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Photon-energy qubit generation by spontaneous emission in a V-type system

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Abstract

We propose a scheme for generating arbitrary pure-sate photon-energy qubits by adiabatic manipulations of a cavity-embedded v-type level system. The dynamics of the system is analyzed as well as the effect of various parameters including decoherence. Our calculations indicate that an equal coupling constant v-type system is the preferable choice. The realization of the proposed scheme is possible in Rb-based implementations—aimed at applications in fiber-optical quantum communication.

(Some figures in this article are in colour only in the electronic version)
using proper parameters of the STIRAP process:

\[ |\psi_{\text{system}}\rangle = (\alpha' |1\rangle + \beta' |3\rangle) \otimes |0_{\text{out}}, 0_{\text{out}}\rangle \]

(2)

where \([0_{\text{out}}, 0_{\text{out}}]\) indicates the photon vacuum state.

The following Jaynes–Cummings Hamiltonian [6] describes the dynamics of the system:

\[ H_{JC} = g_1 \left( \hat{a}^\dagger_1 |2\rangle \langle 1| + \hat{a}_1 |1\rangle \langle 2| \right) + g_3 \left( \hat{a}^\dagger_3 |2\rangle \langle 3| + \hat{a}_3 |3\rangle \langle 2| \right) \]

(3)

where \(\hat{a}_1^\dagger\) and \(\hat{a}_1\) are the respective photon creation and annihilation operators and \(g_{1,3}\) are the coupling constants of the corresponding transitions (figure 1). The evolution of the initial state (equation (2)) according to the Hamiltonian will result in the generation of the desired photon state at a discrete set of times \(t_k = \frac{k}{g_1} \cdot (2 \cdot k + 1)\) where \(k\) is a natural number and \(\hbar\) is Planck’s constant. The dynamics of the system is determined by two oscillation frequencies, and consequently \(k\) has to fulfil an additional condition: \(2 \cdot m + 1 = \left| g_3 / g_1 \right| \cdot (2 \cdot k + 1)\), where \(m\) is a natural number. At these times \(t_k\) the atom–photon system is described by

\[ |\psi_{\text{final}}\rangle = |2\rangle \otimes (\alpha' |\tilde{g}_1\rangle |0_{\text{out}}, 0_{\text{out}}\rangle + \beta' |\tilde{g}_3\rangle |0_{\text{out}}, 0_{\text{out}}\rangle) \]

(4)

where \(\tilde{g}_1 = g_1 / |g_1|\), \(\tilde{g}_3 = g_3 / |g_3|\) are the phases associated with the transition coupling coefficients.

The appropriate \(9 \times 9\) atom–photon density matrix \(\rho = |\psi_{\text{final}}\rangle \langle \psi_{\text{final}}|\) where the electron is described in a \(|1\rangle, |2\rangle, |3\rangle\) space and the photon in the \([0_{\text{out}}, 0_{\text{out}}]|, |1_{\text{out}}, 0_{\text{out}}\rangle, |0_{\text{out}}, 1_{\text{out}}\rangle\) space is block-diagonal at initial and final times \(\rho = \rho_{\text{at}} \otimes \rho_{\text{phot}}\), with the atom reduced matrix in the final state being simply

\[ \rho_{\text{at}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

(5)

and the reduced density matrix of the photon in the final state is

\[ \rho_{\text{phot}} = \begin{pmatrix} 0 & |\alpha' |g_1| \rangle \langle 0 \\ 0 & (\alpha' g_1^* \beta' g_3^*) \end{pmatrix} \]

(6)

with \(|g_1|^2 = |g_3|^2 = 1\), corresponding to the pure photon state

\[ \psi = \alpha |1_{\text{out}}, 0_{\text{out}}\rangle + \beta |0_{\text{out}}, 1_{\text{out}}\rangle \]

(7)

by defining \(\alpha \triangleq \alpha' \tilde{g}_1, \beta \triangleq \beta' \tilde{g}_3\).

Only the magnitude of the state amplitudes is determined by the preparation, while the relative phase is fixed by the coupling constants. To allow an arbitrary selection of the relative phase between \(\alpha_1\) and \(\alpha_2\) components of a photon, a dispersive element can be used after the photon is emitted.

The closed atom–photon system exhibits Rabi oscillations interchanging the energy between the atom and the field. However, using a switchable mirror, it is possible in principle to switch-out photons from the cavity at the desired time. This allows the switch-open timing to be the same for any amplitude relation of the initial electron state.

The evolution of the system for various coupling constants was calculated using the Hamiltonian of equation (3) (figures 2 and 3). The off-diagonal elements of the photon part of the density matrix (figures 2 and 3(a)) are related to the dynamics of the amplitudes of the two-photon frequency components, allowing us to determine a desired qubit state.

The probability of a photon-energy qubit to be emitted is given by population probability of the atomic ground state \(|2\rangle\) (figures 2 and 3(c)), the probability of measuring the emitted photon with one of the frequency components is given by the diagonal elements (figures 2 and 3(b)), while the purity of the photon state is given by the off-diagonal elements of \(\rho_{\text{phot}}\) (figures 2 and 3(a)). In the special case of equal coupling constants (figure 2), the two transitions have equal Rabi frequencies. One advantage of this situation is that in every Rabi cycle the electron populates its ground state with unity probability, in contrast to the different-coupling-constant case (figure 3) where the more complex dynamics makes these events much less frequent. Moreover, for equal coupling, measurement of the electron in the ground state (at any time in the Rabi cycle) results in the photon exactly in the desired superposition state, which is not the case for the different coupling scenario—improper timing will result in a different qubit than desired. The proposed system is feasible with different strengths; however, it is more easily implemented if equal strength can be achieved. It stems from the fact that for equal coupling, the probability of finding the electron in the ground state is path independent at all times, making this configuration flexible.

For evaluating practical realizations, we consider the effect of inaccuracies in the photon extraction time (mirror switch-off time). The time dependence of the reduced photon density matrix is

Figure 1. The proposed energy qubit generation process. (a) A \(\pi\) pulse excites the electron from the ground level \(|2\rangle\) to state \(|3\rangle\). (b) Using two lasers and an additional upper level \(|4\rangle\) a partial STIRAP process is performed on the electron. (c) The electron that is now in a coherent superposition between states \(|1\rangle\) and \(|3\rangle\) decays to state \(|2\rangle\), emitting a photon in coherent superposition of energies.
Figure 2. Time evolution of the photon part of the density matrix for equal coupling strength of the transition paths, with time in units of $\tau = g_1/\hbar$. (a) Real (solid line) and imaginary (dashed line) parts of the off-diagonal element. (b) Absolute values of the diagonal elements for the $\omega_1$ path (solid line) and the $\omega_2$ path (dashed line). (c) Probability for the population of the atomic ground state.

Figure 3. Time evolution of the photon part of the density matrix for different coupling strength of the transition paths, with time in units of $\tau = g_1/\hbar$. (a) Real (solid line) and imaginary (dashed line) parts of the off-diagonal element. (b) Absolute values of the diagonal elements for the $\omega_1$ path (solid line) and the $\omega_2$ path (dashed line). (c) Probability for the population of the atomic ground state.

\[
\rho_{\text{phot}}(t) = \begin{pmatrix}
|\alpha'|^2 \cos^2 \left( \frac{t|g_1|}{\hbar} \right) + |\beta'|^2 \cos^2 \left( \frac{t|g_2|}{\hbar} \right) & 0 & 0 \\
0 & |\alpha'|^2 \sin^2 \left( \frac{t|g_1|}{\hbar} \right) & \eta \sin \left( \frac{t|g_1|}{\hbar} \right) \sin \left( \frac{t|g_2|}{\hbar} \right) \\
0 & \eta^* \sin \left( \frac{t|g_1|}{\hbar} \right) \sin \left( \frac{t|g_2|}{\hbar} \right) & |\beta'|^2 \cdot \sin^2 \left( \frac{t}{\hbar} \cdot |g_2| \right)
\end{pmatrix}
\]

with $\eta = g_2 g_1^* \alpha'' \beta'/|g_1, g_2|$. The purity of the emitted photon at continuous time $t$ can be assessed by applying an atomic ground-state projection operator $P_i = |2\rangle \langle 2| \otimes I_{\text{phot}}$ to the total density matrix, which should, with finite probability, project it onto a one-photon state with two energy components $\tilde{\rho} = P_i \rho P_i / \text{tr}(P_i \rho)$. The photon-reduced matrix after the projection is
of the cycle. According to equation (11) with the parameters
\[ \rho_{\text{phot}}(t) \]
\[ = \mu \left( \begin{array}{ccc} 0 & 0 & |\alpha'|^2 \sin^2 \left( \frac{t}{h} |g_1| \right) \\ 0 & \eta^* \sin \left( \frac{t}{h} |g_1| \right) & \eta \sin \left( \frac{t}{h} |g_1| \right) \times |\beta|^2 \sin^2 \left( \frac{t}{h} |g_2| \right) \\ 0 & |\beta|^2 \sin^2 \left( \frac{t}{h} |g_2| \right) & 0 \end{array} \right) \]  

with \[ |\mu| = \left( |\alpha'|^2 \sin^2 \left( \frac{t}{h} |g_1| \right) + |\beta|^2 \sin^2 \left( \frac{t}{h} |g_2| \right) \right)^{-1}. \]

The emitted photon is in a pure state regardless of the emission time having \( Tr(\rho_{\text{phot}}(t)) = 1 \); however, the amplitudes of the two energy components of the photon are changed by the relative magnitude:
\[ \Delta \alpha(t) = \left| \sin \left( \frac{t}{h} |g_1| \right) \sqrt{|\mu| - 1} \right|^2 \]
\[ \Delta \beta(t) = \left| \sin \left( \frac{t}{h} |g_2| \right) \sqrt{|\mu| - 1} \right|^2 \]  

where \( \Delta \alpha(t) \sim |\alpha'(t) - |\alpha'|^2 / |\alpha'|^2|, \Delta \beta(t) \sim |\beta'(t) - |\beta'|^2 / |\beta'|^2|, \) and \( t = 0 \) is defined as the time when the electron is in the ground state with unitary probability.

An additional physical mechanism, that may deteriorate the purity of the emitted state, is decoherence. It may be included into density matrix formalism [7]:
\[ \frac{d\rho}{dt} = -i/\hbar [H, \rho] - \bar{\Gamma}, \]
\[ \bar{\Gamma}_{ij} = \gamma_{ij} \rho_{ij} \left( \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right) \]  

where \( \gamma_{ij} \) is the electron decoherence rate.

The experimental implementation of the proposal can be quite involved; however, in order to estimate the feasibility of the proposed scheme in principle, we consider a possible practical realization of the system by Rb gas cell, using the states \[ |S_{1/2}, F = 1 \rangle, |S_{1/2}, F = 2 \rangle, |P_{3/2}, F = 2 \rangle \] as the states \( |2 \rangle, |1 \rangle, |3 \rangle \) respectively in our model [8], with transition energies \( E_{1 \rightarrow 2} = h \cdot 3.77 \cdot 10^{14} \text{Hz}, E_{3 \rightarrow 2} = h \cdot 3.84 \cdot 10^{14} \text{Hz} \). For a decoherence rate of \( \gamma_{ij} \sim 1 \) MHz (worst-case scenario) \[ [11, 12] \] assuming the system’s decoherence is determined by the hyperfine splitting of the ground level, coupling energies \( g_1 \sim g_3 \sim h \cdot 0.1 \text{GHz} \) and typical photon lifetime in a single-atom laser cavity \[ [13] \] of \( \sim 1 \) ms. The long Rabi oscillation period allows us to switch open the cavity at the right time with a negligible timing error (figure 2). Several methods for optical switching at rates of \( \sim 10 \) GHz and higher have been reported \[ [14, 15] \], enabling handling a process with a period of the order \( \sim 10 \) ns with a timing precision of \( \sim 1\% \) of the cycle. According to equation (11) with the parameters of the Rb, the estimated relative error in the qubit coefficients is as low as \( \Delta \alpha(\Delta t) \sim \Delta \beta(\Delta t) \sim 10^{-4} \) for a switching time error of \( \Delta t = 0.1 \) ns. This small error is attributed to the fact that the amplitudes change slowly near the ideal coupling-out time \( (t = 0) \). The fidelity of the emitted photon state is generally affected by the fidelity of the STIRAP. Several parameters can deteriorate the fidelity such as laser detuning and pulse delay \[ [16] \]. Theoretical limits on the infidelity for optimized pulse shapes were shown to reach values much lower than those of typical Gaussian pulses \( \sim 10^{-4} \) \[ [17] \], while the highest reported experimental fidelity results are \( \sim 95\% \). Although in the microwave range, the thermally induced photon noise is significant \[ [19] \] even at very low temperatures, in the visible range relevant to our scheme the thermal photon noise is estimated at room temperature to be less than \( 10^{-10} \) photons, and it can therefore be neglected.

In conclusion, we have proposed and analyzed a scheme for generating arbitrary pure-sate photon-energy quibets, based on adiabatic manipulations of a V-type atom–photon system in a cavity. Our analysis including system dynamics and decoherence reveals several advantages of employing an equal coupling constant V-type system such as robustness and flexibility. The proposed system appears to be realizable in Rb-based configurations.

References

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