Cyclic Spectrum Reconstruction and Cyclostationary Detection from Sub-Nyquist Samples

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***Abstract*—In the context of Cognitive Radio (CR), opportunistic transmissions can exploit temporarily vacant spectral bands. Efficient and reliable spectrum sensing is a key in the CR process. CR receivers traditionally deal with wideband signals with high Nyquist rates and low Signal to Noise Ratios (SNRs). Thus, in this paper, we propose sub-Nyquist sampling and cyclostationary detection, which is robust to noise. We first reconstruct the cyclic spectrum or Spectral Correlation Function (SCF) of the signal, which is a characteristic function of cyclostationary signals such as communication signals, from sub-Nyquist samples and then perform detection. We consider both sparse and non sparse signals as well as blind and non blind detection in the sparse case. For each one of those scenarii, we derive the minimal sampling rate allowing for perfect reconstruction of the signal's SCF in a noise-free environment and provide SCF recovery techniques. In the simulations, we show SCF recovery at the minimal rate in noise-free settings as well as the performance of our detector in the presence of noise.**

I. Introduction

Spectral resources are traditionally allocated to licensed or primary users (PUs) by governmental organizations. Today, most of the spectrum is already owned and new users can hardly find free frequency bands. In light of the ever-increasing demand for wireless connectivity, this issue has become critical over the past few years. On the other hand, various studies [1], [2] have shown that the spectrum is usually significantly underutilized and can be described as the union of a few narrowband transmissions spread across a wide spectrum range. This is the motivation behind cognitive radio (CR), which would allow secondary users to opportunistically use the licensed spectrum when the corresponding PU is not active [3]. Even though the concept of CR is said to have been introduced by Mitola [3], the idea of learning machines for spectrum sensing can be traced back to Shannon [4], [5].

One of the most crucial tasks in the CR cycle is spectrum sensing. At the receiver, the CR performs detection to assert which band is unoccupied and can be exploited for opportunistic transmissions. Traditional detection techniques include matched filtering, energy detection and cyclostationary detection [6]–[9]. While the first approach requires *a priori* knowledge of the signal waveform and yields a high complexity architecture, the second is not robust to noise uncertainty and fails to differentiate signal and noise in low SNR regimes. In this paper, in order to detect the active bands efficiently even in the presence of large noise [10], we choose to exploit the cyclostationary property of communication signals and use cyclostationary feature detection. Processes, whose statistical characteristics vary periodically with time, are called cyclostationary [11]. A characteristic function of such processes, referred to as the cyclic spectrum or spectral correlation function (SCF), exhibits spectral peaks at certain frequency locations called cyclic frequencies. When determining the presence or the absence of a signal, cyclostationary detectors exploit one fundamental property of the SCF: stationary noise and interference exhibit no spectral correlation [11]. This renders such detectors highly robust to noise. Today, cyclostationary detection is performed on either the reconstructed SCF as a function of the angular frequency and computed at a specific cycle frequency [12], [13], or on the entire 2D reconstructed SCF from the wideband signal Nyquist samples [14].

In order to minimize the interference that could be caused to PUs, the spectrum sensing task performed by a CR should be reliable and fast [15]–[17]. Moreover, in order to increase the chance of finding an unoccupied spectral band, the CR has to sense a wide band of spectrum. Nyquist rates of wideband signals are high and can even exceed today's best analog-to-digital converters (ADCs) front-end bandwidths. Such high sampling rates generate a large number of samples to process, affecting speed and power consumption. Recently, several new sampling methods have been proposed [18]–[20] that reduce the sampling rate in multiband settings below the Nyquist rate. The authors derive the minimal sampling rate allowing for perfect signal reconstruction in noise-free settings and provide sampling and recovery techniques. However, when the final goal is spectrum sensing and detection, reconstructing the original signal is unnecessary. In [21], the authors propose a method to estimate finite resolution approximations to the true spectrum exploiting multicoset sampling. Spectrum reconstruction is also considered in [22] both in the time and frequency domains. However, no analysis on the minimal sampling rate ensuring perfect spectrum recovery was performed. Both approaches rely on energy detection. Unfortunately, the sensitiveness of energy detection is amplified when performed on sub-Nyquist samples due to aliasing of the noise [23]. In [24]–[26], the authors propose cyclostationary detection from sub-Nyquist samples. However, no analysis on the minimal

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sampling rate ensuring perfect SCF reconstruction was performed. Besides, the two first papers do not deal with the sampling scheme itself.

In this paper, we consider the class of purely wide-sense cyclostationary multiband signals, whose frequency support lies within several continuous intervals (bands). We will consider three different scenarii: (1) the signal is not assumed to be sparse, (2) the signal is assumed to be sparse and the carrier frequencies of the narrowband transmissions are assumed to be known, (3) the signal is assumed to be sparse but we do not assume carrier knowledge. We consider the sampling methods proposed in [18]–[20] and use a similar recovery technique to those derived in [21], [22] but we extend it in order to reconstruct the signal SCF from the sub-Nyquist samples. Our contribution is twofold. First, we derive the minimal sampling rate, allowing for perfect SCF reconstruction in a noise-free environment, for each one of the above three cases. The performance of our detector in noisy settings will be considered in the simulations. We show that the rate required for spectrum reconstruction is a bit higher than half the rate that allows for perfect signal reconstruction, for each one of the scenarii, namely the Nyquist rate, the Landau rate [27] and twice the Landau rate [19], respectively. Second, we provide SCF recovery techniques that achieve these lower bounds.

This paper is organized as follows. In Section II, we present the cyclostationary multiband model and define cyclostationarity and the SCF. Section III describes the sub-Nyquist sampling stage and SCF reconstruction. In Section IV, we derive the minimal sampling rate for each one of the three scenarii described above. Numerical experiments are presented in Section V.

II. System Model and Goal

A. System Model

Let $x(t)$ be a real-valued continuous-time signal, supported on $\mathcal{F} = [-1/2T_{\text{Nyq}}, +1/2T_{\text{Nyq}}]$. Formally, the Fourier transform of $x(t)$ defined by

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} \mathrm{d}t \tag{1}$$

is zero for every $f \notin \mathcal{F}$. We denote by $f_{\text{Nyq}} = 1/T_{\text{Nyq}}$ the Nyquist rate of $x(t)$. We assume that $x(t)$ is composed of up to N_{sig} uncorrelated transmissions with disjoint frequency supports. The bandwidth of each signal does not exceed $2B$ (where we consider both positive and negative frequency bands). Each transmission is assumed to be wide-sense purely cyclostationary with period $T_i, 1 \leq i \leq N_{\text{sig}}$, as defined in Section II-C. We consider three different scenarii.

1) No sparsity assumption: In the first scenario, we assume no *a priori* knowledge on the signal and we do not suppose that $x(t)$ is sparse, namely $2N_{\text{sig}}B$ can be of the order of f_{Nyq}.

2) Sparsity assumption and non blind detection: Here, we assume that $x(t)$ is sparse, namely $2N_{\text{sig}}B \ll f_{\text{Nyq}}$. Moreover, the support of the potentially active transmissions, which corresponds to the frequency support of licensed users defined by the communication standard, is assumed to be known. However, since the PUs' activity can vary over time, we wish to develop a detection algorithm that is independent of a specific known signal support.

3) Sparsity assumption and blind detection: In the last scenario, we assume that $x(t)$ is sparse but we do not assume any *a priori* knowledge on the carrier frequencies. Only the maximal number of transmissions N_{sig} and the maximal bandwidth $2B$ are assumed to be known.

B. Problem Formulation

In each one of the scenarii defined in Section II-A, our goal is to assess which of the N_{sig} transmissions are active from sub-Nyquist samples of $x(t)$. For each signal, we define the hypothesis $\mathcal{H}_{i,0}$ and $\mathcal{H}_{i,1}$, namely the ith transmission is absent and active, respectively.

In order to determine which of the N_{sig} transmissions are active, we first reconstruct the SCF of $x(t)$ defined in Section II-C. In the first and third scenarii, we fully reconstruct the SCF. In the second one, we exploit our prior knowledge and reconstruct it only at the potentially occupied locations. We will then perform detection on the fully or partially reconstructed SCF. For each one of the scenarii, we derive the minimal sampling rate enabling perfect SCF reconstruction in a noise-free environment.

C. Background: Cyclostationarity

In order to detect narrowband signals in the presence of large noise, we propose to exploit the cyclostationary properties of communication signals. A process $x(t)$ is said to be purely cyclostationary with period T_0 in the wide sense if its mean $\mathbb{E}[x(t)] = \mu_x(t)$ and autocorrelation $\mathbb{E}[x(t)x(t+\tau)] = R_x(t,\tau)$ are both periodic with period T_0 [11]:

$$\mu_x(t+T_0) = \mu_x(t), \qquad R_x(t+T_0,\tau) = R_x(t,\tau). \tag{2}$$

Given a wide-sense cyclostationary random process, its autocorrelation $R_x(t,\tau)$ can be expanded in a Fourier series

$$R_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t}, \tag{3}$$

where $\alpha = m/T_0, m \in \mathbb{Z}$ and the Fourier coefficients, referred to as cyclic autocorrelation functions, are given by

$$R_x^{\alpha}(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} R_x(t,\tau) e^{-j2\pi\alpha t} \mathrm{d}t. \tag{4}$$

The SCF is obtained by taking the Fourier transform of (4) with respect to τ, namely

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} \mathrm{d}\tau, \tag{5}$$

where α is referred to as the cyclic frequency and f is the angular frequency [11]. Since the SCF of purely cyclostationary signals exhibits spectral peaks only at specific angular frequencies, $\alpha = m/T_0, m \in \mathbb{Z}$, it is sparse in the α domain. It can be shown [11] that white noise $w(t)$ exhibits no cyclic correlation, that is

$$S_w^{\alpha}(f) = 0 \quad \alpha \neq 0. \tag{6}$$

We will exploit this property in our detection scheme.

III. Sub-Nyquist Sampling and SCF Reconstruction

We consider two different sampling schemes: multicoset sampling [19] and the modulated wideband converter (MWC) [18] which were previously proposed for sparse multiband signals in conjunction with energy detection. We show that the reconstruction stage is identical for both schemes. In this section, we reconstruct the entire SCF. In Section IV-B, we show how we can reconstruct the SCF only at potentially occupied locations when we have *a priori* knowledge on the carrier frequencies and symbol rates.

A. Multicoset sampling

Multicoset sampling can be described as the selection of certain samples from the uniform grid. More precisely, the uniform grid is divided into blocks of N consecutives samples, from which only M are kept. The ith sampling sequence is defined as

$$x_{c_i}[n] = \begin{cases} x(nT_{\text{Nyq}}), & n = mN + c_i, m \in \mathbb{Z} \\ 0, & \text{otherwise}, \end{cases} \quad (7)$$

where $0 < c_1 < c_2 < \cdots < c_M < N-1$. Following the derivations from multicoset sampling [19], we obtain

$$\mathbf{z}(f) = \mathbf{A}\mathbf{x}(f), \qquad f \in \left[0, \frac{1}{NT_{\text{Nyq}}}\right], \quad (8)$$

where $z_i(f) = X_{c_i}(e^{j2\pi f T_{Nyq}}), 0 \le i \le M-1$ is the DTFT of the multicoset samples and

$$x_k(f) = X\left(f + \frac{k}{NT_{Nyq}}\right), -\frac{N}{2} \le k \le \frac{N}{2} - 1. \quad (9)$$

We assume that NT_{Nyq} is an odd multiple of the periods of cyclostationarity $T_i, 1 \le i \le N_{\text{sig}}$ and that N is even.

B. MWC sampling

The MWC [18] is composed of M parallel channels. In each channel, an analog mixing front-end, where $x(t)$ is multiplied by a mixing function $p_i(t)$, aliases the spectrum, such that each band appears in baseband. The mixing functions $p_i(t)$ are required to be periodic with period T_p such that $f_p = 1/T_p \ge B$. The function $p_i(t)$ has a Fourier expansion

$$p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j\frac{2\pi}{T_p}lt}. \quad (10)$$

In each channel, the signal goes through a lowpass filter with cut-off frequency $f_s/2$ and is sampled at the rate $f_s \ge f_p$. For the sake of simplicity, we choose $f_s = f_p$. We assume that it is an odd multiple of the periods of cyclostationarity $T_i, 1 \le i \le N_{\text{sig}}$ and that $N = f_{\text{Nyq}}/f_s > M$ is even. Repeating the calculations in [18], we derive the relation between the known DTFTs of the samples $y_i[n]$ and the unknown $X(f)$

$$\mathbf{z}(f) = \mathbf{A}\mathbf{x}(f), \qquad f \in [0, f_s], \quad (11)$$

where $\mathbf{z}(f)$ is a vector of length N with ith element $z_i(f) = Y_i(e^{j2\pi f T_s})$. The unknown vector $\mathbf{x}(f)$ is given by (9). The $M \times N$ matrix $\mathbf{A}$ contains the coefficients c_{il}:

$$\mathbf{A}_{il} = c_{i,-l} = c^*_{il}. \quad (12)$$

For both sampling schemes, the overall sampling rate is

$$f_{tot} = Mf_s = \frac{M}{N} f_{\text{Nyq}}. \quad (13)$$

C. SCF reconstruction

We note that the system models (8) and (11) are identical for both sampling schemes. The only difference is the sampling matrix $\mathbf{A}$. We assume that $\mathbf{A}$ is full spark in both cases [18], [19]. We thus can derive a method for SCF reconstruction for both sampling schemes together.

We define the autocorrelation matrices $\mathbf{R_z} = \mathbb{E}[\mathbf{z}(f)\mathbf{z}^H(f)]$ and $\mathbf{R_x} = \mathbb{E}[\mathbf{x}(f)\mathbf{x}^H(f)]$. Then from (8), we have

$$\mathbf{R_z} = \mathbf{A}\mathbf{R_x}\mathbf{A^H}. \quad (14)$$

Due to lack of space, the proofs of the following propositions are omitted here and will be detailed in a future paper.

Proposition 1. *Let $x(t)$ be a bandpass wide-sense cyclostationary process with period T_0. Then*

$$\mathbb{E}[X(\omega)X^*(\nu)] = 2\pi \sum_{m=-1}^{1} S_x^{m/T_0}(\omega)\delta\left(\omega - \nu + \frac{m}{T_0}\right), \quad (15)$$

where $S_x^{m/T_0}(\omega)$ is defined in (5).

From Proposition 1, the only non zero elements of $\mathbf{R_x}$ are its diagonal elements $\mathbf{R_x}(i,i) = S_x^0(f + (i - N/2 - 1)f_s)$ [21] and $\mathbf{R_x}(i, N-i+1) = S_x^{(N-2i+1)f_s}(f)$, $-f_s/2 \le f \le f_s/2$, for $1 \le i \le N$. We can then write

$$\mathbf{r_z} = (\mathbf{A}^* \otimes \mathbf{A})\text{vec}(\mathbf{R_x}) = (\mathbf{A}^* \otimes \mathbf{A})\mathbf{B}\mathbf{r_x} \triangleq \mathbf{\Phi}\mathbf{r_x}, \quad (16)$$

where $\mathbf{\Phi} = (\mathbf{A}^* \otimes \mathbf{A})\mathbf{B}$. Here $\mathbf{r_z} = \text{vec}(\mathbf{R_z})$, and $\mathbf{B}$ is a $N^2 \times 2N$ selection matrix that has a 1 in the jth column and $[(j-1)N + j]$th row, and the $[j+N]$th column and $[j(N-1)+1]$th row, $1 \le j \le N$ and zeros elsewhere. It follows that $\mathbf{\Phi}$ is a $M^2 \times 2N$ matrix. Our goal is to recover $\mathbf{r_x}$, that contains the potentially non zero elements of the SCF.

IV. Minimal Sampling Rate

A. No sparsity Assumption

The system defined in (16) is overdetermined for $M^2 \ge N$, if $\mathbf{\Phi}$ is full column rank. The following proposition provides conditions for the system defined in (16) to have a unique solution.

Proposition 2. *Let $\mathbf{A}$ be a full row rank $M \times N$ matrix with N even ($M \le N$) and $\mathbf{B}$ be a $N^2 \times 2N$ selection matrix that has a 1 in the jth column and $[(j-1)N + j]$th row, and the $[j+N]$th column and $[j(N-1)+1]$th row, $1 \le j \le N$ and zeros elsewhere. The matrix $\mathbf{C} = (\mathbf{A}^* \otimes \mathbf{A})\mathbf{B}$ is full column rank if $M^2 \ge 2N$ and $2M > N+1$.*

Since $\mathbf{A}$ is assumed to be full spark, it is full row rank. Therefore, from Proposition 2, (16) has a unique solution if $M^2 \geq N$ and $2M > N+1$. This can happen even for $M < N$ which is our basic assumption. If $M \geq 4$, we have $M^2/2 \geq 2M-1$. Thus, in this case, the values of M for which we obtain a unique solution are $(N+1)/2 < M < N$.

This means that even without any sparsity constraints on the signal, we can retrieve its SCF from sub-Nyquist samples by exploiting its cyclostationary property, whereas the measurement vector $\mathbf{z}$ exhibits no stationary nor cyclostationary properties in general. This was already observed in [25] for a different model, but no proof was provided.

In this case, the minimal sampling rate is

$$f_{(1)} = Mf_s > \frac{N+1}{2}B = \frac{f_{\text{Nyq}} + B}{2}, \tag{17}$$

where $B \ll f_{\text{Nyq}}$.

B. Sparsity Assumption and Non-Blind Detection

We now consider the second scheme, where we have *a priori* knowledge on the frequency support of $x(t)$ and we assume that it is sparse. Instead of reconstructing the entire SCF, we exploit the knowledge of the signal's potential cyclic and angular frequencies in order to reconstruct only the potentially occupied bands. This will allow us to further reduce the sampling rate.

In this scenario, the only non zero elements of $\mathbf{R_x}$ are $K_f = 2N_{\text{sig}}$ diagonal elements and the corresponding K_f off diagonal elements, where $K_f \ll N$. The reduced dimensionality SCF is defined as

$$\hat{\mathbf{r}}_{\mathbf{x}} = \mathbf{M}_f \mathbf{r}_{\mathbf{x}}. \tag{18}$$

Here $\mathbf{M}_f \in \mathbb{R}^{2K_f \times 2N}$ is a matrix with elements equal to 1 at the indices of potential non-zero entries and $\hat{\mathbf{r}}_{\mathbf{x}} \in \mathbb{C}^{2K_f \times 1}$. Furthermore, we define $\mathbf{G}$ to be the $2N \times 2K_f$ matrix that selects the corresponding $2K_f$ columns of $\mathbf{\Phi}$ and $\hat{\mathbf{\Phi}} = \mathbf{\Phi}\mathbf{G}$. The reduced problem can then be expressed as

$$\mathbf{r}_{\mathbf{z}} = \hat{\mathbf{\Phi}} \hat{\mathbf{r}}_{\mathbf{x}}. \tag{19}$$

The following proposition provides conditions for the system defined in (19) to have a unique solution.

Proposition 3. *Let* $\mathbf{A}$ *be a full spark* $M \times N$ *matrix with* N *even* ($M \leq N$) *and* $\mathbf{B}$ *be defined as in Proposition 2. Let* $\mathbf{C} = (\mathbf{A}^* \otimes \mathbf{A})\mathbf{B}$ *and* $\mathbf{G}$ *be the* $2N \times 2K_f$ *that selects* $2K_f < 2N$ *columns of* $\mathbf{C}$ *as defined above. The matrix* $\mathbf{D} = \mathbf{C}\mathbf{G}$ *is full column rank if* $M^2 \geq 2K_f$ *and* $2M > K_f + 1$.

Therefore, from Proposition 2, for $M \geq 4$, if $M > (K_f + 1)/2$, where $K_f \ll N$, we obtain a unique solution for (19). In this case, the minimal sampling rate is

$$f_{(2)} = Mf_s > \frac{2N_{\text{sig}} + 1}{2}B = (N_{\text{sig}} + 0.5)B. \tag{20}$$

Landau [27] developed a minimal rate requirement for perfect signal reconstruction in the non-blind setting, which corresponds to the actual band occupancy. Here, we find that the minimal sampling rate for perfect SCF recovery is equal to half the Landau rate plus half the maximal bandwidth of the narrowband transmissions.

C. Sparsity Assumption and Blind Detection

We now consider the scheme where $x(t)$ is sparse, without any *a priori* knowledge on the support. In the previous section, we showed that $\hat{\mathbf{\Phi}}$ is full column rank, for any choice of K_f columns of $\mathbf{\Phi}$, provided $M^2 \geq 2K_f$ and $2M > K_f + 1$. Thus, for $M \geq 4$, we have spark($\mathbf{\Phi}$) $= 2M - 1$. Therefore, if $\mathbf{r_x}$, is $(M-1)$-sparse, it is the unique sparsest solution of (16), namely we can recover the SCF of any $(M-1)$-sparse signal. In this case, the minimal sampling rate is

$$f_{(3)} = Mf_s > (2N_{\text{sig}} + 1)B, \tag{21}$$

which is twice the rate obtained in the previous case. As in signal recovery, the minimal rate for blind reconstruction is twice the minimal rate for non-blind reconstruction [19].

V. Simulation Results

The first simulation demonstrates SCF reconstruction at the minimal sampling rate derived in Section IV-A. In the second simulation, we show the performance of the proposed detector in the presence of noise. In both cases, we use the MWC analog front-end [18] for the sampling stage.

In order to estimate the autocorrelation matrix $\mathbf{R_z}$, we first compute the estimates of $\mathbf{z}(i), 1 \leq i \leq M$, $\hat{\mathbf{z}}(i)$, using FFT on the samples $z_i[n]$ over a finite time window. We then estimate the elements of $\mathbf{R_z}$ using P realizations of $\hat{z}(\cdot, \cdot)$ as follows

$$\hat{\mathbf{R}}_{\mathbf{z}}(i,j,f) = \sum_{p=1}^{P} \hat{z}_p(i,f)\hat{z}_p^*(j,f), \quad f \in [-f_s/2, f_s/2], \tag{22}$$

where P is the number of frames for the averaging of the SCF.

We then perform cyclostationary detection on the reconstructed SCF. We use a single-cycle detector which computes the energy over 10 frequencies around $f = 0$, at a single cyclic frequency α. In the simulations, we consider AM modulated signals with cyclic features at $\alpha = 2f_c$, where f_c is the carrier frequency of the signal to be detected.

1) Simulation 1: Let $x(t)$ be composed of 40 uncorrelated transmissions. Each transmission is an AM modulated signal with single-sided bandwidth $B = 20MHz$. The Nyquist rate of $x(t)$ is $f_{\text{Nyq}} = 2GHz$. Thus, the occupancy is 80%. We consider $N = 80$ spectral bands and $M = 42$ analog channels, each sampling at $f_s = 25MHz$, following the minimal sampling rate requirement derived in Section IV-A. Figure 1 shows the original and the reconstructed spectrum and Fig. 2 shows the reconstructed SCF (the averages were performed over $P = 500$ frames).

2) Simulation 2: We now consider a non-blind scenario where the carrier frequencies of the signals occupying the wideband channel are known. We consider $N_{\text{sig}} = 3$ potentially active transmissions, with AM modulation and single-sided bandwidth $B = 30MHz$. The Nyquist rate of $x(t)$ is $f_{\text{Nyq}} = 2GHz$. We consider $N = 64$ spectral bands and $M = 4$ analog channels, each sampling at $f_s = 31.25MHz$.

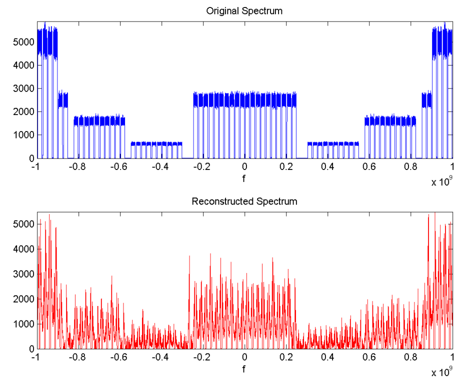

Fig. 1. Original and reconstructed PSD of a non sparse signal with Nyquist rate $2GHz$, sampled at $1.05GHz$.

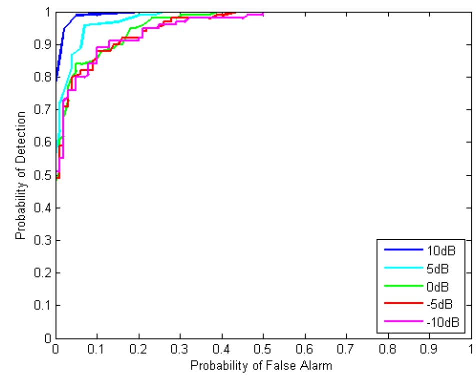

Fig. 3. Receiver operating characteristic (ROC) at SNR=10dB, 5dB, 0dB, -5 dB, -10dB, with sampling rate $125MHz$.

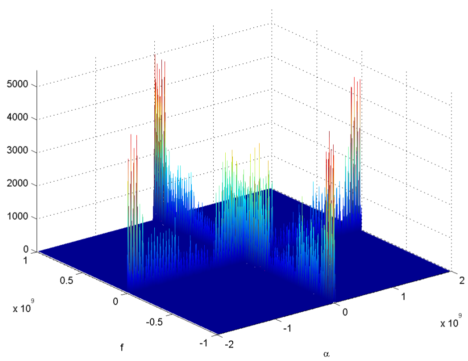

Fig. 2. Reconstructed SCF of a non sparse signal with Nyquist rate $2GHz$, sampled at $1.05GHz$.

The overall sampling rate is $Mf_s = 125MHz$ which is 69% of the Landau rate. The receiver operating characteristic (ROC) curve is shown in Fig. 3 for different SNR regimes (the averages were performed over $P = 15$ frames).

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