Fractional dispersion-modes in a pulsed fiber laser
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Received 13 March 2003; received in revised form 26 May 2003; accepted 26 May 2003

Abstract
We present and demonstrate pulse operation of a fiber laser at fractional dispersion modes. These modes are obtained when the applied modulation frequency of the laser is tuned to match the cavity round-trip to a fraction of the Talbot length. The laser pulse and spectrum behavior and the losses are determined by the repetition rate multiplying aspect of the fractional Talbot effect and the modulation parameters.
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PACS: 42.55.Wd; 42.60.Fc

While pulse operation in a long fiber laser with significant intra-cavity dispersion is problematic, mode-locking was shown to be possible at specific dispersion-modes (D-modes) [1]. These modes are determined by the Talbot effect, which is a pulse-train self-imaging property in a dispersion environment. When the cavity length is a multiplication of half the Talbot length, the pulse train is reproduced after every round trip, thus eliminating the dispersion effects. In this paper we study the operation of this laser at fractions of the D-modes, where the self-consistency condition does not correspond to a single round trip but to a few round trips. Here the spectral width can still be broad, as in the case of regular dispersion modes [1], in contrast to operation of this laser at modulation frequencies not corresponding to fractions of the Talbot length that results in localization in frequency, and a confined spectral width, where pulse operation is problematic [2]. The fractional Talbot resonances are reminiscent of the resonances occurs in quantum kicked rotors when the ratio between the natural rotor frequency and the driving frequency is a rational number [3].

At fractions of the Talbot length, say 1/2m of it, “pseudo-multiplication” by m of the pulse repetition rate occurs. In this case, although the intensity pulse profile is multiplied [4–6], the phases of the pulses in each period are different. Thus, real multiplication of the complex field is not received. In order to fulfill the self-consistency condition the pulse train must make several round trips of the loop, to be reproduced where the number of round-trips depends on the Talbot fraction. In a single round-trip the rate multiplication that does
not match the amplitude modulator can cause extra losses. Thus the combination of the modulation condition with the fractional Talbot effect forces the multiplied pulses to reach some steady state, in which the pulse rate is multiplied to some degree but the pulse train also retains the information of the basic Talbot frequency, defined by the cavity length. We note that the applied modulation frequency must simultaneously support mode locking (be some multiple of the longitudinal mode frequency) and the fractional dispersion modelocking (be some multiple of the longitudinal frequency must simultaneously support the cavity length. We note that the applied modulation frequency is only for long laser cavities, such as those studied here, the basic cavity resonance is only \( f_{\text{longitudinal}} = \frac{v_p}{L} \approx 4 \text{ kHz} \) (for \( L = 50 \text{ km} \)), while the fractional dispersion modes are of order of several \( \text{GHz} \). Therefore, we are dealing with very high multiples of the mode locking frequencies.

These “fractional” modes can be found by generalizing the dispersion modes so the frequency incorporates the fractional Talbot length. The \( D \)-modes are defined by laser cavities of length \( L = \frac{mz_T}{2} = \frac{m}{(2\pi|\beta_2|/f)^2} \), where \( z_T \) is the Talbot length and \( f \) is the modulation frequency. For fractions of the Talbot length the pulse train is reproduced after a few round-trips in the cavity. Therefore, fractional modes can be obtained for \( L = (m/p)z_T/2 \), where \( m \) and \( p \) are coprime integers. The modulation frequencies are given by

\[
f_{m/p} = \sqrt{\frac{m}{p}} \sqrt{\frac{1}{2\pi|\beta_2|L}},
\]

where \( f_{m/1} = \sqrt{m/2\pi|\beta_2|L} \), give the regular dispersion modes. The modulation frequency is \( f \), \( L \) is the cavity length and \( \beta_2 \) is the group velocity dispersion. For instance, for 5.1 km of fiber with high positive dispersion, \( \beta_2 \approx +146 \text{ ps}^2/\text{km} \), the first \( D \)-mode resonance is at \( f_{1/1} = 14.6 \text{ GHz} \), whereas some of the fractional modes are at \( f_{1/2} = 10.3 \text{ GHz} \), \( f_{1/3} = 8.4 \text{ GHz} \) and \( f_{1/4} = 7.3 \text{ GHz} \) which correspond to \( L = \frac{z_T}{4} \), \( L = \frac{z_T}{6} \) and \( L = \frac{z_T}{8} \), respectively. We would then expect to receive for these modulation frequencies rate multiplication by two, three and four. It is obvious that for the fractional modes where the pulse trains are multiplied after one round trip the losses are higher than those received for the regular \( f_{1/1} \) dispersion modes and strongly depend on the modulation parameters. For example, for a standard amplitude modulation, the \( f_{1/2} \) mode inherently experiences about twice the loss of the \( f_{1/1} \) dispersive mode.

The experimental system studied is shown in Fig. 1. It is a ring cavity consisting of a long length of fiber, an erbium-doped fiber amplifier and a LiNbO\(_3\) amplitude modulator. Two types of fiber were used: 75 km of regular fiber with anomalous dispersion of \( \beta_2 \approx -19.4 \text{ ps}^2/\text{km} \) for \( \lambda = 1530 \text{ nm} \) and 5.1 km of fiber with positive high group velocity dispersion \( \beta_2 \approx +146 \text{ ps}^2/\text{km} \) for \( \lambda = 1560 \text{ nm} \). The fiber with the positive group velocity dispersion ensured that no solitons formed during pulsed laser operation.

The output of the long fiber laser operating at fractions of the dispersion modes is shown in Figs. 2–5. The spectra were measured by a spectrum analyzer with 15 pm resolution and the pulses were measured by a photo-detector and digital sampling oscilloscope with 50 GHz bandwidth. We can see that the laser can develop sidebands with a reasonable broad spectral width as occurs for the regular \( D \)-modes, but with additional internal structures in the spectra and the pulse trains, due to the additional losses caused by the multiplication and the modulation. The experimental results are compared to numerical simulations for Figs. 2–4, where the simulation also takes the third order of the dispersion \( \beta_2 \) into consideration, because it is a main factor in limiting the spectral broad-
In these figures the experiments were performed with the fiber with positive high group velocity dispersion with the Talbot fractions $z_T/6, z_T/8$ and $z_T/10$ to give rate multiplications of three, four and five, respectively. As mentioned previously, for $z_T/2m$, where $m$ is an integer,
“pseudo rate multiplication” of $m$ is received. Depending on the modulation, a partial sign of the one round-trip multiplications of the pulses can be observed, as seen in the figures, where the different pulses in a period have different intensities due to the modulation character. This is due to the

Fig. 4. The same as in Fig. 2 for $f_{1/5} = 6.53$ GHz; experiment with bias 7.5 V (a) pulses and (b) spectrum and simulation (c) pulses and (d) spectrum.

Fig. 5. Experimental output and spectrum of the fractional Talbot laser of 75 km ($\beta_2 \approx -19.4$ ps$^2$/km) for $f_{1/3} = 6.08$ GHz and bias 8.03 V (a) and (b), and for $f_{2/3} = 8.6$ GHz and bias 8.1 V (c) and (d). It can be seen that both results show the same multiplication.
tradeoff necessary between the requirement for maximum bandwidth in order to support the desired multiplication and the modulation properties to achieve optimized mode locking. The time window of the amplitude modulation (determined by the modulation frequency, strength and bias) must give preference to some of the pulses in the period as opposed to others, otherwise all the pulses see equal and high losses. This mode of operation of the laser is not optimal with regard to gain and loss. In order to receive higher rate multiplications that reflects lower loss, the bias was shifted slightly from the optimal value (maximum bandwidth) to give preference to the central pulses in each period. The losses that the outer pulses in each period see is symmetrical due to the sinusoidal amplitude modulation. It can be observed in the spectra presented in Figs. 2–4 that the harmonics (sidebands) see different losses, corresponding to the losses of the different pulses of the period. The basic multiplication feature can be observed from oscillograms of the pulses. It can be seen that there is good agreement between the experimental results and the numerical simulations for the fractional \( D \)-modes presented here.

In Fig. 5 the experimental results for the fractional \( D \)-modes \( f_{1/3} \) and \( f_{2/3} \) are presented for \( L = 75 \) km of fiber with anomalous dispersion. It can be seen that there is good agreement between the pulses and the spectrum of the different experiments, where both modes give rate multiplication by three. These experiments and those in Fig. 2, with fiber of positive high group velocity dispersion, show similar results where the rate multiplication in both experiments is by three.

In conclusion, we have demonstrated the operation of the Talbot laser at fractions of the Talbot length (fractional \( D \)-modes) in an actively mode locked laser. At these modes we receive lossy rate multiplication with respect to the basic pulse repetition rate of the laser cavity determined by the Talbot length. Nevertheless, this basic multiplication feature of the pulse repetition rate with corresponding complex spectra can clearly be observed.

**Acknowledgements**

This work was partially supported by the division for research funds of the Israel ministry of science, and by the Fund for Promotion of Research at the Technion.

**References**