Sub-Nyquist SAR via Fourier Domain Processing

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Contributions

- New SAR signal processing algorithm, equivalent to the well-proved Range-Doppler Algorithm (RDA).
- Bypassing interpolation in RDA.
- Avoiding over-sampling which is used in practice.
- Enabling a convenient framework for rate reduction.
- Reconstruction of SAR images using Sub-Nyquist sampling rates at the receiver.
- Based on Xampling mechanism and Compressive Sensing.
- Saving on board memory and overcoming downlink throughput requirements for orbital missions.
- Fast 2D recovery algorithm.
- Exploiting 2D natural structure of image without the use of vectorization.
- Fits to real data sets.

Range-Doppler Processing

Various algorithms have been developed in order to process the SAR received raw data, d[n,m], into an image. RDA is the most widely used approach for high resolution processing of SAR data. RDA contains three main stages: Range compression, Range Cell Migration Correction (RCMC), Azimuth compression.

The RCMC stage is aimed at decoupling the dependency between the azimuth and range axes and to correct the hyperbolic trajectory of the targets’ echoes. The non-constant, non-integer shifts at the RCMC stage are realized by a digital subsequent interpolation which effectively increases the sampling rate of the system.

Goal: Getting rid of interpolation and reduce sampling rate at acquisition.

Fourier Domain Range-Doppler

<table>
<thead>
<tr>
<th>Range Compression</th>
<th>Conventional RDA</th>
<th>Fourier Domain RDA</th>
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<tr>
<td>d[n,m] = a[n,m]h[n,m]</td>
<td>( \tilde{d}_a[l] = T \cdot a[l]h[l] )</td>
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Azimuth DFT

\( S[k,n] = \sum_{m=0}^{M-1} d[n,m]e^{-j2\pi km/M} \)

Azimuth Compression

\( \tilde{S}[k,n] = S[n + n \cdot ak^2,k] \)

Azimuth IDFT

\( \tilde{D}[n,k] = \frac{1}{M} \sum_{m=0}^{M-1} \tilde{S}[k,n]e^{j2\pi km/M} \equiv \frac{1}{M} \sum_{m=0}^{M-1} \tilde{S}[k,n] \delta[n,m] \)

Comparing SAR point spread function (PSF):

Conventional RDA

\( \psi(l) = \psi_0(l) \cdot \delta(l) \)

Fourier Domain RDA

\( \psi(\nu) = \frac{1}{2\pi} \cdot \text{sinc}(\nu) \cdot \delta(\nu) \)

No over-sampling factor is required at the receiver.

Sub-Nyquist SAR

- The returned echoes are sampled in the Fourier domain under the Nyquist rate using Xampling [4]

Fast 2D recovery

- Having the partial Fourier processed measurements, \( \tilde{C}_p \), the image, \( I \), is reconstructed by solving the optimization problem:

\[ \min \| \Psi(I) \|_1 \ s.t. \ \| \tilde{C}_p - F \cdot (\Psi \cdot (F) \) \|_2 < \epsilon \]

- \( F \) – DFT matrix

- Using only 40% of Nyquist rate

- Naturally sparse scene, \( \Psi \) is the wavelets transform.

- Using only 45% of Nyquist rate

References


