MODULATED WIDEBAND CONVERTER WITH NON-IDEAL LOWPASS FILTERS

[1]Yilun Chen, [2]Moshe Mishali, [2]Yonina C. Eldar, [1]Alfred O. Hero III

[1]Department of EECS, University of Michigan, Ann Arbor
[2]Department of Electrical Engineering, Technion – Israel Institute of Technology
{yilun,hero}@umich.edu, moshiko@tx.technion.ac.il, yonina@ee.technion.ac.il

ABSTRACT

We investigate the impact of using non-ideal lowpass filters in the modulated wideband (MWC) converter, which is a recent sub-Nyquist sampling system for sparse wideband analog signals. We begin by deriving a perfect reconstruction condition for general lowpass filters, which coincides with the well-known Nyquist inter-symbol interference (ISI) criterion in communication theory. Then, we propose to compensate for the non-ideal lowpass filters using a digital FIR correction scheme. The proposed solution is validated by experimental results.

Index Terms— Compressed sensing, modulated wideband converter, multiband sampling, non-ideal lowpass filters.

1. INTRODUCTION

Efficient sampling of wideband analog signals is a challenging problem, since their Nyquist rates may exceed the specifications of the best analog-to-digital converters (ADCs) nowadays by orders of magnitude. The modulated wideband converter (MWC) [1, 2] is a recent sub-Nyquist system for sampling multiband signals of wide spectral ranges. The MWC, depicted in Fig. 1 and further described in Section 2, consists of simple mixers and lowpass filters. By exploiting frequency sparsity of multiband signals, the MWC is able to significantly reduce the conversion rate.

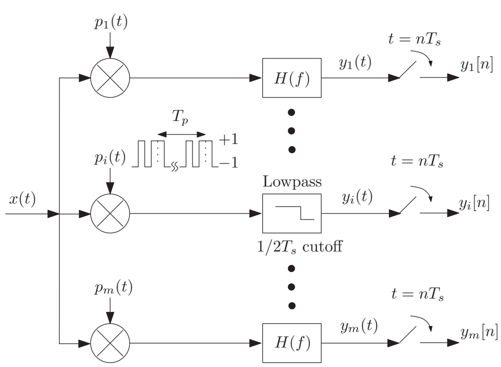

Fig. 1. A block diagram of the modulated wideband converter. The MWC consists of m parallel channels, which mix the input against m periodic waveforms. The mixed signal is then lowpass filtered and sampled at a low rate.

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The original MWC requires ideal analog lowpass filters to accomplish the reconstruction process. In practice, implementing ideal filters is generally difficult and the usual option is to employ high-order Butterworth or Chebyshev filters. Direct use of such off-the-shelf filters does not guarantee perfect reconstruction (PR) in the recovered signal. Indeed, this problem is encountered in the practical implementation of the MWC. Therefore, methods of compensation for the imperfections of non-ideal lowpass filters is an important problem. This is the main motivation of this work.

In this paper we aim at extending the MWC to enable the use of practical lowpass filters. Under the assumption of near perfect stopband response, we show that, with only a moderate amount of oversampling, the imperfections caused by non-ideal filters can be effectively corrected in the digital domain. The contribution of our work is two-fold. First, we derive a perfect reconstruction condition that must be satisfied by lowpass filters in the MWC. We show that the ideal lowpass filter is not the only choice that guarantees PR. Indeed, we prove that perfect reconstruction can be achieved by Nyquist filters [6], which are more general. For cases where the PR condition is not satisfied, we propose a compensation method operating in the digital domain for perfect reconstruction, using a simple bank of finite impulse response (FIR) filters. The coefficients of the FIR filters are designed to meet the PR condition and closed-form expressions for the filter coefficients are provided. Both numerical simulations and real measured data demonstrate that the proposed compensation method can significantly reduce the reconstruction error using low-order FIR filters.

The paper is organized as follows. Section 2 provides a brief introduction to the MWC. In Section 3 we study the MWC with non-ideal filters. We derive the PR condition and propose the digital compensation method. Section 4 provides experimental results.

2. THE MODULATED WIDEBAND CONVERTER

The MWC is a sub-Nyquist sampling system for sampling sparse wideband analog signals. It consists of two stages: sampling and reconstruction. In this section, we briefly introduce the mechanism and principle of the MWC.

2.1. Sampling

In the sampling stage, the signal $x(t)$ enters m channels simultaneously. In the ith channel, $x(t)$ is multiplied by a T_p- periodic mixing function $p_i(t)$. After mixing, the output is lowpass filtered with cutoff frequency $1/(2T_s)$ and then uniformly sampled at rate $1/T_s$. The overall sampling frequency of the MWC is then m/T_s.

The input $x(t)$ is assumed to be a sparse wideband analog signal bandlimited to $[-f_{\mathrm{NYQ}}/2, f_{\mathrm{NYQ}}/2]$, where f_{NYQ} can be very large, much larger than the sampling frequency m/T_s. The support of $x(t)$

resides within N frequency intervals, or bands, such that the width of each band does not exceed B Hz. The band positions are arbitrary and in particular unknown in advance. For example, in communications N represents the number of concurrent transmissions and B is specified by the specific modulation techniques in use.

The sub-Nyquist sampling of the MWC relies on the following key observation. The mixing operation scrambles the spectrum of $x(t)$ such that the baseband frequencies that reside below the filter cutoff contain a mixture of the spectral contents from the entire Nyquist range. To further illustrate this point, let us consider a single channel, and let $P_i(f)$ be the spectrum of the mixing function $p_i(t)$. Since $p_i(t)$ is T_p- periodic, $P_i(f)$ can be expressed as

$$P_i(f) = \sum_{l=-\infty}^{+\infty} c_{il}\delta(f - lf_p), \tag{1}$$

where $f_p = 1/T_p$, c_{il} are arbitrary coefficients and $\delta(\cdot)$ is the Dirac delta function. The spectrum of the mixed signal $\tilde{x}_i(t) = x(t)p_i(t)$ is then

$$\tilde{X}_i(f) = P_i(f) * X(f) = \sum_{l=-\infty}^{+\infty} c_{il}X(f - lf_p), \tag{2}$$

where $X(f)$ is the spectrum of $x(t)$. Lowpass filtering with a filter transfer function $H(f)$ results in a signal $y_i(t)$ with spectrum

$$Y_i(f) = \sum_{l=-\infty}^{+\infty} c_{il}X(f - lf_p)H(f). \tag{3}$$

After sampling the continuous signal $y_i(t)$ at rate $f_s = 1/T_s$, the discrete time Fourier transform (DTFT) of the samples $y_i[n]$ is

$$\begin{aligned} Y_i(e^{j\omega}) &= \sum_{k=-\infty}^{+\infty} Y\left(\frac{f_s}{2\pi}(\omega - 2k\pi)\right) \\ &= \sum_{l,k=-\infty}^{+\infty} c_{il}X\left(\frac{f_s}{2\pi}\omega - lf_p - kf_s\right) H\left(\frac{f_s}{2\pi}\omega - kf_s\right), \end{aligned} \tag{4}$$

for $\omega \in [-\pi, \pi]$. In the standard MWC, $H(f)$ is an ideal rect function with cutoff $f_s/2$. Thus $H(\frac{f_s}{2\pi}\omega - kf_s), \omega \in [-\pi, \pi]$, is nonzero only if $k = 0$. To ease the exposition we further choose[1] $T_s = T_p$. Then, (4) can be expressed as

$$Y_i(e^{j\omega}) = \sum_{l=-L_0}^{L_0} c_{il}X\left(\frac{f_p}{2\pi}\omega - lf_p\right), \ \ \omega \in [-\pi, \pi], \tag{5}$$

where L_0 is the smallest integer satisfying $2L_0 + 1 > f_{\text{NYQ}}/f_p$.

The relation (5) ties the known DTFTs of $y_i[n]$ to the unknown $X(f)$, which is the key to recovery of $x(t)$. For our purpose, it is convenient to write (5) in matrix form

$$\mathbf{y}(e^{j\omega}) = \mathbf{A}\mathbf{z}(\omega), \tag{6}$$

where $\mathbf{y}(e^{j\omega})$ is a $m \times 1$ vector with the ith element $Y_i(e^{j\omega})$, $\mathbf{z}(\omega)$ is an unknown vector of length $L = 2L_0 + 1$ with the ith element $z_i(\omega) = X\left(\frac{f_p}{2\pi}\omega - (i - L_0 - 1)f_p\right)$, and $\mathbf{A}$ is a $m \times L$ matrix containing the coefficients c_{il}. We note that $X\left(\frac{f_p}{2\pi}\omega - lf_p\right)$, for $|l| \leq L_0$, covers all the spectral information of $x(t)$. Therefore, in order to recover $x(t)$, it is sufficient to determine $\mathbf{z}(\omega)$ for every $\omega \in [-\pi, \pi]$.

[1] This choice is relaxed in [1].

The vector $\mathbf{z}(\omega)$ is sparse for each $\omega \in [-\pi, \pi]$ due to the sparse nature of the spectrum of $x(t)$. The sparsity of $\mathbf{z}(\omega)$ assures that the MWC can use a small number of channels to recover $x(t)$, which enables sub-Nyquist sampling. For example, in the basic configuration in which $f_s = f_p = B$, it is shown that [1] with a careful selection of mixing functions, $m = 4N$, where N is the number of bands, is sufficient to recover $\mathbf{z}(\omega)$.

2.2. Reconstruction

The reconstruction stage consists of two steps and is implemented completely in the time domain. First, the spectral support is determined, and then the signal is recovered from the samples by a closed-form expression.

Spectral support recovery relies on recent ideas developed in the context of analog compressed sensing [4] and are implemented by a series of digital computations, which are grouped together under the Continuous-to-Finite (CTF) block [1, 3]. Let the support of $\mathbf{z}(\omega)$ be $S = \bigcup_{\omega\in[-\pi,\pi]} \text{supp}(\mathbf{z}(\omega))$, where $\text{supp}(\cdot)$ is the set of indices of the nonzero entries of a vector. In other words, if $i \notin S$ then $z_i(\omega) = 0$ for all $\omega \in [-\pi, \pi]$. By exploiting the sparsity of $\mathbf{z}(\omega)$, the CTF efficiently infers the support S from a low-complexity finite program.

Once the support S is determined, it follows from (6) that

$$\begin{aligned} \mathbf{z}_S[n] &= \mathbf{A}_S^{\dagger}\mathbf{y}[n] \\ z_i[n] &= 0, \ \ i \notin S, \end{aligned} \tag{7}$$

where $\mathbf{z}[n] = (z_1[n], \ldots, z_L[n])^T$ and $z_i[n]$ is the inverse DTFT of $z_i(\omega)$. $\mathbf{z}_S[n]$ and $\mathbf{A}_S$ mean the subvector and submatrix comprised of the rows of $\mathbf{z}[n]$ and $\mathbf{A}$ indexed by S, respectively. The notation $(\cdot)^{\dagger}$ denotes the pseudo inverse. Equation (7) allows $z_i[n]$ to be generated at the input rate f_s. Every $z_i[n]$ is then interpolated to a continuous baseband signal at rate f_s (*e.g.*, using digital-to-analog devices) yielding (complex valued) $z_i(t)$:

$$z_i(t) = \sum_{n=-\infty}^{\infty} z_i[n]h(t - nT_s), \tag{8}$$

where $h(t) = \text{sinc}(\pi t/T_s)$. Finally, $x(t)$ is reconstructed by modulating $z_i(t)$ to their corresponding bands:

$$\hat{x}(t) = \sum_{i\in S, i>L_0} \text{Re}\left\{z_i(t)\right\}\cos(2\pi i f_p t) + \text{Im}\left\{z_i(t)\right\}\sin(2\pi i f_p t), \tag{9}$$

where $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote the real and imaginary part of their argument, respectively.

3. THE MWC USING NON-IDEAL LOWPASS FILTERS

The lowpass filters in the standard MWC are treated as ideal rect functions in the frequency domain in order to obtain (5) from (4). However, in practice, ideal analog filters are difficult to design and a practical lowpass filter usually has the following imperfections:

1. $H(f)$ is not necessarily flat in the pass band;
2. $H(f)$ does not have sharp edges;
3. The stopband response is not exactly zero.

In this section, we investigate how those imperfections impact perfect recovery of the input $x(t)$ through (9) and propose compensation schemes to correct for these imperfections.

3.1. The perfect reconstruction condition

We start our analysis from (4), which applies to any analog filter $H(f)$. There are two summation operations on the right-hand-side of (4). The sum indexed by subscript l is introduced by the mixing function $p_i(t)$. The sum indexed by k is due to non-ideal stopband of $H(f)$, which is undesirable since it renders the matrix $\mathbf{A}$ in (6) to a function of ω. The MWC with a ω-dependent $\mathbf{A}$ is beyond the scope of this paper; this setting is discussed in [5]. Instead, we assume that the stopband response is designed to be sufficiently small that it can be neglected (*e.g.*, less than -60 dB), based on which we propose to oversample $y_i(t)$ at rate f_s which is larger than the stopband width. This assumption will be continued to the following discussion.

By assuming that $H(f)$ is zero beyond $[-f_s/2, f_s/2]$, (4) can be expressed as

$$Y_i(e^{jw}) = \sum_{l=-L_0}^{L_0} c_{il} Q_l(\omega), \tag{10}$$

where

$$Q_l(\omega) = X\left(\frac{f_s}{2\pi}\omega - l f_p\right) H\left(\frac{f_s}{2\pi}\omega\right). \tag{11}$$

Therefore, from (7) we actually solve for $Q_l(\omega)$ rather than $z_l(\omega)$. After interpolation in (8) and modulation in (9), the resulted spectrum of the reconstructed signal is calculated as

$$\hat{X}(f) = \sum_{l=-L_0}^{L_0} Q_l\left(\frac{2\pi}{f_s}(f + l f_p)\right). \tag{12}$$

Substituting (11) in (12) we obtain

$$\hat{X}(f) = \left(\sum_{l=-L_0}^{L_0} H(f + l f_p)\right) X(f). \tag{13}$$

Since $X(f)$ is only non-zero within $[-f_{\text{NYQ}}/2, f_{\text{NYQ}}/2]$, the PR condition for $H(f)$ is then

$$\sum_{l=-L_0}^{L_0} H(f + l f_p) = 1, \quad f \in \left[-\frac{f_{\text{NYQ}}}{2}, \frac{f_{\text{NYQ}}}{2}\right]. \tag{14}$$

We note that the PR condition in (14) coincides with the well-known Nyquist ISI criterion [6], and any lowpass filter that satisfies (14) is usually referred to as a *Nyquist filter*. Typical examples include raised cosine functions, Kaiser windows and others [6]. Any such filter will lead to PR without requiring any further processing.

3.2. Digital compensating FIR filters

In the above discussion we demonstrated that any Nyquist filter which satisfies (14) ensures PR. For lowpass filters that do not meet the PR condition, we now propose a simple compensation in the digital domain. The compensation scheme is illustrated in Fig. 2 for a single channel. Let $D(e^{j\omega})$ be the digital frequency response of

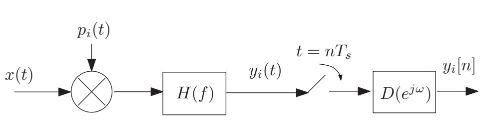

Fig. 2. Illustration of one channel of the digital correction scheme.

the compensation filter, where we use the notation $e^{j\omega}$ to emphasis that the DTFT is 2π-periodic. The relationship in (10) still holds by replacing $Q_l(\omega)$ with

$$Q_l(\omega) = X\left(\frac{f_s}{2\pi}\omega - l f_p\right) H\left(\frac{f_s}{2\pi}\omega\right) D\left(e^{j\omega}\right). \tag{15}$$

Therefore, to ensure perfect reconstruction we need to design a digital filter $D(e^{j\omega})$ such that the frequency response of the corrected analog filter

$$T(f) = H(f) D\left(e^{j2\pi T_s f}\right) \tag{16}$$

satisfies (14).

Here we show that we can implement $D(e^{jw})$ by an FIR filter. Let $\{d_n\}_{n=-N_0}^{N_0}$ be the coefficients of an FIR filter with order $2N_0 + 1$. The digital frequency response $D(e^{j\omega})$ is

$$D(e^{j\omega}) = \sum_{n=-N_0}^{N_0} d_n e^{-j\omega}. \tag{17}$$

Combining (16) and (17),

$$T(f) = \mathbf{h}(f)^H \mathbf{d}, \tag{18}$$

where $\mathbf{h}(f) = H(f)^*(e^{-j2\pi N_0 T_s f}, \ldots, e^{j2\pi N_0 T_s f})^T$, and $\mathbf{d}$ is the coefficient vector $\mathbf{d} = (d_{-N_0}, \ldots, d_{N_0})^T$. The design objective is to seek coefficients $\{d_n\}_{n=-N_0}^{N_0}$ such that $T(f)$ in (18) best meets the PR condition in terms of integrated squared error:

$$\min_{\mathbf{d}} \int_{-f_{\text{NYQ}}/2}^{f_{\text{NYQ}}/2} \left| \sum_{l=-L_0}^{L_0} \mathbf{h}(f - l f_p)^H \mathbf{d} - 1 \right|^2 \mathrm{d}f. \tag{19}$$

Since (19) is a least-squares problem, it has a closed-form solution. It can be shown that the optimal solution is:

$$\mathbf{d}_{\text{opt}} = \left[\int_{-f_{\text{NYQ}}/2}^{f_{\text{NYQ}}/2} \mathbf{g}(f)\mathbf{g}(f)^H \mathrm{d}f\right]^{-1} \int_{-f_{\text{NYQ}}/2}^{f_{\text{NYQ}}/2} \mathbf{g}(f) \mathrm{d}f \tag{20}$$

where $\mathbf{g}(f) = \sum_{l=-L_0}^{L_0} \mathbf{h}(f - l f_p)$. When $\mathbf{h}(f)$ contains $H(f)$ and is not specified analytically, computing the integrals in (20) can be performed using numerical methods.

4. EXPERIMENTAL VALIDATION

In this section we demonstrate the proposed compensation method by experimental results, where two examples are studied.

In the first example we simulate the MWC system with non-ideal filters and evaluate the overall performance of the proposed compensation. The input $x(t)$ is a multiband signal consisting of 3 pairs of bands, each of width $B = 50$ MHz, defined as

$$x(t) = \sum_{i=1}^{3} \sqrt{E_i B}\,\text{sinc}(B(t - \tau_i)) \cos(2\pi f_i (t - \tau_i)), \tag{21}$$

where the energy coefficients $E_i = \{1, 2, 3\}$, the time offsets $\tau_i = \{1.1, 0.3, 0.7\}$ μsecs, and the carriers are set to $f_i = \{1.8, 1.2, 2.8\}$ GHz. The Nyquist rate of $x(t)$ is $f_{\text{NYQ}} = 10$ GHz. We choose $L_0 = 97$ and $f_p = f_{\text{NYQ}}/(2L_0 + 1) \simeq 51.3$ MHz. The number of channels is $m = 50$ and the same mixing functions $p_i(t)$ are used as in [1]. The main difference between the simulation in [1] and the one proposed here is that we use an 8-order Butterworth filter in each channel. The 3-dB bandwidth of the Butterworth filter is set to f_p. With a moderate oversampling, f_s is chosen as $f_s = 5/3 f_p$. Finally, all the continuous signals are represented by a dense grid of 78975 samples observed within $[0, 1.6]$ μsecs, where the time resolution is $1/(5 f_{\mathrm{NYQ}})$. As predicted by our analysis, direct reconstruction using the standard approach yields distortions in the recovered signal, which can be found be comparing Fig. 3(b) with Fig. 3(c). We use a 21-order FIR filter to correct the non-ideal Butterworth filter in each channel. The coefficients are determined by (20) and the reconstructed signal after applying digital corrections is plotted in Fig. 3(c). As expected, near perfect recovery is achieved. For further demonstration, we examine the PR condition of the employed Butterworth filter $H(f)$ and the corrected filter $T(f)$ (obtained by (16)) in Fig. 4, where $\sum_l H(f + l f_p)$ and $\sum_l T(f + l f_p)$ are plotted in dB. It can be seen that for $H(f)$ there exists significant distortions, which illustrates why direct reconstruction does not ensure PR.

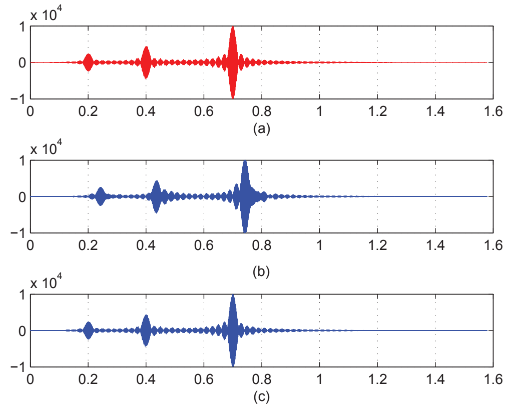

Fig. 3. Reconstructions using Butterworth filters. (a) The multiband input signal $x(t)$. (b) Direct reconstruction signal. (c) Reconstructed signal after digital corrections.

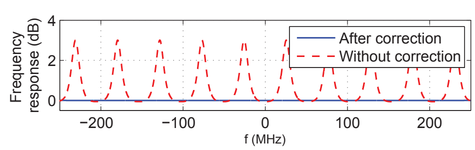

Fig. 4. PR condition. Dot line represents $20\log_{10}|\sum_l H(f + l f_p)|$, and solid line represents $20\log_{10}|\sum_l T(f + l f_p)|$.

In the second example, we employ the proposed method to correct a real analog lowpass filter implemented in a recent hardware realization of the MWC system [7]. The frequency response is measured by an Agilent HP8753E network analyzer and the magnitude is shown in Fig. 5. Here we set $f_p = 60$ MHz and $f_s = 100$ MHz, The results of the correcting FIR filter and the PR condition test are shown in Fig. 6 and Fig. 7. These results indicate that our proposed compensator can be applied to practical applications in signal processing and communications.

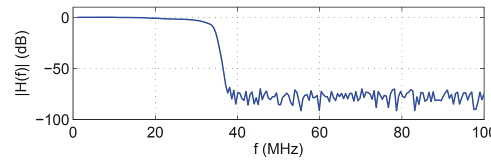

Fig. 5. Frequency response of a real lowpass filter.

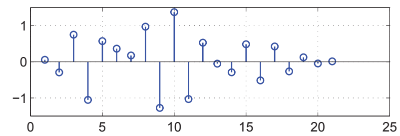

Fig. 6. Coefficients of the correcting FIR filter for the real filter.

5. CONCLUSION

In this paper, we treated the problem of compensating the MWC when the lowpass filters are not ideal. A PR condition for general filters was developed. We demonstrated that, using a moderate oversampling, perfect reconstruction could be achieved by any Nyquist filter. Then, for lowpass filters which do not satisfy the PR condition, we proposed to correct them in the digital domain using FIR filters. We presented a least-squares approach for determining the coefficients of the compensation filter. Both numerical simulations and real measured data demonstrated that the proposed compensation method is effective for recovering near prefect reconstruction.

6. REFERENCES

[1] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals,", *submitted to IEEE Selected Topics on Signal Processing*, Feb. 2009.

[2] M. Mishali, Y. C. Eldar, and J. A. Tropp, "Efficient sampling of sparse wideband analog signals, in *Proc. of IEEEI*, 25th convention, Dec. 2008, pp. 290-294.

[3] M. Mishali and Y. C. Eldar, "Reduce and boost: Recovering arbitrary sets of jointly sparse vectors, *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4692-4702, Oct. 2008.

[4] Y. C. Eldar, "Compressed sensing of analog signals, *submitted to IEEE Trans. Signal Process.*, Jun. 2008.

[5] Y. C. Eldar, "Uncertainty Relations for Shift-Invariant Analog Signals", to appear in IEEE Trans. Inform. Theory.

[6] John G. Proakis, "Digital Communications, 3rd Edition", *McGraw-Hill Book Co.*, 1995. ISBN 0-07-113814-5

[7] The modulated wideband converter, `http://www.technion.ac.il/~moshiko/software.html`

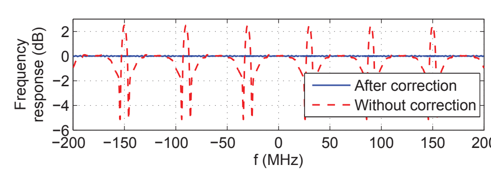

Fig. 7. Tests of the PR condition for the real filter.