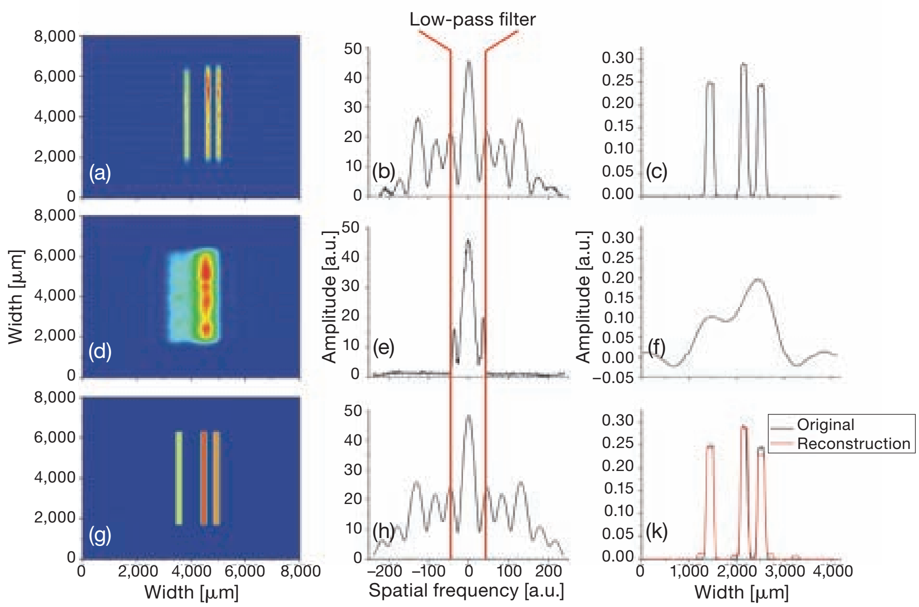
Sparsity-Based Reconstruction of Subwavelength Images from their Optical Far-Field

A. Szameit, Y. Shechtman, S. Gazit, Y.C. Eldar and M. Segev

A fundamental restriction of optical imaging is given by the diffraction limit, stating that the maximal resolution is half of the optical wavelength λ. This is a result of the evanescent nature of waves associated with spatial frequencies exceeding 1/λ.[1] Reaching beyond the diffraction limit is a subject of intense research, culminating in various approaches. However, none captures images in real time: They either require point-by-point scanning in the near-field or necessitate averaging over multiple experiments with florescent particles. Apart from these "hardware solutions," algorithmic attempts have been made to extrapolate the frequency content above the cut-off dictated by the diffraction limit. Such extrapolation methods were extremely sensitive to noise in the measured data and the assumptions made about the information. They all failed in recovering optical sub-λ information.[1]

In recent papers, we have shown that sub-λ information can be recovered from the far-field of an optical image.[2,3] The idea is based on compressed sensing (CS) techniques,[4] which are generically used for efficient data sampling, and are robust to noise in the measured data. Their only condition is that the information is sparse in a known basis (say, in real space). Sparse optical images are common, e.g., living cells, where the fraction of nonzero pixels is about 5 percent.

Subwavelength imaging can be represented as a bandwidth extrapolation problem, where propagation is equivalent to passing through a low-pass filter, with the diffraction limit as the cutoff frequency.[1] The question is how to identify the correct extrapolation, out of the infinite possible extrapolations. Here, sparsity comes into play: CS implies that, in the absence of noise, if the original information is sparse in a basis that is sufficiently uncorrelated with the measurement basis, then the sparsest solution is unique. For sub-λ imaging, this implies there is only one continuation of this truncated spectrum that corresponds to the sparsest image. Hence, if we know that our image is sparse in real-space, and only that, we just need to find the sparsest solution generating the observed far-field image. The uniqueness of the solution guarantees that this is the correct one. In the presence of noise, simulations reveal that the solutions, although not unique anymore, are very close to the original information when the noise is not exceedingly large. Hence, searching for the sparsest solution (that is consistent with the measured data), yields a reconstruction that is very close to ideal under typical experimental conditions. Finding the sparsest solution can be done through various algorithms. For EM fields, we proposed a new algorithm[2] reconstructing amplitude and phase.



(a,b,c) The original information consisting of three vertical stripes. (a) Its Fourier spectrum and (b) a horizontal cross-section, taken through the real-space image (c). (d,e,f) Using a slit, the signal is low-pass filtered at the vertical red lines (e), yielding a highly blurred image (d), where the three stripes merge into one (f). (g,h,k) Sparsity-based reconstruction yields a high-quality recovered image (g) and its Fourier spectrum (h). The strong correspondence between original and recovery is shown in the cross-section (k).

We tested the ideas theoretically with sub-λ information and devised experimental proof-of-concept: a 4-f imaging system with a tunable spatial filter at the Fourier plane. The filter mimics the optical transfer function by eliminating all frequencies above its cutoff. The reconstructed images contain spatial frequencies far beyond the highest frequency passing the low-pass filter. The figure displays the reconstruction of an image of three stripes, whereas the low-pass filtered image was a single broad stripe within which fine features can't be resolved. Recently, we showed the recovery of true sub-λ images of sparse information, reconstructing 100-nm features borne on 532-nm light, at 30-nm resolution.[5]

A. Szameit, Y. Shechtman, S. Gazit, Y.C. Eldar and M. Segev (msegev@techunix.technion.ac.il) are with the Technion, Israel.

References

1. J.W. Goodman. *Introduction to Fourier Optics*, 3rd Ed., Englewood, Roberts & Co., 2005.
2. S. Gazit et al. Opt. Exp. **17**, 23920 (2009).
3. Y. Shechtman et al. Opt. Lett. **35**, 1148 (2010).
4. E.J. Candes et al. IEEE Trans. Info. Theo. **52**, 5406 (2006).
5. A. Szameit et al. CLEO/QELS, paper QTuF1 (2010).