Cyclostationary Detection from Sub-Nyquist Samples for Cognitive Radios

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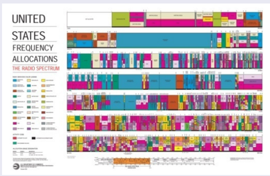
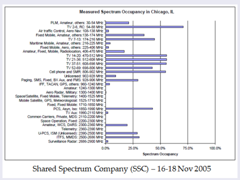
Main Contributions

- Cyclic spectrum reconstruction algorithm from sub-Nyquist samples
- Cyclostationary detection of multiband signals from sub-Nyquist samples
- Comparison of energy detection and cyclostationary detection performance

Cognitive Radios: Between Sparsity and Scarcity

- Address the conflict between spectrum saturation and underutilization
- Grant opportunitisc and non-interfering access to spectrum "holes" to unlicensed users
- Perform spectrum sensing task efficiently, in real-time and reliably

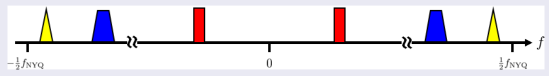




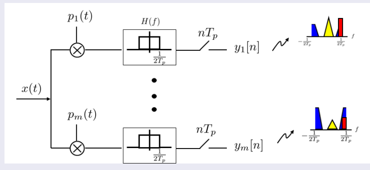
- Nyquist sampling is not an option! $\Rightarrow$ Sub-Nyquist sampling

Sub-Nyquist Sampling - MWC [1]

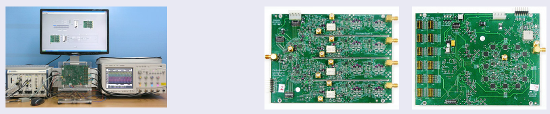
- Modulated Wideband Converter (MWC)
  - Input Signal: multiband model - $x(t)$ with Nyquist rate f_{Nyq} composed of $2N_{\text{sig}}$ bands each with max. bandwidth B



  - Analog front-end: composed of M parallel channels which alias the spectrum, so that each band appears in baseband.

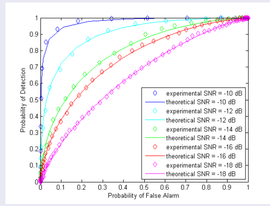


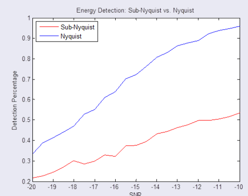
  - Signal reconstruction from sub-Nyquist samples
  - Energy detection
- Hardware implementation



- Minimal sampling rate for signal reconstruction: $2NB$ (twice Landau rate)

Energy Detection in Low SNRs





Input: one signal with bandwidth: 120MHz

Sampling rates: Nyquist 10GHz - Sub-Nyquist 256MHz

- Dramatic decrease in performance!
- Can we adapt our sub-Nyquist hardware with more robust detection?

Cyclostationarity [Gardner]

- Process whose statistical characteristics vary periodically with time
- Cyclic spectrum exhibits spectral peaks at certain frequency locations

AM QAM BPSK MSK

- Active spectral bands can be retrieved from performing detection on the cyclic spectrum

Cyclostationary Multiband Signal

- Signal model:

$$x(t)=\sum_{i=1}^{N_{\text{sig}}}\rho_i s_i(t),$$

where $s_i(t)$ are uncorrelated purely wide-sense cyclostationary.

- Wide-sense cyclostationary:
  - $\mu_s(t)$ and $R_s(t,\tau)$ are periodic with period T_0.
  - Cyclic autocorrelation functions (Fourier coefficients of $R_s(t,\tau)$):

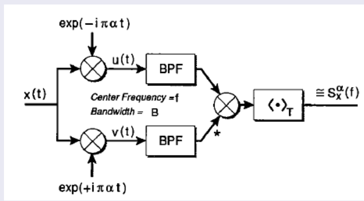
$$R_s^{\alpha}(\tau)=\frac{1}{T}\int_{-\infty}^{\infty}R_s(t,\tau)e^{-j2\pi\alpha t}\mathrm{d}t.$$

  - Cyclic spectrum:

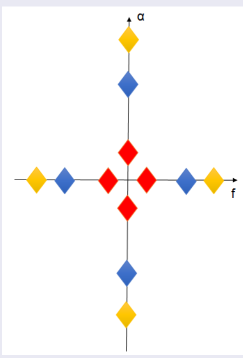
$$S_s^{\alpha}(f)=\int_{-\infty}^{\infty}R_s^{\alpha}(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau.$$

- Alternative interpretation:
  - Cross-spectral density of two frequency-shift versions of $x(t)$:

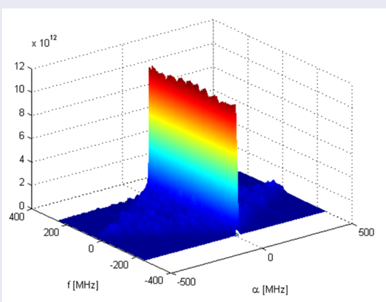
$$S_x^{\alpha}(f)=S_{uv}(f)=\mathbb{E}\left[X(f+\frac{\alpha}{2})X^*(f-\frac{\alpha}{2})\right],$$



- Cyclic spectrum and detection:
  - Support region of a multiband signal cyclic spectrum



  - Cyclic spectrum of white gaussian noise

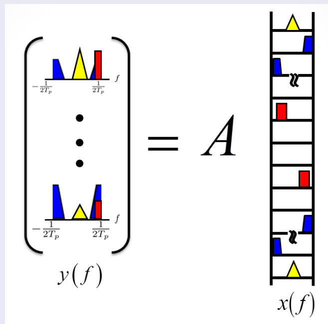


- Cyclostationary detectors exploits signals cyclic correlation.

Sub-Nyquist Sampling and Cyclic Spectrum Reconstruction

- Relation between known discrete time Fourier transforms (DTFTs) of the samples and unknown signal Fourier transform $X(f)$:

$$\mathbf{y}(f)=\mathbf{A}\mathbf{x}(f),\quad f\in[-f_s/2,f_s/2].$$

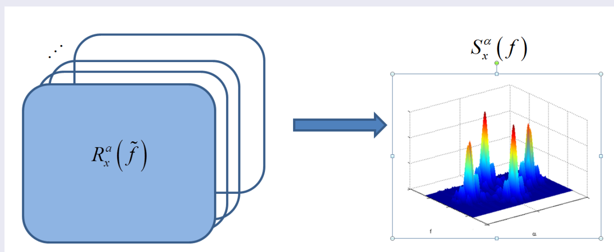


- Relation between correlations of the slices $\mathbf{x}(f)$ and the cyclic spectrum $S_x^{\alpha}(f)$:

$$\mathbf{R}_{\mathbf{x}}^{a}(\tilde{f})=\mathbb{E}\left[\mathbf{x}(\tilde{f})\mathbf{x}^H(\tilde{f}+a)\right],\quad a\in[0,f_s],\tilde{f}\in[-f_s/2,f_s/2-a],$$

It holds that

$$\begin{aligned}\mathbf{R}_{\mathbf{x}}^{a}(\tilde{f})_{(i,j)}&=S_x^{\alpha}(f),\\ \text{for }\alpha&=(j-i)f_s+a\\ \text{and }f&=-\frac{f_{\text{Nyq}}}{2}+\tilde{f}-\frac{f_s}{2}+\frac{(j+i)f_s}{2}+\frac{a}{2}.\end{aligned}$$

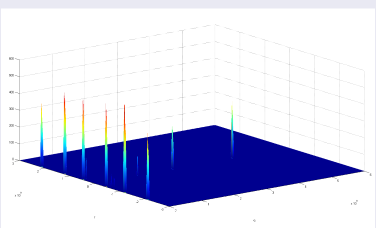


- Relation between $\mathbf{R}_{\mathbf{x}}^{a}(\tilde{f})$ and correlations of $y(f)$:

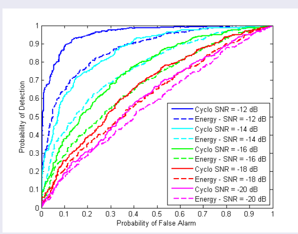
$$\mathbf{R}_{\mathbf{y}}^{a}(\tilde{f})=\mathbf{A}\mathbf{R}_{\mathbf{x}}^{a}(\tilde{f})\mathbf{A}^H,\qquad a\in[0,f_s],\tilde{f}\in[-f_s/2,f_s/2-a],$$

- By recovering $\mathbf{R}_{\mathbf{x}}^{a}(\tilde{f})$ from $\mathbf{R}_{\mathbf{y}}^{a}(\tilde{f})$, we reconstruct the cyclic spectrum $S_x^{\alpha}(f)$.

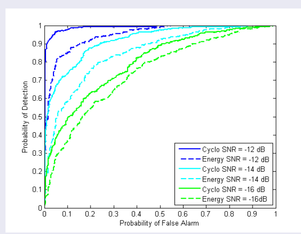
Simulations: Cyclic Spectrum Reconstruction and Cyclostationary Detection



- Number of signals: 3 (AM)
- Nyquist Rate: 6GHz
- Sampling Rate: 830MHz



- Number of signal: 1
- Nyquist Rate: 10GHz
- Sampling Rate: 620MHz



- Number of signals: 3
- Nyquist Rate: 10GHz
- Sampling Rate: 1.09GHz

References

[1] M. Mishali and Y. C. Eldar, "From Theory to Practice: Sub-Nyquist sampling of Sparse Wideband Analog Signals", *IEEE Journal of Selected Topics on Signal Proc.*, vol. 4, no. 2, pp. 375-391, Apr. 2010.

[2] D. Cohen, E. Rebeiz, Y. C. Eldar and D. Cabric, "Cyclic Spectrum Reconstruction and Cyclostationary Detection from Sub-Nyquist Samples", *SPAWC*, pp. 425-429, Jun. 2013.

[3] D. Cohen, and Y. C. Eldar, "Cyclic Spectrum Reconstruction from Sub-Nyquist Samples", *submitted for publication, GLOBECOM*, Dec. 2014.