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Joint Spectrum Sensing and Direction of Arrival Recovery from sub-Nyquist Samples

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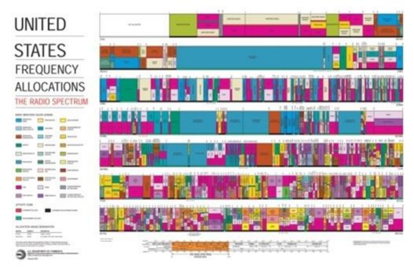
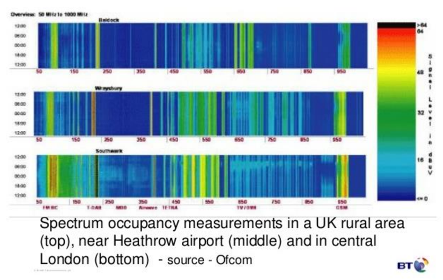
Contributions

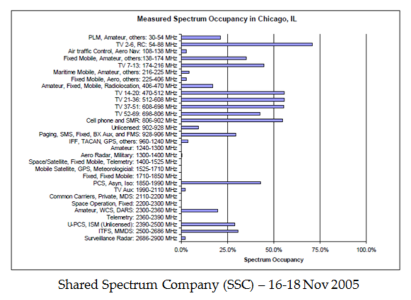
- Description of a sub-Nyquist L-shape uniform linear array (ULA) system based on the modulated wideband converter (MWC) [1]
- Formulation of the joint spectrum sensing and direction of arrival (DOA) estimation from sub-Nyquist samples problem
- Derivation of sufficient conditions for perfect recovery of the carrier frequencies, DOAs and input signals
- Derivation of an ESPRIT-based joint frequency and DOA recovery algorithm that achieves the minimal derived sampling rate

Cognitive Radio (CR)

- Address the conflict between spectrum saturation and underutilization
- Grant opportunistic access to spectrum "holes" to unlicensed users
- Perform spectrum sensing task efficiently, in real-time and reliably



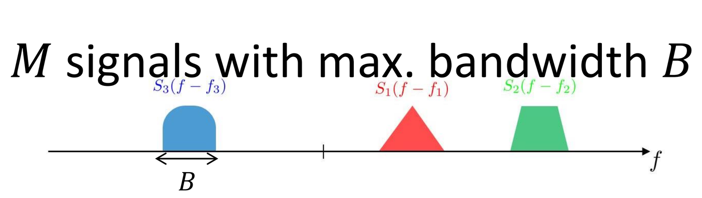




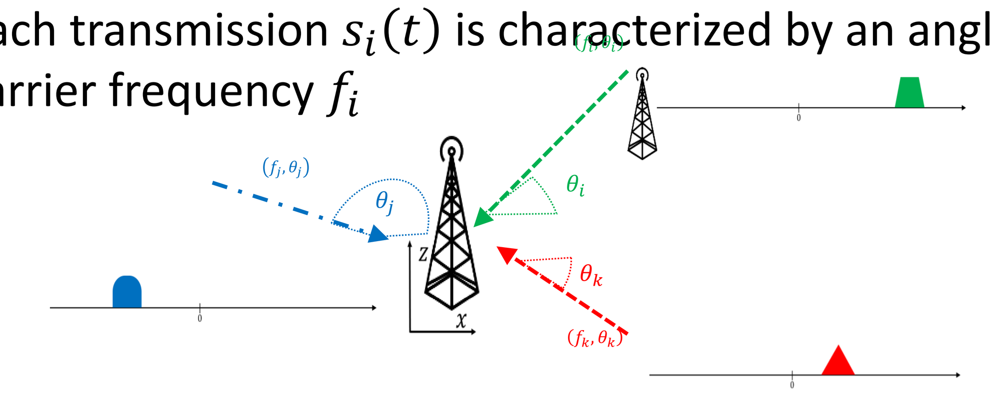
- Nyquist sampling is not an option ⇒ sub-Nyquist sampling
- Joint DOA and spectrum sensing increase CR efficiency

Sparse Multiband Signal Model

- Multiband model: M signals with max. bandwidth B and max. frequency $\frac{f_{Nyq}}{2}$.



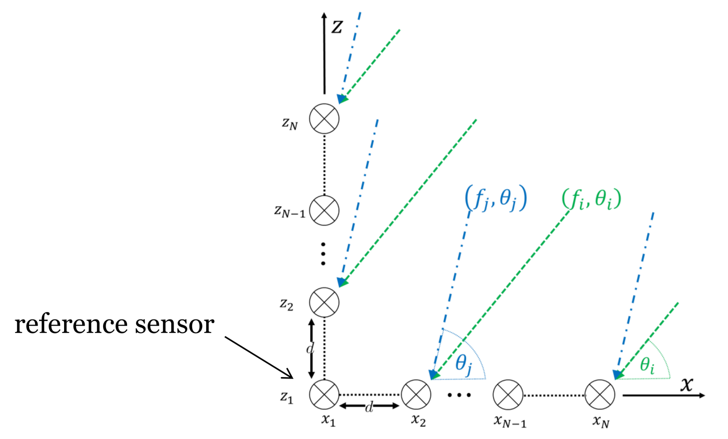
- Each transmission $s_i(t)$ is characterized by an angle of arrival (AOA) θ_i and carrier frequency f_i



Goal: perfect blind reconstruction of θ_i, f_i and $s_i(t)$

System Description

- L-shape ULA with N sensors in x axis and $N+1$ sensors in z axis:



- Phase accumulation in x axis:

$$\Delta\phi_{X_n}(f_i,\theta_i) = \frac{2\pi d}{c}\cdot n \cdot f_i\cos(\theta_i)$$

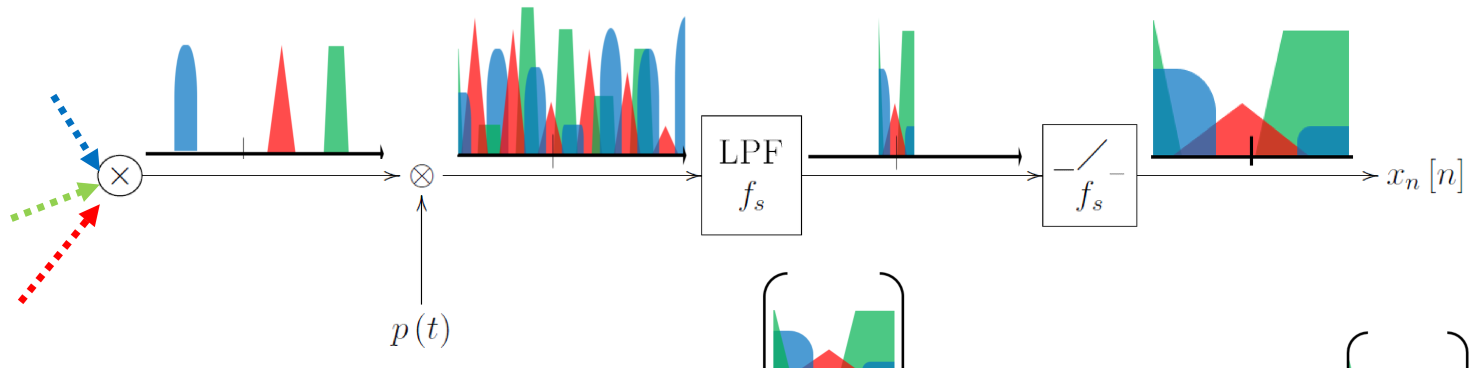
- Phase accumulation in z axis:

$$\Delta\phi_{Z_n}(f_i,\theta_i) = \frac{2\pi d}{c}\cdot n \cdot f_i\sin(\theta_i)$$

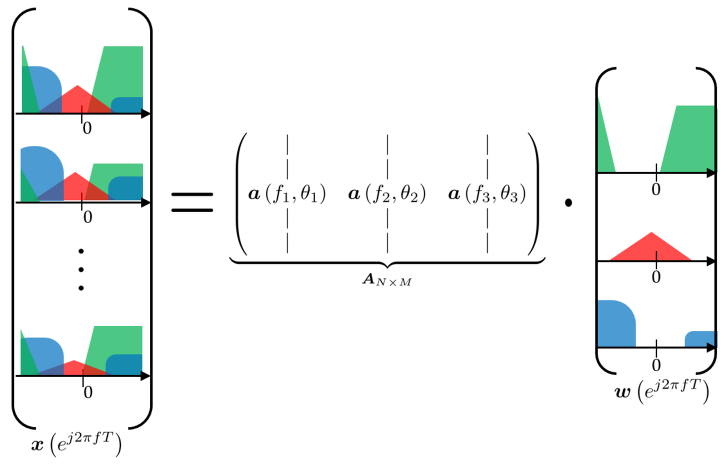
- Received signal at n[th] sensor in x axis: $U_n(f) = \sum_{i=1}^{M} S_i(f-f_i)e^{j\Delta\phi_{X_n}(f_i,\theta_i)}$

Sampling Scheme

- Analog front-end: modified MWC sampling chain [1]
- In each sensor, a unique channel aliases the spectrum so that each band appears in baseband using the same mixing function:



- Relation between known discrete time Fourier transforms (DTFTs) of the samples from the ULA in x axis and unknown signal Fourier transform:



Recovery conditions

Theorem

Let $x(t)$ be a multiband signal composed of M transmissions each with maximal bandwidth B and Nyquist rate $f_{\rm Nyq}$. If:

- $N \geq M+1$
- $f_s \geq f_p \geq B$
- $d < \frac{c}{f_{\rm Nyq}}$
- $c_l = f_p\int_0^{1/f_p} p(t)e^{-j2\pi l f_p t}dt \neq 0, \forall l : |lf_p| \leq \frac{f_{\rm Nyq}}{2}$

then, $\{f_i,\theta_i,s_i(f)\}$, for $1 \leq i \leq M$ can be perfectly reconstructed.

Sufficient condition to recover M signals: $2M+1$ sensors, each sampling at rate B

Pairing Issue

- ESPRIT algorithm

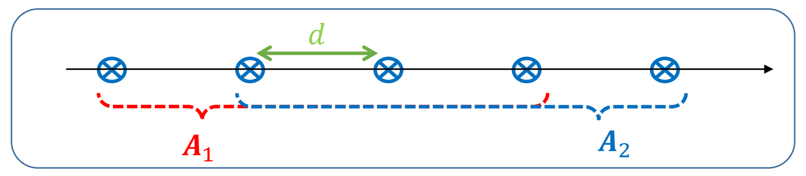
Input: $\boldsymbol{x}$ – Sensors measurements

1. Calculate the covariance matrix $\boldsymbol{R} = \boldsymbol{E}[\boldsymbol{x}\,\boldsymbol{x}^T]$
2. Decompose $\boldsymbol{R}$ (using SVD) to its eigenvectors
3. Calculate $\widetilde{\boldsymbol{\Phi}} = \mathbf{eig}(\mathbf{U}_1^{\dagger}\mathbf{U}_2)$.

Output : $\widetilde{\boldsymbol{\Phi}}$ – diagonal matrix

Eigenvectors of R spans the same subspace as $\boldsymbol{A}$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_1\Phi \end{bmatrix} T$$



$$A_2 = A_1 \cdot \underbrace{\begin{pmatrix} e^{j\frac{2\pi d}{c}f_1\cos(\theta_1)} & 0 & \cdots & 0 \\ 0 & e^{j\frac{2\pi d}{c}f_2\cos(\theta_2)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & e^{j\frac{2\pi d}{c}f_M\cos(\theta_M)} \end{pmatrix}}_{\Phi}$$

- Apply ESPRIT on x axis ULA: $f_i\cos(\theta_i) = \frac{c}{2\pi d}\angle\widetilde{\boldsymbol{\Phi}}_{ii}$
- Apply ESPRIT on z axis ULA: $f_j\sin(\theta_j) = \frac{c}{2\pi d}\angle\widehat{\boldsymbol{\Psi}}_{jj}$

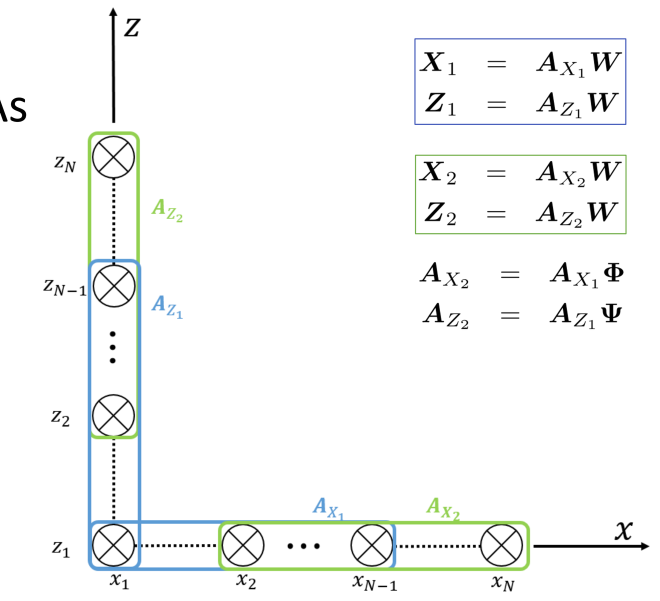
Problem: $\widetilde{\boldsymbol{\Phi}}, \widehat{\boldsymbol{\Psi}}$ suffer from different permutations. How can we pair the eigenvalues?

Joint Frequency – Angle Estimation (Joint SVD ESPRIT)

- To overcome the pairing problem:

Compute cross correlation matrices between ULAs

$$\mathbf{R_1} = E\left[\boldsymbol{X}_1\boldsymbol{Z}_1^H\right] = \mathbf{A}_{X_1}\mathbf{R}_W\mathbf{A}_{Z_1}^H$$
$$\mathbf{R_2} = E\left[\boldsymbol{X}_2\boldsymbol{Z}_1^H\right] = \mathbf{A}_{X_1}\boldsymbol{\Phi}\mathbf{R}_W\mathbf{A}_{Z_1}^H$$
$$\mathbf{R_3} = E\left[\boldsymbol{X}_1\boldsymbol{Z}_2^H\right] = \mathbf{A}_{X_1}\Psi^H\mathbf{R}_W\mathbf{A}_{Z_1}^H$$



$$\boldsymbol{X}_1 = \boldsymbol{A}_{X_1}\boldsymbol{W}, \quad \boldsymbol{Z}_1 = \boldsymbol{A}_{Z_1}\boldsymbol{W}$$
$$\boldsymbol{X}_2 = \boldsymbol{A}_{X_2}\boldsymbol{W}, \quad \boldsymbol{Z}_2 = \boldsymbol{A}_{Z_2}\boldsymbol{W}$$
$$\boldsymbol{A}_{X_2} = \boldsymbol{A}_{X_1}\boldsymbol{\Phi}, \quad \boldsymbol{A}_{Z_2} = \boldsymbol{A}_{Z_1}\boldsymbol{\Psi}$$

- Perform joint SVD on the cross-correlations to create same permutations for $\widetilde{\boldsymbol{\Phi}}$ and $\widetilde{\boldsymbol{\Psi}}$

$$\mathbf{U}_S = \begin{bmatrix}\mathbf{U}_1\\ \mathbf{U}_2 \\ \mathbf{U}_3\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{X_1}\\ \mathbf{A}_{X_1}\boldsymbol{\Phi} \\ \mathbf{A}_{X_1}\Psi^H\end{bmatrix}\mathbf{T} \Rightarrow \begin{matrix}\mathbf{U}_1^{\dagger}\mathbf{U}_2 = \mathbf{T}^{-1}\boldsymbol{\Phi}\mathbf{T} \\ \mathbf{U}_1^{\dagger}\mathbf{U}_3 = \mathbf{T}^{-1}\boldsymbol{\Psi}^H\mathbf{T}\end{matrix}$$

- Compute θ_i and f_i from the paired eigenvalues:

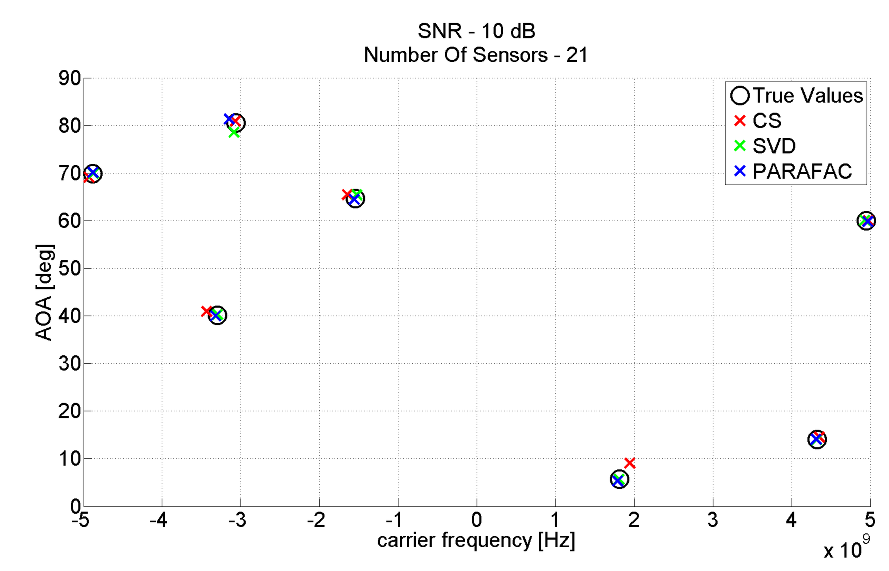
$$\theta_i = \tan^{-1}\left(\frac{\angle\widetilde{\Phi}_{ii}}{\angle\widetilde{\Psi}_{ii}}\right)$$
$$f_i = \frac{c}{2\pi d\cos(\theta_i)}\cdot\angle\widetilde{\Phi}_{ii}$$

- Reconstruct $\boldsymbol{A}(\boldsymbol{f},\boldsymbol{\theta})$ and compute $\boldsymbol{w} = \boldsymbol{A}^{\dagger}\boldsymbol{x}$

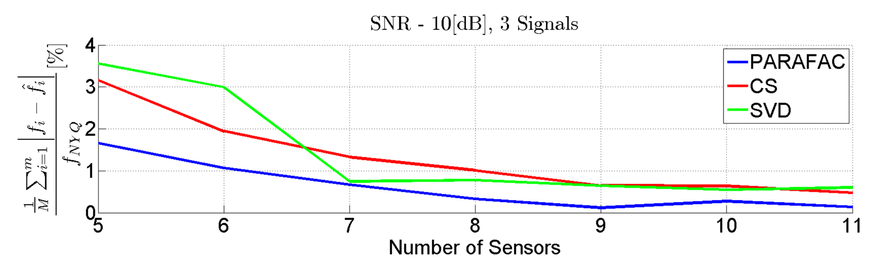
Simulation Results

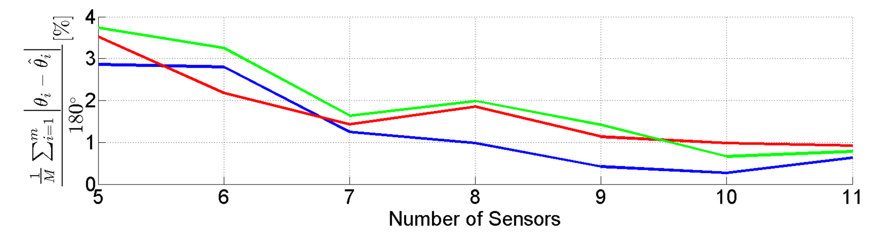
Compared methods:

- PARAFAC: iterative algorithm (based on alternating least squares) [3]
- Compressed sensing (CS): exploiting the spectrum sparsity
- Joint SVD ESPRIT (SVD): analytic solution (as presented)



SNR	10dB
Number Of Sensors	21
Number of Snapshots	2,000
Number of Signals	7
f_{NYQ}	$10GHz$
Sampling Rate	$21\cdot 50MHz \approx 1GHz$





SNR	10dB
Number of Snapshots	2,000
Number of Signals	3
f_{NYQ}	$10GHz$
Sampling Rate	$N\cdot 50MHz$ (N – number of sensors)

References

- [1] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals," IEEE Journal of Selected Topics in Signal Processing, 2010
- [2] A. Paulraj, R. Roy, and T. Kailath, "Estimation of signal parameters via rotational invariance techniques - ESPRIT," Information Systems Laboratory, 1986
- [3] D. Liu and J. Liang, "L-shaped array-based 2D DOA estimation using parallel factor analysis," WCICA, 2010