fourier transform of \( h(x, y) \) in this case is:

\[ H(u, v) = \frac{\sin(\pi d u)}{\pi d u} \cdot \sin(\pi d v) \]

(4)

The amplitude of \( H(u, v) \) is characterized by periodic zeros on the u axis, which occur at:

\[ u = \pm \frac{1}{d}, \pm \frac{2}{d}, \pm \frac{3}{d}, \cdots \]

(5)

For out-of-focus blur, \( h(x, y) \) is assumed here to be given by the cylinder:

\[ h(x, y) = \begin{cases} 1 & \text{if } x^2 + y^2 < R^2 \\ 0 & \text{otherwise} \end{cases} \]

(6)

Where \( R \) is the "blur radius" and \( R \) is proportional to the extent of defocusing. The Fourier transform of \( h(x, y) \) in this case is:

\[ H(u, v) = \frac{\pi R}{u^2 + v^2 + R^2} \]

(7)

Where \( R^2 = u^2 + v^2 \) and \( R^2 \) is the first order Bessel function. The
amplitude of $H_{\text{in}}(x, y)$ is characterized by a "almost periodic" circular of radius $r$, in which $H_{\text{in}}(x, y)$ takes the value zero. This occurs at values of $r$ satisfying:

$$r = \frac{k}{1 + k^2}$$

It is seen therefore that these two types of dish are each characterized by a PSF which requires only a single parameter for its complex

determination.

2.2 Cylindrical Representation

The spectral representation of the blur function $B(x, y)$ is given by:

$$C_{\text{out}}(m, n) = | \mathcal{F}^{-1}( \log |H_{\text{in}}(x, y)|) |$$

where $\mathcal{F}^{-1}$ denotes the inverse Fourier Transform. Since $H_{\text{in}}(x, y)$ is real and even, so is $C_{\text{out}}(m, n)$. The Morlet's value, $\mu$, is equal to the distance between adjacent zeros of the blur function spectrum is $\mu = \frac{1}{2\pi}$. Representing this function by the spectral domain results in a disturbance negative pulse of distance $d = \mu$ from the origin and replica of this point at integer multiples of $d$.

(12)

For our purpose that $H_{\text{in}}(x, y)$ has "almost-periodic" circular of radius $r$ where the distances between adjacent circles is approximately $\mu(2r)$. Representation of the blur function in the spectral domain gives a distinct circle of negative amplitude with a radius of approximately $2r$ and replicas of this circle with radii which are approximately multiples of $2r$.

3. CANNON'S APPROACH

In his work [13], Cannon found out that the noise added to the input image is the main cause for the "disappearance" of some of the pixels in the spectral domain and therefore prevents their identification in the spectral domain. He proposed the following scheme:

The assumption that the image and the noise are samples of a stationary process gives (from (2))

$$P_{\text{in}}(x, y) = P_{\text{out}}(x, y) * H(x, y)^2 = P_{\text{out}}(x, y)$$

where $P_{\text{in}}(x, y)$, $P_{\text{out}}(x, y)$ and $P_{\text{in}}(x, y)$ denote the power spectra of $x, y$ and $z$, respectively.

An estimate of $P_{\text{in}}(x, y)$ can be obtained using Welch's algorithm [14]. This is done by dividing the image into sub-images, multiplying each sub-image by a 2-D Hann window function, computing its magnitude of magnitude Fourier transform (simplified periodogram), and averaging over all sub-images, resulting in

$$P_{\text{est}}(x, y) = P_{\text{out}}(x, y) \cdot |H(x, y)|^2 + P_{\text{in}}(x, y)$$

where the overhat indicates the estimation of the true power of the original power spectra.

Assuming that the noise is white, $P_{\text{in}}(x, y)$ converges (as the number of sub-images grows) to a constant which equals the noise variance. It is evident that if the noise variance is sufficiently small, complex noise can be expected to be a complex noise which is equal to the true power spectra, as explained above, and will dominate the form of $C_{\text{out}}(m, n)$ of the estimated representation of $P_{\text{out}}(x, y)$. Identification can thus be achieved by a careful examination of the (semi-dimensional) spectrum $C_{\text{out}}(m, n)$ in the spectral domain.

As was mentioned earlier, the main drawback of this algorithm is its high sensitivity to additive noise. The gain level of the paper is therefore to modify the above approach and make it more robust in presence of high level noise. The first step in our proposed algorithm is noise reduction by spectral subtraction - as explained next.

4. NOISE REDUCTION BY SPECTRAL SUBTRACTION

A detailed development and description of the spectral subtraction algorithm is given in [13]. Here we briefly present the main result. Let $x(x, y)$ denote the filtered output (noise free) image. From equation (1) takes the form:

$$g(x, y) = x(x, y) + n(x, y)$$

Elimination of the Fourier transform of $x(x, y)$, $B(x, y)$, from $C_{\text{out}}(m, n)$ by using the power spectrum method [1] results in the following formulation of the spectral subtraction algorithm [13]:

$$\hat{P}_{\text{in}}(x, y) = \frac{1}{2} \left( B(x, y) + |C_{\text{out}}(m, n)| \right)$$

(13)

where $\hat{P}_{\text{in}}(x, y)$ is a coefficient used to control the subtraction extent and is in a small constant in order to avoid numerical difficulties when taking the logarithm of (13).

The method of estimating $\hat{P}_{\text{in}}(x, y)$ is explained in Section 5. Note that in this application no use is made of the phase, as we need not remembrace the enhanced image.

Because of image nonstationarity, it is impossible to apply the spectral subtraction technique to sub-images of the given image and then recalculate the enhanced sub-images to achieve the final image. It appears therefore that the integration of this computational approach with the spectral averaging of sub-images is used in Cannon's algorithm can be extended from (2). However, the performance of this approach for blur parameters is less accurate as well as other integration approaches we examined, was found to be lower than using the spectral subtraction approach on the whole image, as described in Section 5. It should be emphasized that since the goal here is not the maximization of the image but the restoration of the blur function parameters, our judgment of the noise reduction technique applied, is based only on the ability to identify the true parameters and not on the enhancement of the image.

6. ALGORITHM DESCRIPTION

The proposed algorithm has two stages. In the first stage a form of the spectral subtraction model is employed for noise reduction. In the second stage the enhanced spectral magnitude function is transferred to the spatial domain and the identification procedure is performed using a "counter-peak" filter (not shown).

6.1 Stage I: Noise Reduction

The noise reduction procedure in this stage follows the spectral subtraction formulation given in (13) with $\hat{P}_{\text{in}}(x, y)$ the value of which is carefully selected to enhance performance, but is applied to the whole image, without dividing it into sub-images.

It is possible to get a good estimate of $\hat{P}_{\text{in}}(x, y)$. This is done here by using a "median-complement" image, defined below, to get an estimation of the noise, $\alpha(x, y)$, from which $\hat{P}_{\text{in}}(x, y)$ is computed. A median filter [13] is defined as follows:

$$\hat{P}_{\text{est}}(x, y) = \text{median}[\{x(y, z) - \alpha(x, y)\}]$$

(14)

where $\text{median}$ is the median of the central element of the rank ordered sequence $\{x(y, z)\}$.

A median-complement filter is thus defined by:

$$\hat{P}_{\text{est}}(x, y) = \text{median-complement}[\{x(y, z)\} - \alpha(x, y)]$$

Applying this procedure to the problem at hand results in:

$$\hat{P}_{\text{est}}(x, y) = \text{median-complement}[\{x(y, z)\}]$$

(15)

and we get

$$\hat{P}_{\text{in}}(x, y) = \hat{P}_{\text{est}}(x, y) - \alpha(x, y)$$

(16)
where \( F_x(\tau) \) is the Fourier transform of \( f(x, y) \). The filter used was a 2-dimensional Gaussian filter of length \( b \), i.e., \( R = 2 \).

The block diagram of this stage appears in Fig. 3a. Note that a 2-D Hanning window is used and a complex number is represented by the magnitude and phase angle.

5.2 Stage II: Blue Parameter Identification

Although it is already clear that the cepstral representation of the PSF is dominated by its form \( C_{\text{PSF}}(p, q) \) — the cepstral representation of the blurring image — the motion blur case is a distinctively different process, since the blur parameter B varies with the position of the object in the image. In the case of a stationary blur, the cepstral function is calculated as a function of the relative position of the object in the image. In the case of a moving blur, the cepstral function is calculated as a function of the relative position of the object in the image.

Thus, it is seen that the cepstral representation of the PSF is dominated by its form \( C_{\text{PSF}}(p, q) \), which is a function of the relative position of the object in the image. In the case of a stationary blur, the cepstral function is calculated as a function of the relative position of the object in the image. In the case of a moving blur, the cepstral function is calculated as a function of the relative position of the object in the image.

Since a 2-Dimensional function is usually used for identifying the blur parameter, we examined the number of computations required for the estimation of the blur parameters. We examined the number of computations required for the estimation of the blur parameters.

For the motion blur case, an attempt to apply a similar theorem in polar coordinates (to obtain \( C_{\text{PSF}}(p, q) \)) did not lead to the required reduction in the number of computations.

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6. RESULTS

An experiment was conducted in which the images were synthesized by a computer, where the extent of blur and noise corruption was completely controllable.

The algorithm tested on various types of edge blur and noise of various types of edge blur and noise of various types of edge blur and noise. For each type of blur and noise parameter, noise was added in order to determine the minimum value of input SNR (SNR) that still allowed identification of the blur parameter.

For motion blur, the parameter of blur length was given values such as 11, 19, 31, and 31 for each type of blur, which are the parameters used in the experiment. The Signal-to-Noise Ratio is defined here as the ratio of the signal and noise variance for each noise type. Some of the results obtained in the experiment appear in table 1.

Table 1: Minimum SNR Required for Blue Parameter Identification

<table>
<thead>
<tr>
<th>Blue Type</th>
<th>Distance d</th>
<th>Motion d</th>
<th>Deflections</th>
<th>Deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR (dB)</td>
<td>0.7</td>
<td>-0.4</td>
<td>3.1</td>
<td>2.8</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

The proposed algorithm combines processing in the space, spectral, and cepstral domain, taking advantage of the signal characteristics in each domain, and results in adequate identification of the blurring function parameter from blurred images with SNR of down to 0dB, for motion blur, and 3dB for zero-bulk blur. In previous work, this was achieved only with SNR of over 20dB.
REFERENCES


[Figures and equations]