Supervised Learning of Market Making Strategy

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April 2, 2012

Abstract

Many economic markets, including most major stock exchanges, employ market-makers to aid in the transactions and provide a better quality market. This Study is aimed to establish an analytical foundation for electronic market making strategy, by giving a probabilistic interpretation to the Bid-Ask spread. The suggested strategy will be optimized with supervised learning from the high frequency data of the TASE (Tel Aviv Stock Exchange) order book. Based on this foundation, we wish to create an automated securities dealer that will perform the task of providing liquidity to the markets efficiently, and with low downturn risk. We compare the expected performance of the automated dealer with several bench mark measures of market liquidity such as those presented in Roll (1984) and Glosten & Milgrom (1985).

1 Introduction

The main function of a security market is to provide a place where buyers and sellers can transact. The efficiency of the market is measured by its liquidity, which defined as ability to execute transactions quickly without

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causing significant movements in price. For that reason exactly, most stock exchanges over the world have dealers. The role of a dealer is to provide liquidity on the exchange by quoting bid and ask prices at which he is willing to buy and sell a specific quantity of assets. Traditionally, this role has been filled by market-makers or specialist firms. In recent years, with the growth of electronic exchanges, anyone willing to submit limit orders in the system can effectively play the role of a dealer. In our paper, we study the submission strategies of bid and ask quotes in such a limit order book.

The difference between the ask and the bid quotes (bid-ask spread) and its influence on market’s liquidity have been studied extensively in the microstructure literature. Early works on the bid-ask spread by Benston and Hagerman (1974), Demsetz (1968), Hamilton (1976), and Tinic (1972) examined the cross-sectional relation between quoted spreads and variables such as trading volume, security price, and security risk. Following them, many studies among the field concentrated on the different costs faced by liquidity suppliers. Stoll (1978), Amihud and Mendelson (1980), and Ho and Stoll (1981) emphasized the inventory holding costs of liquidity suppliers. Copeland and Galai (1983), Glosten and Milgrom (1985) and Easley and OHara (1987) focused on the adverse selection costs which takes major part in markets with informed traders.

Several statistical models empirically measured the components of the bid-ask spread. Roll (1984) pioneers this type of analysis by introducing a serial covariance estimator of the bid-ask spread that relies only on a sequence of transaction prices and provides an estimate only of order processing costs. Other covariance spread models include Choi, Salandro and Shastri (1988), Stoll (1989) and George, Kaul, and Nimalendran (1991). Other category of models estimates bid-ask spread on the basis of a trade indicator regression model. Glosten and Harris (1988) proposed this idea to measure the adverse selection spread component by maximum likelihood. Later, Madhavan, Richardson, and Roomans (1996) suggested a Bayesian approach to estimate inventory and adverse selection costs. Other known papers in the regression models category are Huang and Stoll (1997), Ball and Chordia (2001) and Hasbrouck (2004).

Our work falls into the first category. We intend to find the proportion of the informed trading populations based on the bid-ask quotes as presented by Glosten Milgrom model (1985). Given the informed population the stock
correct price can also be revealed and used as a leading indicator for future price movements. We enhance this strategy by a supervised learning procedure. We intend to evaluate the liquidity in Tel-Aviv stock exchange by serial covariance models, and implement optimized market-maker upon this market.

This Paper is organized in seven parts. In Section 2, we first give a brief overview of the Bid-Ask spread properties and importance as it appears in the classical literature. Following in Section 3, we describe the data set and market microstructure of Tel Aviv Stock Exchange (TASE). Section 4 presents several important theoretical market-making models such as Roll (1984) and Glosten & Milgrom (1985), which form the basis for our research. In Section 5 we device and develop an learning methods of market making strategy for our automated dealer. Simulation analysis and empirical results are carried out in Section 6 to assess market liquidity in TASE, and evaluate the performance of our market making strategies. Finally, Section 7 contains our conclusions.

2 Properties of Bid-Ask Spread

Market-makers in securities markets provide immediacy services for traders who want to transact promptly. To compensate himself for his services, the market-maker sets two different quotes for the underlying security - an ask quote in which he is willing to sell the asset, and a bid quote in which he is willing to buy it. The gap between these buy and sell prices is called bid-ask spread. The bid ask spread will ultimately be the market maker profit for every stock purchased and sold at a later date.

The bid-ask spread is the subject of much interest traders, regulators, and researchers. The size of the spread measures the market liquidity and ultimately determines market-maker success. Existing market microstructure models show that the quoted bid-ask spread generally has three components: order processing costs, adverse selection costs and inventory-holding costs.

Order processing costs represent a fee arising from order execution like administrative cost and compensation for the market maker’s time. Inventory costs originate from holding unwanted risky securities in inventory by the
market maker. Adverse selection costs exist due to asymmetric information between the market maker and informed traders. Informed traders are individuals who are better informed about the expected price movement of the underlying security. Early studies in microstructure literature have shown negative correlation between the bid-ask spread and security price, trading volume and market capitalization. On the other hand, bid-ask spread is positive correlated with the volatility of the security price.

3 Market Structure and Data Set

3.1 TASE Trading System

TACT (Tel-Aviv Continuous Trading) is the TASE’s automated system for continuous and simultaneous trading. All securities and derivatives in Tel-Aviv trade via the TACT system. The system is based on a computerized order book accessible to all investors (order-driven), similar to the trading systems used by European stock exchanges.

Trading in TASE starts with a call auction followed by a continuous phase and culminates with a closing phase. The day starts with an empty order book at 9:00 AM, when traders start submitting limit and market orders. At a random time between 9:45AM and 9:50AM all the submitted orders are crossed, using an auction mechanism with time and price priority rules, and the continuous trading phase begins. The continuous phase is a limit order book, with designated market makers in some securities. All traders observe the best three prices and quantities on each side. Traders may post either market or limit orders, and those are executed according to price and time priority rules. Each transaction is executed at the price determined by the intersection of bid and ask orders. There is no price fluctuation limit during the continuous trading phase. The closing phase begins at 4:15 PM. All unexecuted orders are cancelled at the end of the trading day and the next day starts, again, with an empty book. Equities, bonds, index options, futures contracts and ETNs are all traded on the same TASE platform, with some minor differences in hours, minimum order size and tick size.

Only since 2003, TASE trading system has started to support the integration
of professional market makers in the trading process. Market makers operate under the existing TASE rules, without any preference over other investors in terms of trading or information. However, they get significant discounts on clearing and trading fees which place them in a better position at the market.

3.2 Data

The data set was purchased from Tel Aviv Stock Exchange (TASE). It covers one day of transactions in the TA-25 stock index - 03/01/2010. The data is divided into two lists: transactions list, which details transaction time, price and volume, and orders list which details order type, time, price, volume and status.

We used Matlab environment for analyzing our data. First, we have designed virtual clearing system which receives TASE historical data and maintains updated order book. Later, we built our chosen microstructure models and evaluated relevant parameters for our research. Finally, we tried to estimate liquidity parameters in real-time, learn market current state and perform efficient market maker.

3.3 TASE Order Book

An order book is a list of orders available to be matched and maintained for each security trading through an order driven system. Market participants can post two types of orders: A limit order which is an order to trade a certain amount of a security at a given price, and market order which is an order to trade a certain amount of a security immediately at the best available current price.

Limit orders arrive to electronic trading system, summarized and updated at each price level: this is known as the limit order book. The lowest price of limit sell order is called the ask price, and the highest price of limit buy order is called the bid price. When a market order arrives it is matched with the best available price in the limit order book and a transaction occurs. The quantities available in the limit order book are updated accordingly.
A limit order sits in the order book until it is either executed against a market order or it is canceled. A limit order may be executed very quickly if it corresponds to a price near the bid and the ask, but may take a long time if the market price moves away from the requested price or if the requested price is too far from the bid/ask. Alternatively, a limit order can be canceled at any time.

We consider a market where limit orders can be placed on a different price ticks \( \{0, 1, \ldots, n\} \). We chose upper boundary \( n \) to be large enough so probability for orders with prices higher than \( n \) approaches zero. Lower boundary is chosen to be the minimum price of security (zero). We track the state of each side of the order book with a continuous time processes:

\[
\text{ASK}(t) = (\text{ASK}_0(t), \text{ASK}_1(t), \ldots, \text{ASK}_n(t))_{t \geq 0}, \text{ where } \text{ASK}_p(t) \text{ is the volume of limit ask orders at price level } P \{0 \leq P \leq n\}.
\]

\[
\text{BID}(t) = (\text{BID}_0(t), \text{BID}_1(t), \ldots, \text{BID}_n(t))_{t \geq 0}, \text{ where } \text{BID}_p(t) \text{ is the volume of limit bid orders at price level } P \{0 \leq P \leq n\}.
\]

The ask price at time \( t \) is defined by:

\[
P_{\text{ASK}}(t) = \inf\{P = 0, 1, \ldots, n|\text{ASK}_p > 0\} \land n
\]

The bid price at time \( t \) is defined by:

\[
P_{\text{BID}}(t) = \sup\{P = 0, 1, \ldots, n|\text{BID}_p > 0\} \land 0
\]

Notice that when there are no ask orders in the book we force an ask price of \( n \), and when there are no bid orders in the book, we force a bid price of 0.

Now, we will describe how the limit book is updated by the inflow of new orders. Assuming order book at time \( t \) is in state \( \text{ASK}(t), \text{BID}(t) \in Z^n \). A buy/sell order which arrives at the market operates as follows:

- A limit buy order at price level \( P < P_{\text{ASK}} \) and volume \( V \) increases the volume at level \( P \) of the bid vector \( \text{BID}(t) \).

- A limit buy order at price level \( P \geq P_{\text{ASK}} \) and volume \( V < V_{\text{ASK}} \) decreases the volume at level \( P_{\text{ASK}} \) of the ask vector \( \text{ASK}(t) \) (transaction occurs).
A limit buy order at price level $P \geq P_{ASK}$ and volume $V \geq V_{ASK}$ decreases the volume at level $P_{ASK}$ to 0. Then, the ask price is updated to the next level, and we can assume a new limit buy order has arrived at the market at price level $P$ and volume $V - V_{ASK}$.

A limit sell order at price level $P > P_{BID}$ and volume $V$ increases the volume at level $P$ of the ask vector $ASK(t)$.

A limit sell order at price level $P \leq P_{BID}$ and volume $V < V_{BID}$ decreases the volume at level $P_{BID}$ of the bid vector $BID(t)$ (transaction occurs).

A limit sell order at price level $P \leq P_{BID}$ and volume $V \geq V_{BID}$ decreases the volume at level $P_{BID}$ to 0. Then, the bid price is updated to the next level, and we can assume a new limit sell order has arrived at the market at price level $P$ and volume $V - V_{BID}$.

A market buy order with volume $V < V_{ASK}$ decreases the volume at level $P_{ASK}$ of the ask vector $ASK(t)$ (transaction occurs).

A market buy order with volume $V \geq V_{ASK}$ decreases the volume at level $P_{ASK}$ to 0. Then, the ask price is updated to the next level, and we can assume a new market buy order with volume $V - V_{ASK}$ has arrived at the market.

A market sell order with volume $V < V_{BID}$ decreases the volume at level $P_{BID}$ of the bid vector $BID(t)$ (transaction occurs).

A market sell order with volume $V \geq V_{BID}$ decreases the volume at level $P_{BID}$ to 0. Then, the ask price is updated to the next level, and we can assume a new market sell order with volume $V - V_{BID}$ has arrived at the market.

A cancellation of a limit buy order at price level $P < P_{ASK}$ and volume $V$ decreases the volume at level $P$ of the bid vector $BID(t)$.

A cancellation of a limit sell order at price level $P > P_{BID}$ and volume $V$ decreases the volume at level $P$ of the ask vector $ASK(t)$.

The evolution of the order book is driven by the incoming flow of market orders, limit orders and cancellations at each price level.
4 The Models

This section gives a brief summary to the basic liquidity models we will analyze in our paper. We will use these models to infer the relative frequency of the informed trading population. This frequency will provide us as a leading indicator for our market making strategies.

4.1 Roll model

In order to infer the relative frequency of the informed trading population under transaction costs we first present the Roll model (1984) that produces a basic measure of market liquidity. The main appeal of Roll model is its simplicity in capturing an approximated bid-ask spread from the stock dynamics. Roll proposed serial covariance model of transaction prices to derive an estimator of the bid-ask spread.

In an efficient market the price dynamics may be stated as:

\[ m_t = m_{t-1} + \epsilon_t \]  
\[ p_t = m_t + \frac{S}{2} q_t \]

Where \( m_t \) is the unobservable efficient price, \( \epsilon_t \) is the serially uncorrelated innovation in the true value at time \( t \), \( p_t \) is the transaction price observed at time \( t \), \( S \) is the bid-ask spread, and \( q_t \) is the trade direction indicator which take the values \{+1\} for buy orders or \{-1\} for sell orders with equal probabilities.

Figure 4.1: Possible paths of observed market price.
The change in transaction prices between two successive trades is:

\[ \Delta p_t = \frac{S}{2} \Delta q_t + \Delta m_t = \frac{S}{2} \Delta q_t + \epsilon_t \]  

(4.3)

Roll examined the serial covariance of the security price:

\[
\text{Cov}(\Delta p_t, \Delta p_{t-1}) = E[\Delta p_t \Delta p_{t-1}] - E[\Delta p_t]E[\Delta p_{t-1}] \\
= E\left[\frac{S^2}{4} \Delta q_t \Delta q_{t-1}\right] + E\left[\frac{S}{2} \Delta q_t \epsilon_{t-1}\right] \\
+ E\left[\frac{S}{2} \Delta q_{t-1} \epsilon_t\right] + E[\epsilon_t \epsilon_{t-1}] - E[\Delta p_t]E[\Delta p_{t-1}] \\
\]

In order to solve the last equation Roll made several assumptions:

1. Bid-Ask spread \( S \) is constant.

2. Expected change in security price is zero:

\[ E[\Delta p_t] = 0. \]

3. Successive transaction types are independent:

\[ E[q_t q_{t-1}] = 0. \]

4. Transaction type is uncorrelated with the innovation in the true value:

\[ E[q_t \epsilon_t] = E[q_t \epsilon_{t-1}] = E[q_{t-1} \epsilon_t] = 0. \]

5. Uncorrelation between innovations in the true value:

\[ E[\epsilon_t \epsilon_{t-1}] = 0. \]

Thus,

\[
\text{Cov}(\Delta p_t, \Delta p_{t-1}) = \frac{S^2}{4} E[\Delta q_t \Delta q_{t-1}] = -\frac{S^2}{4} E[q_{t-1}^2] \\
= -\frac{S^2}{4} \left[0.5 \times (-1)^2 + 0.5 \times (+1)^2\right] \\
= -\frac{S^2}{4}
\]

(4.4)
Solving for $S$ yields Roll’s spread estimator:

$$Spread = 2\sqrt{-Cov(\Delta p_t, \Delta p_{t-1})}$$  \hspace{1cm} (4.5)

As we can see Rolls measure is simple, intuitive, and easy to compute. It gives us a good perspective on the average bid-ask spread of the security, and helps us estimate the liquidity in the market.

### 4.2 Glosten and Milgrom model

The Glosten Milgrom model (1985) is a sequential trade model which tries to infer the trading population from the bid-ask spread. The population is represented by two main groups, informed (insider) and uninformed (liquidity) traders. In their model, Glosten and Milgrom analyze market-makers decision problem with informed and uninformed traders who arrive at the market randomly, singly and independently.

Glosten and Milgrom model makes few assumptions:

- The market is a pure dealership and market - no transaction costs.
- The market is an asymmetric information market - there are only adverse selection costs.
- The market-maker knows the probabilistic structure of the arrival process, and he makes correct statistical inferences from observed data.
- The investors arrive one by one, randomly and anonymously at the market makers post.
- Zero profit condition is set - The market-maker is risk neutral and competitive.

The process begins with the market-maker who posts a bid price and an ask price, at which he agrees to buy one unit of security and sell one unit of security respectively. Only one unit can be transact at each point in time and all orders are market orders. Now, a trader arrives at the market and submits an order to either buy or sell one unit of security at the posted
bid and ask prices. After the transaction, the market maker may revise the posted prices. Then another trader arrives at the market and so on.

The value of the security in some specified future time is uncertain, it may be high \( \bar{V} \), or low \( V \). The probability of low value is \( \delta \). The trading population consists of informed and uninformed traders. We define the proportion of informed traders in the market as \( \mu \). Informed trader has private information about the future value of the security - when \( V = \bar{V} \) he buys the security, and when \( V = V \) he sells it. Uninformed trader has no private information about the security - he buys with probability \( \gamma \) and sells with probability \( 1 - \gamma \). In our market we assume symmetric buy and sell orders, thus \( \gamma = 0.5 \).

During his trading process, the market-maker faces an adverse selection problem - the informed trader has more information about the real value of the security when he submits his orders. Hence, the market maker looses on trading with informed traders. In response, market-maker revises bid and ask prices through Bayesian updating process about his expectations. He posts higher prices for buyer-initiated transactions (Ask) and lower prices for seller-initiated transactions (Bid).

Considering zero profit condition the quoted ask and bid prices will be:

\[
Ask = E[V|Buy] = Pr(V = \bar{V}|Buy) \cdot \bar{V} + Pr(V = V|Buy) \cdot V \quad (4.6)
\]
\[ Bid = E[V|Sell] = Pr(V = \bar{V}|Sell) \cdot \bar{V} + Pr(V = \bar{V}|Sell) \cdot V \]  

(4.7)

The unconditional buy and sell probabilities are:

\[ Pr(Buy) = \frac{1}{2} \delta(1 - \mu) + (1 - \delta)\mu + \frac{1}{2}(1 - \mu)(1 - \delta) \]  

(4.8)

\[ = \frac{1}{2}[1 + \mu(1 - 2\delta)] \]

\[ Pr(Sell) = \delta\mu + \frac{1}{2}\delta(1 - \mu) + \frac{1}{2}(1 - \mu)(1 - \delta) \]  

(4.9)

\[ = \frac{1}{2}[1 - \mu(1 - 2\delta)] \]

The conditional probabilities are:

\[ Pr(Buy|V = \bar{V}) = \mu \cdot 1 + (1 - \mu) \cdot \frac{1}{2} = \frac{1}{2}(1 + \mu) \]

\[ Pr(Buy|V = \bar{V}) = \mu \cdot 0 + (1 - \mu) \cdot \frac{1}{2} = \frac{1}{2}(1 - \mu) \]

\[ Pr(Sell|V = \bar{V}) = \mu \cdot 0 + (1 - \mu) \cdot \frac{1}{2} = \frac{1}{2}(1 - \mu) \]

\[ Pr(Sell|V = \bar{V}) = \mu \cdot 1 + (1 - \mu) \cdot \frac{1}{2} = \frac{1}{2}(1 + \mu) \]

Evaluating the value distribution of security by Bayes’ rule:

\[ Pr(V = \bar{V}|Buy) = \frac{Pr(Buy|V = \bar{V})Pr(V = \bar{V})}{Pr(Buy)} = \frac{(1 + \mu)(1 - \delta)}{1 + \mu(1 - 2\delta)} \]

\[ Pr(V = \bar{V}|Buy) = \frac{Pr(Buy|V = \bar{V})Pr(V = \bar{V})}{Pr(Buy)} = \frac{(1 - \mu)\delta}{1 + \mu(1 - 2\delta)} \]

\[ Pr(V = \bar{V}|Sell) = \frac{Pr(Sell|V = \bar{V})Pr(V = \bar{V})}{Pr(Sell)} = \frac{(1 - \mu)(1 - \delta)}{1 - \mu(1 - 2\delta)} \]

\[ Pr(V = \bar{V}|Sell) = \frac{Pr(Sell|V = \bar{V})Pr(V = \bar{V})}{Pr(Sell)} = \frac{(1 + \mu)\delta}{1 - \mu(1 - 2\delta)} \]
Finally, the market maker’s bid and ask prices:

\[ \text{Ask}^{GM} = \frac{(1 + \mu)(1 - \delta)\bar{V} + (1 - \mu)\delta V}{1 + \mu(1 - 2\delta)} \] (4.10)

\[ \text{Bid}^{GM} = \frac{(1 - \mu)(1 - \delta)\bar{V} + (1 + \mu)\delta V}{1 - \mu(1 - 2\delta)} \] (4.11)

And the bid-ask spread is:

\[ \psi^{GM} = \text{Ask}^{GM} - \text{Bid}^{GM} = \frac{4\mu\delta(1 - \delta)(\bar{V} - V)}{1 - \mu^2(1 - 2\delta)^2} \] (4.12)

### 4.3 Extended Glosten and Milgrom model

Adding transaction costs to Glosten and Milgrom model described at the previous section creates a new combined bid-ask model, in which both Roll and GM take part. Now, in addition to adverse selection problem the market maker has to pay the transaction cost component \( c \) on each trade. On the other hand, the bid and ask quotes are set to recover \( c \) as well as asymmetric information costs:

\[ \text{Ask} = E[V|Buy] + c \] (4.13)

\[ \text{Bid} = E[V|Sell] - c \] (4.14)

In asymmetric information market, the evolution of the efficient price is given by:

\[ m_t = m_{t-1} + \lambda q_t + \epsilon_t \] (4.15)

Where \( \lambda \) represents the adverse selection component of the trade. The actual trade price is:

\[ p_t = m_t + cq_t \] (4.16)

And bid-ask spread of the extended model is:

\[ \text{Spread} = \text{Ask} - \text{Bid} = 2(\lambda + c) \] (4.17)
Where $c$ represents the fixed transaction cost of the security in the market. We can see that in case of $\lambda = 0$, i.e. adverse selection component is zero, the model reduces to the basic Roll model.

For the original Roll model, we developed moving average and autoregressive representations that were useful in parameter estimation and forecasting. Here, we examine the time series structure of the model.

The change in transaction prices between two successive trades:

$$\Delta p_t = \Delta m_t + c \Delta q_t = (\lambda + c)q_t - cq_{t-1} + \epsilon_t$$  \hspace{1cm} (4.18)

The serial covariance of the security price:

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = E[\Delta p_t \Delta p_{t-1}] - E[\Delta p_t]E[\Delta p_{t-1}]$$

$$= E[(\lambda + c)q_t - cq_{t-1} + \epsilon_t]E[(\lambda + c)q_{t-1} - cq_{t-2} + \epsilon_{t-1}]$$

$$- E[\Delta p_t]E[\Delta p_{t-1}]$$

In expectation, under the same assumption specified in Roll model, all of the cross-products vanish except for $-c(c + \lambda)q_{t-1}^2$ expression.

Then:

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = -c(c + \lambda)E[q_{t-1}^2] = -c(c + \lambda)$$  \hspace{1cm} (4.19)

The adverse selection component in Glosten and Milgrom terms is $\lambda = \frac{1}{2} \psi^{GM}$. Thus:

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = -\frac{1}{2} \psi^{GM} c - c^2$$  \hspace{1cm} (4.20)

And this is the serial covariance of the security price in the combined Roll-GM model.

5 The Market Making Strategy

In this section, we first describe a basic market making strategy and discuss its performance in details. Later we suggest advanced strategy based on supervised learning market maker.
5.1 Basic Approach

The basic market maker strategy we have developed uses our knowledge about informed traders in the market. As we mentioned earlier, a market maker who arrives at the market faces an adverse selection problem - informed traders in the market have more information about the fundamental value of the stock. This situation causes the market maker to lose money when he trades with informed trader. In response, the market-maker can take several steps to mitigate his losses:

- Increasing the bid-ask spread. By setting higher quotes of ask price and lower quotes of bid price, the market maker balances his losses with profits from uninformed traders (zero sum game).
- Changing quotes volumes can discourage further trades on the same side, and encourage trades on the other side.
- Stopping his trading work until the proportion of the informed decreases to reasonable number.

As a basic approach for our research we chose to create an automatic agent which trades only when proportion of informed traders at the market is low. During his trading work, our agent places buy and sell orders of the same volume from both sides of the effective spread - buy order at the last best bid price, and sell order at the last best ask price. When the agent encounters high proportion of informed traders, he stops submitting orders to the market until informed traders number drops to reasonable value. Then he sets new quotes according to current effective price and continues his trading. For completion of our basic algorithm we must determine a threshold of informed trader proportion in the market. This threshold limits between trading state of our agent, and non-trading state.

Here is an intuitive example of how this strategy works. Assume we determine informed threshold to be 30%. The current price of stock A is 15 NIS, and the last bid and ask quotes are 14.99 NIS and 15.01 NIS respectively. Now, our market maker uses extended GM model to evaluate stock’s value distribution and current proportion of informed traders in the market. In case informed proportion is less than 30%, the market maker submits buy order of 14.99
NIS and sell order of 15.01 NIS. Otherwise, if informed proportion is higher than 30%, our market maker stops setting quotes and waits until informed proportion drops below 30%.

5.2 Supervised Learning Approach

Supervised learning is the machine learning task of computing functional relationship between input and output data. The supervised learning algorithm takes a set of training examples which consist of input and output pairs, and produces an inferred function using mathematical solutions. The inferred function tries to predict the correct output value for any valid input object.

Earlier, we saw that market maker who arrives at the market faces a decision problem - what kind of buy and sell orders he should submit each time period. Considering current conditions at the market, his portfolio, and his major goal of profit maximization, the market maker must define a clear strategy for his trading work. In this section, we will try to model the market making problem with supervised learning algorithm and define a corresponding strategy. In order to solve the given problem of supervised learning, we will perform the following steps:

1. Determine the type of training examples.
2. Gather a training set.
3. Determine the input feature representation of the learned function.
4. Determine the structure of the learned function and corresponding learning algorithm.
5. Evaluate the accuracy of the learned function on a test set.

Each step during his trading, the market maker should consider many parameters: market depth, proportion of informed traders at the market, last transaction price, bid and ask prices, order book state, his inventories etc. These variables characterize the temporal market maker state and have major impact upon his next action.
The model in this paper focuses on four fundamental state variables: last bid price ($BID$), last ask price ($ASK$), informed traders proportion at the market ($INFORMED$) and probability for low security value ($VLOW$), which define market maker’s state at time $t$:

$$X_t = (BID_{t-1}, ASK_{t-1}, INFORMED_{t-1}, VLOW_{t-1})$$

The first two variables, BID and ASK, are measured by observing the order book, while the last two, INFORMED and VLOW, are estimated by the extended Glosten and Milgrom model. Given his current state, the market-maker reacts by adjusting his orders to the market. Permissible actions by the market-maker include the following:

- Change the bid price.
- Change the ask price.
- Set the bid size.
- Set the ask size.
- Hold his trading until market state changes.

Our market maker changes his bid and ask prices each time period, and we assume fixed bid and ask sizes (e.g. one share). Thus, the action vector at time $t$ is defined as:

$$Y_t = (BID_t, ASK_t)$$

And the algorithm’s training set is:

$$D = \{X_t, Y_t\}_{t=1}^n$$

Where $X_t$ represents the market-maker’s state vector, and $Y_t$ represents the market-maker’s action vector. Next stage of the process is determining the training set. We consider one trading day as full data set. The data set consists of total observation of the trading day, and training period is chosen to be the first 30% observations. The remaining 70% quotes will be used later as our test set.
after determining and gathering output and input pairs, we have to characterize the structure of our learned function. As many famous studies within microstructure field, our supervised learning algorithm uses multilinear regression:

\[
BID_t = \beta_0 + \beta_1 BID_{t-1} + \beta_2 ASK_{t-1} + \beta_3 \text{INFORMED}_{t-1} + \beta_4 VLOW_{t-1} + \epsilon \tag{5.1}
\]

\[
ASK_t = \gamma_0 + \gamma_1 BID_{t-1} + \gamma_2 ASK_{t-1} + \gamma_3 \text{INFORMED}_{t-1} + \gamma_4 VLOW_{t-1} + \epsilon \tag{5.2}
\]

Running the regression algorithm on the training set with least squares method, tunes the coefficient vectors \((\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)\) and \((\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4)\), and produces our algorithm’s learned function.

The learned function shows us market maker’s optimal behavior for each scenario during his work. As a last step, we run a simulation on the test set, calculate test outputs, and evaluate them by comparison to the real history data. Possible evaluation of the learning market maker includes the following:

- "In the money" forecast - The market maker aims to make money by buying share at the bid price and then selling it at the higher ask price. The market maker loses money when the ask price drops below his bid price before he sells the share, or when the bid price rises above its ask price before he buys it. In both situations the security mid-price goes out of the bid-ask spread, and the market maker is in loosing position. One indicator to examine our learned function is calculating winning positions of our market maker comparing to his total trading positions.

- Next-step forecast - Another option to examine the learned function is tracking the quoted bid and ask prices against the predicted bid and ask prices. The number of times our strategy succeeded following after the bid-ask spread could indicate our ability to be a profitable market maker in this market.

- Total revenue simulation - More complicated solution to evaluate the performance of our learning market maker is building real-time market simulator in which one can perform as market maker, submit orders to the market and watch his profit each stage. Such simulator could help us in the evaluation task of our strategy.
6 Empirical Results

This chapter describes the final results of the analysis we have done throughout this paper. Our data consists of time-series sequences of trades and orders of one of the largest and leading shares in TASE - Leumi Bank share. The observations include 2805 samples of the share from 03/01/2010 (Continuous trading phase). At the beginning, we introduce the intraday evolution of the share price and its general behavior. Later, we show the properties of the share's transactions and order book. Then, we will run the different models on Leumi's data and try to characterize TASE market with the obtained parameters. Finally, we implement learning market maker with the theories presented earlier, and try to conclude whether it is possible to perform as a profitable market maker in the TASE market.

6.1 Price Process

![Figure 6.1: Leumi’s price process.](image)

Figure 6.1: Leumi's price process.
The schematic above illustrates the paths of observed transaction price of Leumi’s share between successive time periods. As we can see, the transaction price jumps between the ask and the bid prices in different levels of the order book.

### 6.2 Order Book Statistics

An important stage of exploring liquidity in the market is gathering the trading data and observe orders and transactions statistics.

![Daily distribution of bid-ask spread](image)

Figure 6.2: Daily distribution of bid-ask spread.

The measured spread ranged from 1 tick (about 60% of total observations) to 5 ticks. This information provides us fundamental insight into liquidity level at the market.

The next diagram shows the distribution of transactions levels in the order book. The 1st level represents transactions at the 1st bid (best bid) and 1st ask (best ask) prices, the 2nd represents the next level of each side and so forth.
We can see that all trading takes place at the first 3 levels of the order book, and about 90% of transactions occur at the best bid or best ask.

The schematic below describes the bid-ask spread dynamic against transactions’ order book levels, best bid and best ask volumes.

Figure 6.3: Daily distribution of transaction levels.

Figure 6.4: Comparison between Leumi’s bid-ask spread, transaction levels and order-book volumes.
This comparison leads us to important conclusions about the components of the spread, and the trading mechanism at the market. We can see that the spread increases only when the transaction executes at the 2nd or 3rd level of the order book. This fact suggests that the reason for the spread opening is large orders arrived on the market, and not any market makers strategy.

6.3 Roll Model

We took the serial covariance of price changes and used it to examine effective bid-ask spread of the share according to Roll.

Figure 6.5: Leumi’s Roll spread against observed and expected spread.

The measured Roll spread is about 0.043% of the underlyned price, and the expected bid-ask spread is about 0.086%.

6.4 Basic Glosten and Milgrom Model

Given Glosten and Milgrom theory, we have implemented simulation which estimates proportion of informed trader in the market and probability of direction of fundamental value (low or high) at each time period.
6.5 Extended Glosten and Milgrom Model

Extended GM model adds another element to the adverse selection component of the spread - transaction costs.

The average value of transaction costs across all time periods is 11.08% of the spread, the other 88.92% represents the expected adverse selection costs.
6.6 Basic Market Making Strategy

The final stage in the study is creating an automated securities market maker under a given strategy. Our basic strategy uses our information about \( \mu \) (proportion of informed traders) and \( \delta \) (probability of low value) parameters, which are estimated each time period according to extended GM formulas.

Our first basic strategy exploits the \( \mu \) parameter. First, we set a \( \bar{\mu} \) threshold which determines dealer’s participation on trading. The dealer waits until new arriving of order on the market, and then calculates the current \( \mu \) proportion. In case the measured \( \mu \) is higher than \( \bar{\mu} \), the automated dealer cancels all his open orders immediately and does not submit new orders to the market. Otherwise, in case \( \mu \leq \bar{\mu} \), the dealer continues his trading work.
and revises his orders according to the last best bid and best ask prices at the market.

Figure 6.9: Evaluation of basic market making strategy with $\mu$ threshold.

The graphs above evaluate the performance of the market maker under different $\tilde{\mu}$ thresholds, by comparing it to the trivial market making strategy - imitating Bid-Ask prices each step of the trading day. For example, when choosing $\tilde{\mu} = 0.2$ threshold, our market maker is "in the money" position in about 92.5% of his trades and he successfully predict next-step bid and ask prices in about 84% of his trades.

The second basic strategy uses the $\delta$ parameter. We set a $\tilde{\delta}$ threshold which determines dealer’s participation on trading. Now, instead of $\mu$ parameter, the dealer evaluates $\delta$ each time period and continues his trading only in case of $|\delta - 0.5| \leq \tilde{\delta}$. 

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6.7 Supervised Learning Market Making

In this section we have developed an advanced automated market maker who learns the market at the beginning of the trading day, and uses his knowledge to forecast intraday changes in the share price.

The learning functions we have drafted in the previous chapter:

\[
BID_t = \beta_0 + \beta_1 BID_{t-1} + \beta_2 ASK_{t-1} + \beta_3 INFORMED_{t-1} + \beta_4 VLOW_{t-1} + \epsilon
\]

\[
ASK_t = \gamma_0 + \gamma_1 BID_{t-1} + \gamma_2 ASK_{t-1} + \gamma_3 INFORMED_{t-1} + \gamma_4 VLOW_{t-1} + \epsilon
\]

Those regression functions suggest that knowing the last bid and ask prices, in addition to informed traders proportion and value probability distribution, can help our market maker choosing optimal next action and defining profitable trading strategy.

Figure 6.10: Evaluation of basic market making strategy with \(\delta\) threshold.
The regression results create our automated market making strategy:

\[
BID_t = 2545.85 + 0.866 \cdot BID_{t-1} + 0.12 \cdot ASK_{t-1} + 28.64 \cdot INFORMED_{t-1} + 90.2 \cdot VLOW_{t-1}
\]

\[
ASK_t = -476.83 - 0.02 \cdot BID_{t-1} + 1.02 \cdot ASK_{t-1} + 101.68 \cdot INFORMED_{t-1} + 150 \cdot VLOW_{t-1}
\]
Running the learned strategy on the input test set:

Figure 6.12: Supervised learning - Test period

And then we can evaluate the strategy results against the real historical results. In this case, where training is 30%:

- "In the money" forecast = 94.75%.
- Next-step forecast = 81.42%
Regression results of the different training size:

\[ BID_t = \beta_0 + \beta_1 BID_{t-1} + \beta_2 ASK_{t-1} + \beta_3 INFORMED_{t-1} + \beta_4 VLOW_{t-1} \]

<table>
<thead>
<tr>
<th>Training</th>
<th>( \beta_0 ) (CONST)</th>
<th>( \beta_1 ) (BID)</th>
<th>( \beta_2 ) (ASK)</th>
<th>( \beta_3 ) (INFORMED)</th>
<th>( \beta_4 ) (VLOW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>927.859</td>
<td>0.862</td>
<td>0.133</td>
<td>39.235</td>
<td>97.820</td>
</tr>
<tr>
<td>20%</td>
<td>1982.551</td>
<td>0.855</td>
<td>0.134</td>
<td>37.761</td>
<td>96.119</td>
</tr>
<tr>
<td>30%</td>
<td>2545.846</td>
<td>0.866</td>
<td>0.119</td>
<td>28.637</td>
<td>90.202</td>
</tr>
<tr>
<td>40%</td>
<td>1236.043</td>
<td>0.880</td>
<td>0.113</td>
<td>29.030</td>
<td>80.996</td>
</tr>
<tr>
<td>50%</td>
<td>1484.706</td>
<td>0.888</td>
<td>0.104</td>
<td>19.484</td>
<td>74.905</td>
</tr>
<tr>
<td>60%</td>
<td>1256.310</td>
<td>0.899</td>
<td>0.093</td>
<td>12.796</td>
<td>72.175</td>
</tr>
<tr>
<td>70%</td>
<td>1059.246</td>
<td>0.930</td>
<td>0.063</td>
<td>-9.466</td>
<td>61.336</td>
</tr>
<tr>
<td>80%</td>
<td>795.052</td>
<td>0.924</td>
<td>0.0712</td>
<td>-10.876</td>
<td>56.139</td>
</tr>
<tr>
<td>90%</td>
<td>717.614</td>
<td>0.917</td>
<td>0.0787</td>
<td>-6.2547</td>
<td>55.317</td>
</tr>
</tbody>
</table>

\[ ASK_t = \gamma_0 + \gamma_1 BID_{t-1} + \gamma_2 ASK_{t-2} + \gamma_3 INFORMED_{t-1} + \gamma_4 VLOW_{t-1} \]

<table>
<thead>
<tr>
<th>Training</th>
<th>( \gamma_0 ) (CONST)</th>
<th>( \gamma_1 ) (BID)</th>
<th>( \gamma_2 ) (ASK)</th>
<th>( \gamma_3 ) (INFORMED)</th>
<th>( \gamma_4 ) (VLOW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>-1621.769</td>
<td>-0.0539</td>
<td>1.062</td>
<td>160.445</td>
<td>187.100</td>
</tr>
<tr>
<td>20%</td>
<td>-400.908</td>
<td>-0.031</td>
<td>1.032</td>
<td>111.226</td>
<td>160.024</td>
</tr>
<tr>
<td>30%</td>
<td>-476.827</td>
<td>-0.0165</td>
<td>1.019</td>
<td>101.679</td>
<td>149.997</td>
</tr>
<tr>
<td>40%</td>
<td>-1046.449</td>
<td>-0.031</td>
<td>1.036</td>
<td>121.582</td>
<td>143.860</td>
</tr>
<tr>
<td>50%</td>
<td>-369.017</td>
<td>-0.015</td>
<td>1.017</td>
<td>103.861</td>
<td>131.055</td>
</tr>
<tr>
<td>60%</td>
<td>295.999</td>
<td>0.013</td>
<td>0.987</td>
<td>74.375</td>
<td>113.935</td>
</tr>
<tr>
<td>70%</td>
<td>536.940</td>
<td>0.030</td>
<td>0.967</td>
<td>62.391</td>
<td>99.928</td>
</tr>
<tr>
<td>80%</td>
<td>342.641</td>
<td>-0.006</td>
<td>1.004</td>
<td>74.016</td>
<td>85.622</td>
</tr>
<tr>
<td>90%</td>
<td>155.046</td>
<td>-0.011</td>
<td>1.009</td>
<td>79.560</td>
<td>84.232</td>
</tr>
</tbody>
</table>

Figure 6.13: Multi-linear Regression on different training size.
Finally, we will run our supervised strategies on the test set and evaluate their performance:

![Figure 6.14: Evaluation of supervised leaning market making strategy.](image)

**7 Conclusions**

In this paper, we derive an adaptive learning model of market-making in Tel-Aviv Stock Exchange (TASE). We develop explicit trading strategies, achieving multiple objectives under the trading system in Israel. Our research relies on two of the most famous and important works in microstructure literature - Roll model (1984) and Glosten and Milgrom model (1985).

Roll model provides us insights into the liquidity of a specific market, and evaluates the fixed transaction costs of our trades. Glosten and Milgrom model adds the asymmetric information component to our model, and estimates the adverse selection costs we must pay due to informed trades in the market. Our automated market-maker uses Roll and GM models for parameters estimation, sets the measured parameters in the learned strategies, and submits corresponding buy and sell orders to the market.
Our experiment stage begins with price process and statistical analysis of TASE. The analysis shows that bid-ask spread in TASE ranges between minimum to low levels. Further comparison between observed spread and submitted orders implies a strong dependency between spread width and orders’ volume. These results prove high liquidity level in Tel-Aviv market, and suggest low chance of playing efficient market maker in this market.

In next step of our experiment, we have implemented Roll and GM models over TASE historical data. The measured Roll spread was about 0.043% of the underlyned price, and the expected bid-ask spread was about 0.086%. Both results are significantly lower than the minimum transaction fee in TASE for buying and selling an asset - 0.14%. In other words, a private market-maker in TASE pays on average more than he earns. We can therefore conclude that most of the trading in Tel-Aviv are carried out by privileged professional market-makers.

As a final stage, we have integrated our theoretical algorithms with the estimated parameters, and ran the market-making strategies on TASE data. Our simulation results showed that knowing informed-traders population in TASE does not significantly improve market maker’s position at the market. Our basic strategy which is based on informed traders parameter, shows similar results to the trivial strategy. However, our learning market-making strategy has delivered better results than the trivial, and predicted successfully the next step of trading.

In conclusion, our study has shown success in bringing learning techniques to building market-making algorithms in a simple simulated market. It offers an alternative solution for playing the role of market-maker on trading. Despite the assertion that there is no place for private market-maker in TASE, we can assume our model would show a better performance in more competitive markets. Future extensions of this paper may include the more competitive and complex market environments, the introduction of additional objectives to the market-making model, and refinement of the learning techniques in the model.
References


