Adaptive Compensation of Periodic Disturbance in Permanent Magnet DC-Motor

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2001/2
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Abstract – This is an undergraduate final year term project concerning the usage of Lyapunov function synthesis to adaptively cancel out angular position dependent oscillations resulting from cogging and eccentricity in a permanent magnet DC-motor system. The project presents the reduced order model of the motor we’re concerned with and compares the sinusoidal position dependent disturbance model we use to a time dependent one. We show the noticeable effects of the disturbance in our model and the non-validity of treatment of it as a time dependent sinusoidal disturbance for inconstant angular velocity reference profiles. We demonstrate the use of an adaptive periodic disturbance compensator (AEC) derived from Lyapunov synthesis of the system and the position dependent disturbance, as depicted in [1]. The resulting AEC controllers are nonlinear and we illustrate their ability to cancel out a sinusoidal position dependent disturbances resulting in a steady state tracking error of zero. We show the use of similar synthesis by us to cancel out a disturbance constituting of two position dependent harmonic disturbances and to create a position feedback control loop vs. an angular velocity one demonstrated in [1], with satisfying results.

I Introduction

Cogging and eccentricity present in many electro-mechanical systems result in oscillatory angular position dependent disturbances. Irregularities and asymmetries on the rotation axes in rolls and bearings are the cause for eccentricity while cogging is caused by the tendency of the rotor to be locked in specific points. During operation, permanent magnet motors experience sine-shaped disturbance forces, due to cogging. Cogging forces are caused by the magnetic interaction between the permanent magnets on the stator and iron cores mounted in the coils of the translator.

Figure 1  Cogging - Tendency of the rotor to be locked at specific points results in angular position dependent oscillations
The cogging force tries to align the magnets and cores to stable positions of the translator. The cogging force and disturbances resulting from eccentricity can be approximated by a discrete some of position dependent harmonics. Entering the disturbance approximation in a feed-forward manner with equal magnitude and opposing phase is a manner by which such oscillations can be rejected. The nature of these oscillations, being position and not time dependent in motors, causes modeling of them as linear time dependent disturbances to be invalid for all but constant angular velocity reference profiles as will be demonstrated.

The main tool that will be employed by us to design the control loop is Lyapunov function synthesis. This method will be used to help the design a nonlinear observer capable of estimating the disturbance model with asymptotic convergence, and to design a globally stable controller capable of completely canceling out the disturbance in the model system. In part we reconstruct the work of Carlos Canudas de Wit and Laurent Praly presented in their article Adaptive Eccentricity Compensation [1] and later we demonstrate similar synthesis employed by us. The usage of Lyapunov function synthesis enables us to prove the validity of our results for the system model.

II Problem Definition

We are interested in canceling out angular position dependent oscillations resulting from cogging and eccentricity in a permanent magnet DC-motor system. This is required both in an angular velocity state feedback and in an angular position one. Previous analysis of the system by our supervisor yielded that oscillations are present mainly at a constant frequency of 13 oscillations per rotation and by a smaller degree at harmonics of this frequency, mainly 26 oscillations per rotation. Furthermore it led to an estimate on the magnitude of the disturbance of roughly $2 \cdot 10^{-3} \text{ Nm}$. The exact disturbance magnitude is unknown and may vary from one motor to another of the same kind. The disturbance phase is also unknown. Thus it is required that we demonstrate the ability to cancel out a fixed number of constant angular position frequency disturbances of unknown phase and magnitude in a model of the required motor. It is also required that we prove that the system is stable and that we prove that the disturbance is significantly reduced.
III Angular Position Periodic Disturbance Compensation Methods

We will try to classify some of the methods for angular position periodic disturbance compensation and part of their advantages and disadvantages in a general way using as described in [4].

**Off-line methods with a-priori knowledge**

The eccentricities are identified or measured with several methods before operation starts. When operation starts, a correction signal is fed to compensate for the eccentricity.

**Advantages:**
- No stability problem of the compensation can occur, because there is no closed control loop.
- If the eccentricity is well-known, it can be compensated completely.

**Disadvantages:**
- Essential time loss to determine the eccentricities
- Does not work, if the eccentricities change in amplitude or phase.
- Needs exact position information which can be hard to determine.

**On-line methods with identification and simultaneous compensation**

Here the eccentricities are detected during operation and compensated at the same time. For the identification, several ways are possible. Some include direct measurement of the eccentricity and therefore require additional gear. Others like the method we employ use the output signal used for closing the control loop and therefore do not require additional gear.

**Advantage:**
- No a-priori knowledge is necessary.

**Disadvantages:**
- Stability problems may occur because of the closed loop principle;
- A lag period in the identification process may be present.
- Identification may be inaccurate.
- Identification gear is may be required

**Time sinusoidal disturbance model:**

Here the periodic disturbance is treated as time sinusoidal.

**Advantages:**
- Maintaining a time linear model of the system
- Simple disturbance model

**Disadvantages:**
- The disturbance is time sinusoidal only for constant angular velocity output profiles. Compensation using a wrong disturbance model will not completely cancel the disturbance and may cause an increase in the actual disturbance.

**Angular position dependent model:**

Here the periodic disturbance is treated as an angular position dependent disturbance.

**Advantages:**
- A more true representation of the disturbance allows for better compensation.

**Disadvantages:**
- The resulting model is non-linear which is a cause for greater engineering complexity.
- Stability may be more difficult to prove.

Eccentricity and cogging which cause angular position dependent disturbances can also be compensated in physical and mechanical ways which will not be discussed here.
1 Permanent Magnet DC-Motor Model

This part introduces the reduced order model of the motor that we will work with in our project and its validity with respect to a more detailed model that we will derive first.

Variables:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_a, I_a$</td>
<td>Armature Current</td>
</tr>
<tr>
<td>$v_a, V_a$</td>
<td>Armature Voltage</td>
</tr>
<tr>
<td>$v_b, V_b$</td>
<td>Back EMF</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular Velocity</td>
</tr>
<tr>
<td>$T_d$</td>
<td>Disturbance Torque</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Load Torque</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Motor Torque</td>
</tr>
</tbody>
</table>

Parameters:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a$</td>
<td>Armature Resistance</td>
<td>1[\Omega]</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Armature Inductance</td>
<td>3[mH]</td>
</tr>
<tr>
<td>$J$</td>
<td>Total Inertia Moment</td>
<td>$1 \cdot 10^{-6}[Kg \cdot m^2]$</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Armature Constant</td>
<td>$50 \cdot 10^{-3}[Nm/A]$</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Back EMF Constant</td>
<td>$50 \cdot 10^{-3}[V \cdot \text{sec/rad}]$</td>
</tr>
<tr>
<td>$b$</td>
<td>Friction Torque</td>
<td></td>
</tr>
</tbody>
</table>

1.1 Second order differential equations of motor

Electrical part:

\[
T_m(t) = K_m \cdot i_a(t) \tag{1.1}
\]

\[
v_a(t) = R_a \cdot i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) \tag{1.2}
\]

Mechanical part:

\[
T_L(t) = J \frac{d\omega(t)}{dt} + b\omega(t)
\]

where $b$ is very small therefore neglected:

\[
T_L(t) = J \frac{d\omega(t)}{dt} \tag{1.3}
\]

and:

\[
T_L(t) = T_m(t) - T_a(t) \tag{1.4}
\]

DC-Motor scheme:

![DC-Motor scheme graphically depicting some of the variables]

Figure 1.1 DC-Motor scheme graphically depicting some of the variables
1.2 Transfer function

In s plane:

\[
T_m(s) = K_m \cdot I_a(s) \tag{1.5}
\]

\[
V_a(s) = (R_a + L_a \cdot s) \cdot I_a(s) + V_b(s) \tag{1.6}
\]

\[
T_e(s) = T_m(s) - T_d(s) = Js^2 \theta(s) \tag{1.7}
\]

Isolating \( I_a(s) \) from (1.6) we get:

\[
I_a(s) = \frac{V_a(s) - K_b \cdot \omega(s)}{R_a + L_a(s)} \tag{1.8}
\]

where \( K_b \) is the back EMF constant.

Substituting \( T_m(s) \) in (1.7) from (1.5) and then substituting \( I_a(s) \) from (1.8) results in:

\[
\theta(s) \cdot Js^2 = K_m \cdot \frac{V_a(s) - K_b \cdot \omega(s)}{R_a + L_a \cdot s} - T_d(s) \tag{1.9}
\]

Placing \( T_d(t) = 0 \) and using \( \omega(s) = s \cdot \theta(s) \) we get the required transfer function:

\[
G(s) = \frac{\omega(s)}{V_a(s)} = \frac{K_m}{Js \cdot (R_a + L_a \cdot s) + K_b \cdot K_m} \tag{1.10}
\]

1.3 Simulink Model

![Simulink Model Diagram](image)

**Figure 1.2** Non-reduced Simulink model of permanent magnet DC-Motor

![Simulation Graph](image)

**Figure 1.3** Step response of non-reduced model of permanent magnet DC-Motor
1.4 Order Reduction

For many DC motors $\tau_a = \frac{L_a}{R_a}$, the time constant of the armature, is negligible.

This produces a reduced transfer function:

$$G(s) = \frac{1}{\tau_i s}$$  \hspace{0.5cm} (1.11)

where $\tau_i = \frac{R_a J}{K_m K_a}$. We can see that the time constant of the non-reduced model [Fig 1.3] is almost equal to the reduced system’s time constant $\tau_i = 4.4\text{[msec]}$ and that the two systems’ Bode diagrams are very much alike for frequencies lower than $\omega_{BW}$ - [Fig 1.4]:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{bode_diagram.png}
\caption{Figure 1.4 Reduced and original non-reduced transfer function Bode diagram}
\end{figure}
The armature’s resistance changes from one motor to another and during motor operation due to wire heating. In addition, the armature constant $K_a$ is related to physical properties of the motor, such as magnetic field strength which can also vary from one motor to another. We add a high bandwidth current-loop control to compensate for the properties mentioned above and for the resistor-inductor relatively fast electrical dynamics:

![Diagram](image1)

**Figure 1.5** Non-reduced model with an additional high bandwidth current-loop

The current-loop operates at much higher frequency than the EMF feedback and this allows us to remove the EMF feedback and treat it as a negligible disturbance:

![Diagram](image2)

**Figure 1.6** Non-reduced model with an additional bandwidth loop and without the EMF feedback

In addition we can remove the current-loop because it operates at much higher frequency than the mechanical system. The current-loop therefore follows relatively fast the reference current and as such can be treated as a constant compared to the mechanical system dynamics we are interested in.

![Diagram](image3)

**Figure 1.7** Reduced order model of permanent magnet DC-motor
2 Linear Control Design in Frequency Domain
This part deals with the design of a simple linear controller that will allow us in later parts to
demonstrate the effect of several disturbance types on the system.

Control Requirements:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{bw} )</td>
<td>Bandwidth</td>
<td>100 [Hz]</td>
</tr>
<tr>
<td>( M_r )</td>
<td>Resonant Peak</td>
<td>2 dB</td>
</tr>
<tr>
<td>( e_{ss} )</td>
<td>Steady State Error</td>
<td>0 [rad/sec]</td>
</tr>
</tbody>
</table>

2.1 Simulink Model of DC-Servo:

![Figure 2.1 Reduced order model of permanent magnet DC-motor](image)

2.2 Design of a Simple PI Controller:
Closing the control loop with a PI controller we get the following diagram:

![Figure 2.2 Reduced order model with PI controller](image)

The corresponding input/output transfer function is:

\[
G(s) = \frac{\frac{K1}{J} + \frac{K2}{J}}{s + \frac{K1}{J} + \frac{K2}{J}}
\]  
(2.1)
Bandwidth is obtained when solving the transfer function for a –3dB output:

$$\omega_{bw} = \sqrt{2 \left( \sqrt{8 \cdot J^2 \cdot K2^2 + 4 \cdot J \cdot K1^2 \cdot K2 + K1^4 + 2 \cdot J \cdot K2 + K1^2} \right)}$$

(2.2)

To obtain the required $M_r$ and $\omega_{bw}$ we place $K2 = 0.039$ and $K1 = 5.6 \cdot 10^{-4}$. The resulting transfer function is now:

$$G(s) = \frac{560s + 39000}{s^2 + 560s + 39000}$$

(2.3)

In the pole-zero map of $G(s)$ we can see the near cancellation of the slower mechanical pole:

![Figure 2.3 Pole-zero plot of system with desired PI controller](image)

The resulting transfer’s function bode diagram is similar to a 1st order system:

![Figure 2.4 Bode diagram of system with desired PI controller](image)
2.3 Disturbance – Output Transfer Function:
From the model presented in [Fig 2.2] we obtain the following disturbance-output transfer function:

\[ P(s) = \frac{\omega(s)}{Td(s)} = \frac{-s / J}{s^2 + \frac{K_1}{J}s + \frac{K_2}{J}} \]  

(2.4)

The corresponding Bode diagram is:

![Bode Diagram](Figure 2.5 Bode diagram of disturbance-output transfer function)

We can see that the disturbance has the greatest effect at 150 – 350 [rad/sec].
At lower frequencies the disturbance is reduced due to the integrator that strongly damps and cancels lower frequencies. At frequencies higher than \( \omega_n \) the disturbance is reduced due to the form of the characteristic polynomial of \( P(s) \).
3 Simulation in Time Domain, Reaction to Angular Velocity Input Command

In this part we show the characteristics of the system model with the designed PI controller so that later we can demonstrate how it is effected by the angular position disturbance.

Model Parameters:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>Proportional Gain</td>
<td>$5.6 \cdot 10^{-4} [Nm \text{ sec/ rad}]$</td>
</tr>
<tr>
<td>K2</td>
<td>Integral Gain</td>
<td>$3.9 \cdot 10^{-2} [Nm / rad]$</td>
</tr>
</tbody>
</table>

Simulated Model:

![Reduced order model with PI controller](image)

**Figure 3.1** Reduced order model with PI controller

### 3.1 Constant Angular Velocity Input Command:

Step response for a constant angular velocity input command:

![Step response for a constant angular velocity input command](image)

**Figure 3.1** Step response for a constant angular velocity input command

We can see that when no disturbance is involved the steady state error is zero as expected.
3.2 Sinusoidal Angular Velocity Input Command:

![Figure 3.2 Sinusoidal angular velocity input command near $\omega_{BW} = 100$Hz frequency](image)

We can see that the tracking error at bandwidth frequency is very large mainly due to large phase difference (-51°) and 3dB damping near $\omega_{BW}$.

![Figure 3.3 Sinusoidal angular velocity input command near 10Hz frequency](image)

The tracking error is much smaller for a lower 10Hz frequency. Error is mainly due to phase difference and does not result from damping. This can be seen in the transfer function’s bode diagram [Fig 2.4] at a frequency of 63[rad/sec].
4 Reaction to Torque Disturbance

This part demonstrates the reaction of the model with the designed PI controller to a constant and sine torque disturbance.

4.1 Constant Torque Disturbance:

Simulated Model:

![Simulated Model Diagram](image)

**Figure 4.1** Reduced order model with PI controller and a constant torque disturbance

**Figure 4.2** Reaction to a constant torque disturbance

We can see that a constant torque disturbance is canceled out at steady state. The model is linear so we can deduce from the superposition principle that this reaction will be added to the resulting output linearly for other reference inputs.
4.2 Sinusoidal Torque Disturbance:
Simulated Model:

Figure 4.3 Reduced order model with PI controller and a sinusoidal torque disturbance

Figure 4.4 Reaction to a sinusoidal torque disturbance

This corresponds to a near maximal gain increase as is visible in [Fig 2.5] disturbance-output stiffness Bode diagram at 400[rad/sec].
4.3 Sinusoidal Torque Disturbance and Sinusoidal Angular Velocity Input Command

Simulated Model:

Figure 4.5 Sinusoidal input command and a sinusoidal torque disturbance

Figure 4.6 Tracking error for sinusoidal input command and sinusoidal disturbance

We can see that for a small reference amplitude the tracking error is mainly due to the sinusoidal disturbance.
5 Angle Dependent Periodic Torque Disturbance Effect and Compensation:

Variables:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>Angular Position</td>
</tr>
</tbody>
</table>

Model Parameters:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>Position Dependent Disturbance Amplitude</td>
<td>$2 \cdot 10^{-3}$ [$Nm$]</td>
</tr>
<tr>
<td>Phi</td>
<td>Position Dependent Disturbance Phase</td>
<td>0</td>
</tr>
</tbody>
</table>

5.1 Comparison Between Angle Dependent Periodic Disturbance and Sinusoidal Disturbance:

Simulated Model:

![Simulated Model Diagram]

**Figure 5.1** Reduced order model with PI controller and a position dependent torque disturbance

**Figure 5.2** Tracking error for position dependent disturbance and sinusoidal 100[rad/sec] input command

We can see when comparing [Fig 4.6] that the time dependent sinusoidal model of the disturbance is very different from the position dependent model for non-constant reference velocity profiles. The main frequency of the resulting disturbance depends on the input magnitude which changes for a sinusoidal input command. The resulting tracking error contains one main frequency but as is visible there are additional frequencies present.
5.2 Compensation for Known Position Dependent Disturbance:

Simulated Model:

Figure 5.3 Reduced order model with PI controller and a position dependent torque disturbance and fixed parameter disturbance compensation

5.3 System Tracking Error With Constant Compensation and Different Disturbance Parameters:

Figure 5.4 Tracking error results for different phase position dependent torque disturbance and fixed parameter disturbance compensation
Constant compensation only partially helps to reduce the disturbance for small phase variations and may increase the tracking error if the phase is opposite to the real disturbance phase.

\[ A=1 \times 10^{-3} \]
\[ A=2 \times 10^{-3} \text{ (same as Compensation)} \]
\[ A=4 \times 10^{-3} \]

**Figure 5.4** Tracking error results for different amplitude position dependent torque disturbance and fixed parameter disturbance compensation

Constant compensation only partially helps to reduce the disturbance for small disturbance torque variations and may increase disturbance if the real disturbance is smaller than half it’s size.
6 Adaptive Controller

This part presents Lyapunov function synthesis such that we finally get an adaptive eccentricity compensator capable of asymptotically reducing the oscillatory disturbance to a theoretical zero value.

Variables:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Λ</td>
<td>Disturbance Amplitude</td>
</tr>
<tr>
<td>φ</td>
<td>Disturbance Phase</td>
</tr>
</tbody>
</table>

6.1 Lyapunov Function Introduction

We present the following analysis in order to solve the problem of rejecting single frequency oscillatory position-dependent disturbances with unknown amplitude and phase.

System model:

We use the permanent magnet DC motor model that was introduced in part 1.

\[
\begin{align*}
J\dot{\omega} &= u + T_d(x) \\
\omega &= \dot{x}
\end{align*}
\]  

(6.1)

Disturbance model:

We use a constant position frequency of 13 as the major disturbance frequency present in our motor.

\[
T_d(x) = \Lambda \cos(13x + \phi)
\]  

(6.2)

where \( \phi \) and \( \Lambda \) are unknown.

Internal model for \( T_d \):

Let \( z = [z1, z2]^T \), be defined as:

\[
\begin{align*}
z1 &= \Lambda \cos(13x + \phi) \\
z2 &= -\frac{\Lambda}{13} \sin(13x + \phi)
\end{align*}
\]  

(6.3)

this gives the following state space representation for \( T_d \):

\[
\begin{align*}
\dot{z}_1 &= \omega \cdot 13^2 z_2 \\
\dot{z}_2 &= -\omega \cdot z_1 \\
T_d &= z_1
\end{align*}
\]  

(6.4)

Observer structure:

The following observer structure is introduced where \( f_\omega(\cdot) \) are open functions later to be defined as to fit the requirements of a negative semi-definite Lyapunov derivative:

\[
\begin{align*}
J\dot{\hat{\omega}} &= u + \hat{z}_1 + f_\omega(\cdot) \\
\dot{\hat{z}}_1 &= \omega \cdot 13^2 \cdot \hat{z}_2 + f_{z_1}(\cdot) \\
\dot{\hat{z}}_2 &= -\omega \cdot \hat{z}_1 + f_{z_2}(\cdot)
\end{align*}
\]  

(6.5)

where \( u \) is the control input to be designed.
Error equations:
We introduce the following error definitions:
\[
\hat{\omega} = \dot{\omega} - \omega \\
e = \omega_d - \omega \\
\hat{z}_1 = \dot{z}_1 - z_1 \\
\hat{z}_2 = \dot{z}_2 - z_2
\] (6.6)
The closed-loop error equations are therefore:
\[
J\hat{\omega} = \hat{z}_1 + f_w(\cdot) \\
J\dot{e} = J\omega_d - u - z_1 \\
\hat{z}_1 = \omega \cdot 13^2 \cdot \hat{z}_2 + f_{z_1}(\cdot) \\
\hat{z}_2 = -\omega \cdot \hat{z}_1 + f_{z_2}(\cdot)
\] (6.7)

Lyapunov function:
We introduce Lyapunov function \( V \) as:
\[
V = \frac{1}{2} \left[ k_1(\hat{\omega}^2 + p \cdot e^2) + \hat{z}_1^2 + 13^2 \hat{z}_2^2 \right] \] (6.8)
where \( p \) distinguishes between an adaptive eccentricity predictor if \( p=0 \), or if the observer is to be used in closed loop as an eccentricity compensator \( p=1 \).

This gives the following Lyapunov derivative and leads to the following synthesis:
\[
\dot{V} = k_1 \hat{\omega} [\hat{z}_1 + f_w(\cdot)] + k_1 p \cdot [J\omega_d - u - z_1] + \hat{z}_1 [\omega \cdot 13^2 \cdot \hat{z}_2 + f_{z_1}(\cdot)] + 13^2 \hat{z}_2 [\omega \cdot \hat{z}_1 + f_{z_2}(\cdot)] \Rightarrow \\
\dot{V} = k_1 \hat{\omega} [\hat{z}_1 + f_w(\cdot)] + k_1 p \cdot [J\omega_d - u - z_1] + \hat{z}_1 f_{z_1}(\cdot) + 13^2 \hat{z}_2 f_{z_2}(\cdot)
\]
Choosing \( f_{z_1}(\cdot) = -k_1 \hat{\omega} + f_{\omega}^*(\cdot) \) we get:
\[
\dot{V} = k_1 \hat{\omega} f_{\omega}(\cdot) + k_1 p \cdot [J\omega_d - u - z_1] + \hat{z}_1 f_{z_1}(\cdot) + 13^2 \hat{z}_2 f_{z_2}(\cdot)
\]
Choosing \( f_{\omega}(\cdot) = -k_0 \hat{\omega} + f_{\omega}^*(\cdot) \) we get:
\[
\dot{V} = -k_1 k_0 \hat{\omega}^2 + k_1 \hat{\omega} f_{\omega}^*(\cdot) + k_1 p \cdot [J\omega_d - u - z_1] + \hat{z}_1 f_{z_1}(\cdot) + 13^2 \hat{z}_2 f_{z_2}(\cdot)
\]
Choosing \( u = J\omega_d - p \hat{z}_1 + u^* \) and choosing \( f_{z_1}(\cdot) = -k_1 p e + f_{z_1}^*(\cdot) \) we get:
\[
\dot{V} = -k_1 k_0 \hat{\omega}^2 + k_1 \hat{\omega} f_{\omega}^*(\cdot) - k_1 p e u^* + \hat{z}_1 f_{z_1}^*(\cdot) + 13^2 \hat{z}_2 f_{z_2}(\cdot)
\]
Choosing \( f_{\omega}^*(\cdot) = 0 \) and \( u^* = k_2 e + u^* \) we get:
\[
\dot{V} = -k_1 k_0 \hat{\omega}^2 - k_1 k_2 p e^2 - k_1 p e u^* + \hat{z}_1 f_{z_1}^*(\cdot) + 13^2 \hat{z}_2 f_{z_2}(\cdot)
\]
Finally choosing \( u^* = 0 \) and \( f_{z_1}^*(\cdot) = 0 \) and \( f_{z_2} = 0 \) we get:
\[
\dot{V} = -k_1 k_0 \hat{\omega}^2 - k_1 k_2 p e^2
\]
It can be deduced from the observer model that when \( \hat{\omega} \to 0 \) and \( e \to 0 \) we get \( \hat{z}_1 \to 0 \) and \( \hat{z}_2 \to 0 \).

Summary:
The previous synthesis results in the following observer structure
\[
J\hat{\omega} = u + \hat{z}_1 - k_0 (\hat{\omega} - \omega) \\
\hat{z}_1 = \omega \cdot 13^2 \cdot \hat{z}_2 - k_1 (\hat{\omega} - \omega) - k_1 p (\omega_d - \omega) \] (6.10)
\[
\hat{z}_2 = -\omega \cdot \hat{z}_1
\]
And in the following control law:
\[
u = J\omega_d - k_1 (\omega - \omega_d) - p \cdot \hat{z}_1 \] (6.11)
where \( p \) is used to select between two operation modes, \( p=0 \) for eccentricity prediction or \( p=1 \) for compensation.
6.2 System and Observer Simulation Model:

Figure 6.1 Model of permanent magnet DC motor with an AEC controller and a position dependent disturbance

Figure 6.2 A model of the adaptive eccentricity compensator
7 Adaptive Controller Simulation

This part presents the results of a simulation done using the model constructed in part 6.

Model Parameters:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_v$</td>
<td>Controller Gain</td>
<td>$628 \cdot 10^{-6} [Nm \cdot sec/ rad]$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Observer Constant</td>
<td>5</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Observer Constant</td>
<td>1</td>
</tr>
<tr>
<td>$W$</td>
<td>Position dependent frequency</td>
<td>13</td>
</tr>
</tbody>
</table>

Simulation of the model preformed with a desired angular velocity profile $\omega_d = 10 \sin(100/13t)$.
Compensation begins at $t=0.5[sec]$ while prediction is active beginning at $t=0$.

![Adaptive Eccentricity Compensation - $W_d = 10 \cdot \sin(100/13)t$](image)

**Figure 7.1** Tracking error for adaptive eccentricity compensator

The AEC results in complete canceling of the disturbance in the model system.
Again the nature of the disturbance as not a time sinusoidal disturbance is visible.
Figure 7.2 Desired and Actual Angular Velocity for adaptive eccentricity compensator

The great effect of the disturbance on the actual angular velocity compared to the desired one is visible clearly in a graph presenting them both. We can see the complete cancellation of the disturbance after compensation begins at $t=0.5$[sec].

Figure 7.3 Predicted and actual disturbance using an adaptive eccentricity compensator

We can see that the disturbance observer $\tilde{z}_1$ converges to the actual disturbance before compensation begins but does not do so completely yet. After compensation begins the observer fully matches the actual disturbance.
8 Angular Position State Feedback Design
This part presents Lyapunov function synthesis similar to the one accomplished in chapter 6 using an angular position state feedback with a goal of achieving a theoretical position tracking error of zero.

8.1 Lyapunov Function Introduction
We present the following analysis in order to solve the problem of rejecting single frequency oscillatory position-dependent disturbances with unknown amplitude and phase in an angular position state feedback. We use the Lyapunov function developed in part 6 as the basis for a new function that includes angular position observer error and angular position tracking error.

System model:
We use the permanent magnet DC motor model that was introduced in part 1.

\[
\begin{align*}
J\dot{\omega} &= u + T_d(x) \\
\omega &= \dot{x}
\end{align*}
\]  

(8.1)

Disturbance model:
We use a constant position frequency of 13 as the major disturbance frequency present in our motor.

\[
T_d(x) = \Lambda \cos(13x + \phi)
\]  

(8.2)

where \( \phi \) and \( \Lambda \) are unknown.

Internal model for \( T_d \):
Let \( z = [z_1, z_2]^T \) be defined as:

\[
\begin{align*}
z_1 &= \Lambda \cos(13x + \phi) \\
z_2 &= -\frac{\Lambda}{13} \sin(13x + \phi)
\end{align*}
\]  

(8.3)

this gives the following state space representation for \( T_d \):

\[
\begin{align*}
\dot{z}_1 &= \omega \cdot 13^2 z_2 \\
\dot{z}_2 &= -\omega \cdot z_1
\end{align*}
\]  

(8.4)

Error equations:
We use the following error definitions as defined in part 6:

\[
\begin{align*}
\tilde{\omega} &= \dot{\omega} - \omega \\
e &= \omega - \omega \\
\tilde{z}_1 &= \dot{z}_1 - z_1 \\
\tilde{z}_2 &= \dot{z}_2 - z_2
\end{align*}
\]  

(8.5)

and we include the next definitions for angular position observer error and angular position error:

\[
\begin{align*}
\tilde{x} &= \dot{x} - x \\
e_s &= x_d - x
\end{align*}
\]  

(8.6)
Observer structure:
To the observer structure in part 6 we include in this part an angular position observer:

\[
\begin{align*}
J_\dot{\omega} &= u + \dot{z}_1 - k_s(\dot{\omega} - \omega) \\
\dot{z}_1 &= \omega \cdot 13^2 \cdot \dot{z}_2 - k_i(\dot{\omega} - \omega) - k_l p(\omega_d - \omega) \\
\dot{z}_2 &= -\omega \cdot \dot{z}_1 \\
\dot{x} &= f_s(\cdot)
\end{align*}
\] (8.6)

Lyapunov function:
We introduce Lyapunov function \( V^* \) as:

\[
V^* = V + \frac{1}{2} \begin{bmatrix}
\hat{x} \\ e_s
\end{bmatrix}^T \begin{bmatrix}
\frac{1}{\sqrt{\alpha \cdot p}} & \beta \cdot p
\end{bmatrix} \begin{bmatrix}
\hat{x} \\ e_s
\end{bmatrix}
\] (8.7)

where \( V \) is Lyapunov function used in chapter 6:

\[
V = \frac{1}{2} \left[ Jk_4 (\ddot{\omega}^2 + p \cdot e^2) + \dot{z}_1^2 + 13^2 \ddot{z}_2^2 \right]
\]

and \( \alpha, \beta \) are constants later to be defined satisfying \( \beta > \alpha > 0 \) to maintain \( V^* \) positive definite.

We use the following analysis such that the resulting Lyapunov derivative will be negative definite with respect to \( \ddot{x} \) and \( e_s \), and remain so with respect to \( \dot{\omega} \) and \( e \).

We designate \( u^{**} \) to the additional control output we intend to add.

\[
\begin{align*}
\dot{V}^* &= \dot{V} + \dddot{x} + p\sqrt{\alpha \dddot{x}} \dddot{e} + p\sqrt{\alpha \dddot{x}} e + p\dddot{e} \dddot{e} \\
\dot{V}^* &= -k_s k_4 \dddot{\omega} - k_i k_s \dddot{e} - k_i \dddot{e} \dddot{u} \dddot{u} + \dddot{x} f_s(\cdot) - \omega + p\sqrt{\alpha \dddot{x}} e + p\dddot{e} \dddot{e} \\
\text{choosing } f_s(\cdot) = \omega + f^*_s(\cdot) \text{ we get:} \\
\dot{V}^* &= -k_s k_4 \dddot{\omega} - k_i k_s \dddot{e} - k_i \dddot{e} \dddot{u} \dddot{u} + \dddot{x} f^*_s + p\sqrt{\alpha \dddot{x}} e + p\sqrt{\alpha f^*_s} e + p\dddot{e} \dddot{e} \\
\text{choosing } u^{**}(\cdot) = \frac{\sqrt{\alpha}}{k_1} \dddot{x} + \frac{\beta}{k_1} e + u^{***}(\cdot) \text{ we get:} \\
\dot{V}^* &= -k_s k_4 \dddot{\omega} - k_i k_s \dddot{e} - k_i \dddot{e} \dddot{u} \dddot{u} + (\dddot{x} + p\sqrt{\alpha e}) f^*_s \\
\text{choosing } f^*_s(\cdot) = -k_x \begin{cases}
\dddot{x} & \dddot{x} \geq p\sqrt{\alpha e} \\
\frac{\sqrt{\alpha e}}{\dddot{x}} & \dddot{x} < p\sqrt{\alpha e}
\end{cases} + f^*_s(\cdot) \text{ we get:} \\
\dot{V}^* &= -k_s k_4 \dddot{\omega} - k_i k_s \dddot{e} - k_i \dddot{e} \dddot{u} \dddot{u} + \begin{cases}
\dddot{x} (\dddot{x} + p\sqrt{\alpha e}) & \dddot{x} \geq p\sqrt{\alpha e} \\
\frac{\dddot{x}^2}{\dddot{x}} & \dddot{x} < p\sqrt{\alpha e}
\end{cases} + k_i \dddot{e} \dddot{u} \dddot{u} + (\dddot{x} + p\sqrt{\alpha e}) f^*_s \\
\text{finally choosing the remaining open functions to be zero we get:} \\
\dot{V}^* &= -k_s k_4 \dddot{\omega} - k_i k_s \dddot{e} - k_i \dddot{e} \dddot{u} \dddot{u} + \begin{cases}
\dddot{x} (\dddot{x} + p\sqrt{\alpha e}) & \dddot{x} \geq p\sqrt{\alpha e} \\
\frac{\dddot{x}^2}{\dddot{x}} & \dddot{x} < p\sqrt{\alpha e}
\end{cases}
\end{align*}
\]

When \( x = \sqrt{\alpha e} \) and compensation is set then from the observer \( \dddot{x} \) model we can deduce it converges for each of the two control modes and we get a smaller \( \dddot{x} \) and subsequently surly a smaller angular position tracking error \( e_s \). For solutions that include both control modes analysis was not carried out.
Summary:
The previous analysis results in the following observer structure

\[
\begin{align*}
J\dot{\omega} &= u + \dot{z}_1 - k_0 (\dot{\omega} - \omega) \\
\dot{z}_1 &= \omega \cdot 13^2 \cdot \dot{z}_2 - k_1 (\dot{\omega} - \omega) - k_i p (\omega_d - \omega) \\
\dot{z}_2 &= -\omega \cdot \dot{z}_1 \\
\dot{\tilde{x}} &= \omega - k_2 \cdot \begin{cases} 
\tilde{x} & |\tilde{x}| \geq p\sqrt{\alpha e_x} \\
\tilde{x} + p\alpha e_x & |\tilde{x}| < p\sqrt{\alpha e_x}
\end{cases}
\end{align*}
\]  
(8.9)

And in the following control law:

\[
u = J\dot{\omega}_d - k_4 (\omega - \omega_d) - p \cdot \dot{z}_1 + \frac{\sqrt{\alpha}}{k_1} \tilde{x} + \frac{\beta}{k_1} e_x
\]  
(8.10)

where \( p \) is used to select between two operation modes, \( p=0 \) for eccentricity prediction or \( p=1 \) for compensation. The constants must be positive and satisfy \( \beta > \alpha \).
8.2 System and Observer Simulation Model:

Figure 8.1 AEC simulink model for an angular position state feedback

Figure 8.2 AEC simulink model for an angular position state feedback
8.3 Adaptive Controller Simulation

Model Parameters:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_v$</td>
<td>Controller Gain</td>
<td>$628 \cdot 10^{-6} [Nm \cdot sec/\text{rad}]$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Observer Constant</td>
<td>10</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Observer Constant</td>
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<tr>
<td>$k_2$</td>
<td>Observer Constant</td>
<td>360</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Controller Gain</td>
<td>$200 [Nm/\text{rad}]$</td>
</tr>
<tr>
<td>$\sqrt{\alpha}$</td>
<td>Controller Gain</td>
<td>$10 [Nm/\text{rad}]$</td>
</tr>
</tbody>
</table>

![Graph](image_url)

**Figure 8.3** Angular position tracking error for adaptive eccentricity compensator

The error is reduced significantly by approximately 40dB, 3 seconds after compensation begins at $t=2[\text{sec}]$. We cannot see complete rejection of the disturbance and it is not clear whether this is due to numerical error, or analytical result of perhaps a phase delay.
Prediction is accomplished up to a magnitude of roughly 40dB. When compensation begins at t=2[sec] the error increases due to a sudden change in the system at that moment by the compensator.
We can see that there remains an oscillatory error in the prediction. The error is perhaps due to a phase delay present or due to numeric calculations. An increase in the prediction error is present when compensation begins and is due to the immediate change in the system at the compensation switching moment at $t=2$[sec].
9 Position Dependent Disturbance Consisting of Two Harmonics

In this part we deal with a position dependent disturbance consisting of two harmonics.

Variables:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_A, \Lambda_B )</td>
<td>Disturbance Amplitudes</td>
</tr>
<tr>
<td>( \phi_A, \phi_B )</td>
<td>Disturbance Phases</td>
</tr>
<tr>
<td>( W_A, W_B )</td>
<td>Disturbance Angular Position</td>
</tr>
<tr>
<td></td>
<td>Frequencies</td>
</tr>
</tbody>
</table>

9.1 Lyapunov Function Introduction

We present the following analysis in order to solve the problem of rejecting single frequency oscillatory position-dependent disturbances with unknown amplitude and phase.

System model:
We use the permanent magnet DC motor model that was introduced in part 1.

\[
J \ddot{\omega} = u + T_d(x) \\
\dot{\omega} = \dot{x} \tag{9.1}
\]

Disturbance model:
We use a constant position frequency of 13 as the major disturbance frequency present in our motor and an additional frequency of 26 as a harmonic of this frequency present.

\[
T_d(x) = \Lambda_A \cos(13x + \phi_A) + \Lambda_B \cos(26x + \phi_B) \tag{9.2}
\]

where \( \phi_A, \phi_B \) and \( \Lambda_A, \Lambda_B \) are unknown.

Internal model for \( T_d \):
Let \( z = [z_{1A}, z_{2A}, z_{1B}, z_{2B}]^T \), be defined as:

\[
\begin{align*}
    z_{1A} &= \Lambda_A \cos(13x + \phi_A) \\
    z_{2A} &= -\frac{\Lambda_A}{13} \sin(13x + \phi_A) \\
    z_{1B} &= \Lambda_B \cos(26x + \phi_B) \\
    z_{2B} &= -\frac{\Lambda_B}{26} \sin(26x + \phi_B)
\end{align*}
\tag{9.3}
\]

this gives the following state space representation for \( T_d \):

\[
\begin{align*}
    \dot{z}_{1A} &= \omega \cdot 13^2 z_{2A} \\
    \dot{z}_{2A} &= -\omega \cdot z_{1A} \\
    \dot{z}_{1B} &= \omega \cdot 26^2 z_{2B} \\
    \dot{z}_{2B} &= -\omega \cdot z_{1B} \\
    T_d &= z_{1A} + z_{1B}
\end{align*} \tag{9.4}
\]
Observer structure:
We shall use the following observer structure designed in a similar way to the one designed in part 6.

\[
J \dot{\omega} = u + \hat{z}_{1A} + \hat{z}_{1B} - k_0 (\dot{\omega} - \omega) \\
\dot{\hat{z}}_{1A} = \omega \cdot 13^2 \cdot \hat{z}_{2A} - k_1 (\dot{\omega} - \omega) - k_1 p(\omega_d - \omega) \\
\dot{\hat{z}}_{2A} = -\omega \cdot \dot{\hat{z}}_{1A} \\
\dot{\hat{z}}_{1B} = \omega \cdot 26^2 \cdot \hat{z}_{2B} - k_1 (\dot{\omega} - \omega) - k_1 p(\omega_d - \omega) \\
\dot{\hat{z}}_{2B} = -\omega \cdot \dot{\hat{z}}_{1B}
\]  
(9.5)

where \( u \) is the control input to be shortly introduced.

Error equations:
We introduce the following error definitions:

\[
\tilde{\omega} = \dot{\omega} - \omega \\
e = \omega_d - \omega \\
\tilde{z}_{1A} = \hat{z}_{1A} - z_{1A} \\
\tilde{z}_{2A} = \hat{z}_{2A} - z_{2A} \\
\tilde{z}_{1B} = \hat{z}_{1B} - z_{1B} \\
\tilde{z}_{2B} = \hat{z}_{2B} - z_{2B}
\]  
(9.6)

The closed-loop error equations are therefore:

\[
J \dot{\tilde{\omega}} = \tilde{z}_{1A} + \tilde{z}_{1B} - k_0 (\dot{\omega} - \omega) \\
J \dot{e} = J \dot{\omega}_d - u - \tilde{z}_{1B} \\
\dot{\tilde{z}}_{1A} = \omega \cdot 13^2 \cdot \tilde{z}_{2A} - k_1 A (\dot{\omega} - \omega) - k_1 p(\omega_d - \omega) \\
\dot{\tilde{z}}_{2A} = -\omega \cdot \tilde{z}_{1A} \\
\dot{\tilde{z}}_{1B} = \omega \cdot 26^2 \cdot \tilde{z}_{2B} - k_1 B (\dot{\omega} - \omega) - k_1 p(\omega_d - \omega) \\
\dot{\tilde{z}}_{2B} = -\omega \cdot \tilde{z}_{1B}
\]  
(9.7)
Control Law:
We shall use the following control law:
\[ u = J\dot{\omega_d} - k_e (\omega - \omega_d) - p \cdot \dot{z}_1 - p \cdot \dot{z}_2 \]  \hspace{1cm} (9.8)
where \( p \) is used to select between two operation modes, \( p=0 \) for eccentricity prediction or \( p=1 \) for compensation.

Lyapunov function:
We introduce Lyapunov function \( V \) as:
\[ V = \frac{1}{2} \left[ J(k_{1A} + k_{1B})(\dot{\omega}_2^2 + p \cdot e_2^2) + \frac{k_{1A} + k_{1B}}{k_{1A}} \left[ \ddot{z}_{1A}^2 + 13^2 \ddot{z}_{2A}^2 \right] + \frac{k_{1A} + k_{1B}}{k_{1B}} \left[ \ddot{z}_{1B}^2 + 13^2 \ddot{z}_{2B}^2 \right] \right] \hspace{1cm} (9.9)\]
where \( p \) distinguishes between an adaptive eccentricity predictor if \( p=0 \), or if the observer is to be used in closed loop as an eccentricity compensator \( p=1 \).

This gives the following Lyapunov derivative:
\[ \dot{V} = J(k_{1A} + k_{1B})(\ddot{\omega}_2 + p \cdot e_2) + \frac{k_{1A} + k_{1B}}{k_{1A}} \left[ \dot{\ddot{z}}_{1A} + 13^2 \dot{\ddot{z}}_{2A} \right] + \frac{k_{1A} + k_{1B}}{k_{1B}} \left[ \ddot{z}_{1B} + 13^2 \ddot{z}_{2B} \right] \Rightarrow \]
\[ \dot{V} = (k_{1A} + k_{1B})(\ddot{\omega}_2 + p \cdot e_2) + \frac{k_{1A} + k_{1B}}{k_{1A}} \left[ \dot{\ddot{z}}_{1A} + 13^2 \dot{\ddot{z}}_{2A} - k_{1A}(\hat{\omega} - \omega) + p \left[ J\dot{\omega}_d - J\dot{\omega}_d - k_e (\omega - \omega_d) - p \cdot \dot{z}_1 - p \cdot \dot{z}_2 \right] - \dot{z}_{1A} - \dot{z}_{1B} \right] + \]
\[ + \frac{k_{1A} + k_{1B}}{k_{1A}} \left[ \dot{\ddot{z}}_{1A} + 13^2 \dot{\ddot{z}}_{2A} - k_{1A}(\hat{\omega} - \omega) - k_{1A} p(\omega_d - \omega) \right] + 13^2 \ddot{z}_{2A} \left[ - \omega \cdot \dot{z}_{1A} \right] \]
\[ \Rightarrow \dot{V} = (k_{1A} + k_{1B}) \left[ \ddot{\omega}_2 + p \cdot e_2 \right] \hspace{1cm} (9.10)\]
It can be deduced from the observer model that when \( \hat{\omega} \to 0 \) and \( e \to 0 \) we get \( \ddot{z}_{1A}, \ddot{z}_{1B} \to 0 \) and \( \ddot{z}_{2A}, \ddot{z}_{2B} \to 0 \).
9.2 System and Observer Simulation Model:

Figure 9.1 Model of permanent magnet DC motor with an AEC controller and a position dependent disturbance constituting of two frequencies

Figure 9.2 A model of the adaptive eccentricity compensator for two angular position disturbances
9.3 Adaptive Controller Simulation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{1A}, k_{1B}$</td>
<td>Observer Constant</td>
<td>5</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Observer Constant</td>
<td>1</td>
</tr>
<tr>
<td>$W_A, W_B$</td>
<td>Position dependent frequency</td>
<td>13,26</td>
</tr>
<tr>
<td>$\phi_A, \phi_B$</td>
<td>Disturbance Phases</td>
<td>$0[\text{rad}], \frac{\pi}{3}[\text{rad}]$</td>
</tr>
<tr>
<td>$\Lambda_A, \Lambda_B$</td>
<td>Disturbance Amplitudes</td>
<td>$2 \cdot 10^{-4}[\text{Nm}], 5 \cdot 10^{-4}[\text{Nm}]$</td>
</tr>
</tbody>
</table>

Simulation of the model performed with a desired angular velocity profile $\omega_d = 10 \sin(100/13 t)$. Compensation begins at $t=0.5[\text{sec}]$ while prediction is active beginning at $t=0$.

![Image of Figure 9.1](image.png)

**Figure 9.1** Tracking error for adaptive eccentricity compensation of two harmonic position dependent disturbances

The AEC results in complete cancellation of the disturbance in the model system. The compensation takes slightly longer for two harmonic position dependent disturbances than for one using the same constants and the magnitude of the total disturbance is larger.
Figure 9.2 Desired and Actual Angular Velocity for adaptive eccentricity compensator

The great effect of the disturbance on the actual angular velocity compared to the desired one is visible clearly in a graph presenting them both. We can see the complete cancellation of the disturbance after compensation begins at $t=0.5[\text{sec}]$.

Figure 9.3 Predicted and actual disturbance using an adaptive eccentricity compensator

We can see that the disturbance observers $\hat{\tilde{z}}_{iA} + \hat{\tilde{z}}_{iB}$ converge to the actual disturbance before compensation begins but do not do so completely yet. After compensation begins the observers fully matches the actual disturbance.
10 Summary

Throughout this project we have tackled the problem of periodic disturbance compensation in a DC motor.

A common approach was to consider the disturbance as time dependent maintaining an LTI model of the system. In that framework, compensation worked for only constant angular velocity profiles, as we showed. Previous knowledge of the disturbance profile allowed us to use inverse dynamics and cancel it, yet this advantage is partial, since a small variation in the disturbance (e.g. a phase change) turned out to be counterproductive. So, although disturbance compensation in this manner is theoretically possible, in fact it is not a good engineering approach.

Then, and as the main purpose of our project, we treated the torque disturbance as a position-dependent periodic function, which seems to fit more accurately to reality, and led to a non-linear model of the system. An adaptive controller based on disturbance estimation was used, and the results consist of an estimation period first and a compensation period later.

First, the velocity controller was designed. Lyapunov analysis yielded the asymptotic stability of velocity tracking and estimations (velocity and disturbance) errors. After that the solution was extended for the angular position stabilization problem. The results were not conclusive about zero convergence of the position tracking error $e_x$, only about it's boundness (since $\dot{x} \rightarrow \tilde{\omega}$). A hybrid control scheme was used and an additional analysis of $e_x$ showed its convergence for two control modes but detailed analysis of solutions spanning across two of these modes was not carried out. For both cases, the observer estimations are well behaved, asymptotically and globally stable.

Finally, we presented an analysis regarding a disturbance model composed of 2 harmonics with different amplitudes and phases (since our model was non-linear, the superposition theorem is not valid), which led to a slightly more complex Lyapunov Function $V$. But, asymptotic stability and convergence was achieved for $\tilde{\omega}$ and $e$ in both disturbance estimation and compensation, as desired.
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