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Complex Networks of Interacting Solitons: Noise-Enhanced Memory and Self-Synchronization

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Abstract: We propose complex networks constructed from interacting vector solitons. Within soliton-based networks, we demonstrate memory effects that are greatly enhanced by noise, as well as spontaneous entire network self-synchronization effects.

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Complex networks have been fascinating scientists for more than a decade by now. They appear in diverse areas, ranging from natural systems such as food chains and metabolic networks to man-made systems like electrical power grids and the internet [1,2,3]. At the heart of complex networks is the fact that a complex nonlinear system does not behave as a superposition of its building blocks. Rather, a network can display dynamics of its own, either collective – where the networks behaves as one entity, partially collective, or sometimes highly fragmented, where different sections behave in completely uncorrelated fashion. The internal dynamics of complex networks can take on many forms, some conceived as crucial to the existence of life (e.g., DNA repair systems [4]), some posing hazards to civilization (spread of infectious diseases [5]), and some entertaining, linking seemingly unrelated events [1,6]. Much of the work on complex networks concentrates on their structure, not addressing the evolution of signals within the networks [2]. However, networks do have complex internal dynamics, which is very interesting in its own right [1]. In fact, the internal dynamics of complex networks often plays a crucial role in the structural formation of the network itself, as happens even in natural weblike structures [7]. Perhaps the largest challenge in current research on complex networks is the one highlighted by Strogatz in his keynote review [1]: "**The greatest challenge today …. is the accurate and complete description of complex systems**."

We here take the next step in this vision, and propose complex networks constructed from interacting fields, where the interaction dynamics at each individual node in the system has infinite degrees of freedom. We use solitons as the "carriers of interactions" between nodes in the network. In doing that, we take advantage of the generic properties of solitons [8], and construct networks made of interacting fields, in which the dynamical parameter characterizing the internal evolution is the ratio between amplitudes of the fields comprising the solitons, while all other properties (number of solitons, power and momentum they carry, etc.) are conserved. Since the solitons are fields, the number of degrees of freedom for the interaction at each node in the network (the "interaction dimension") is in principle infinite, which could render the problem intractable. However, the conservation laws of solitons imply that the number of different solitons propagating within the network is uniquely defined by initial conditions. This feature is what makes the problem tractable. As examples, we study networks within which trains (sequences) of two vector solitons propagate and interact with one another at every node. We study memory effects in such soliton-based networks, and show that the memory is enormously enhanced by noise. We also find that such networks, with an infinite-dimensional dynamics, can exhibit spontaneous self-synchronization effects.

Let us discuss first the dimensionality of the nonlinear interaction as each node of the network, in terms of the possible degrees of freedom. In complex networks studied thus far, the number of available degrees of freedom for the interaction is limited, and is typically very small. Examples range from the simplest case of simple binary interactions (bits), to other kinds of interactions in man-made networks and in networks simulations [1]. More importantly, in all networks studies the number of degrees of freedom for the interaction at each node is finite [1,7]. In a sharp contrast, here we study networks in which the interaction at each node is between complex fields, hence the interaction outcome at each node can yield any complex value. Consequently, **the number of degrees of freedom for the interaction at each node is infinite**. In principle, this could make such networks theoretically intractable. However, we use solitons as the carrier of interaction. Solitons, self-localized wavepackets which behave and interact with one another as real particles do [9], obey conservation laws; in particular, for integrable systems the number of conservation laws is infinite. A natural (simplest) choice for using solitons in networks could be solitons of the cubic nonlinear Schroedinger equation (Kerr solitons). However, such solitons have been studied for computation purposes, and it was proven that the interactions between them are "oblivious", in terms of information processing (i.e., computation machines made up from Kerr solitons would never be Turing-equivalent) [9]. Manakov solitons, on the other hand, being comprised of two interacting fields, are ideal for this purpose, as they can construct a Turing machine [9]. More so, the interaction between a pair of Manakov solitons can be expressed as a simple bilinear transformation between complex numbers [9], which actually makes these simulations of large fields-based networks possible [otherwise, with non-soliton fields, the interaction at each node of the network would require solving a dynamic nonlinear PDE; this would render simulations of large networks unmanageable]. The interaction between Manakov solitons at a given node is sketched in Fig. 1a. The variables x,a,y,b are complex numbers representing the ratios between the fields comprising the "red" and "green" Manakov solitons at the input (x and a) and output (y and b) of the node.

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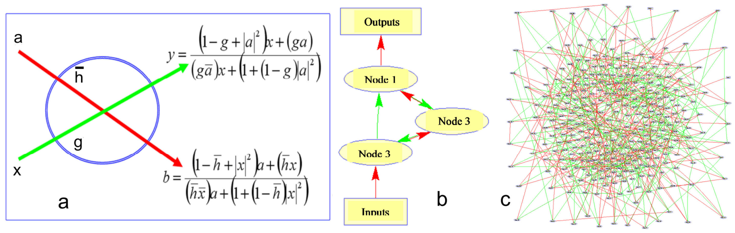

Fig.1. (a) Interaction between two Manakov solitons, and the bilinear transformations describing it. (b,c) Small and large soliton-based complex networks.

One of the consequences of integrability of Manakov solitons, is that the number of solitons is conserved, and so is the power and momentum of each one of them. This means that, in a network, the kinds of Manakov solitons circulating in the network is uniquely defined by the stream of incoming solitons into the network. The minimum number would be one: launching a train of identical solitons into a network. In this case, however, the network has very limited dynamics, because the outcome of the collision between two identical Manakov solitons is too simple [10]. The next option is launching two kinds of Manakov solitons into the network, say, two solitons of identical power but different velocities. This is exactly what we do here. Figures 1b, 1c depict a small and a large network into which two kinds of Manakov solitons are launched: the green soliton is circulating in the system, whereas the red soliton enters at one node and leaves at another. We emphasize that the quantity circulating in these networks is just the ratio between the field constituents of the solitons: the net number of solitons in the network at any time is fixed, hence the power in circulating in the network is constant in time. **What circulates in the network, reflecting its complex internal dynamics, is information alone.** In such fields-based network the number of degrees of freedom at each node is uniquely defined by the input, and could be in principle infinite.

Having constructed soliton-based networks, the question is, what are they good for? It is yet unknown if soliton-based networks could provide a better means for information processing. Irrespective of potential applications, we demonstrate several unique features of soliton-based complex networks. Figure 2 shows noise-enhanced memory effects in small and large networks. A sequence of (identical) red solitons is launched into the network between t=0 and some other specific time, and we examine the amplitudes of the red solitons remaining in the network at some node. After the input sequence of red solitons is terminated, red solitons continue to circulate in the network (blue curves in Fig. 2). This in itself not surprising, because both networks contain closed loops, so it is expected that some "memory" will survive from the time-limited stream of red solitons. However, when we superimpose some small-amplitude stochastic noise on the input stream of red solitons, the memory is enhanced by many orders of magnitude (red curves): in the small network it brings the memory to the level of the input signal. The memory enhancement via noise is even larger (>6 orders of magnitude) in the large network.

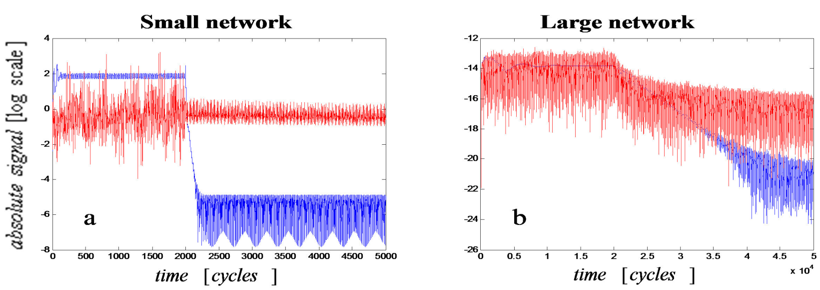

Fig.2. Memory effects in complex networks (blue curves) and their huge enhancements (red curves) by superimposing stochastic noise on the input signals. Shown are the absolute values of the ratios between fields comprising the solitons at some node, as they evolve in time.

We observe another unique feature of soliton-based complex networks: **spontaneous self-synchronization** (not shown here due to lack of space). When we launch two random time-sequences of red and green solitons into the networks, after some time the entire network operates in a synchronous fashion, as if a clock is timing it (or as if the solitons are launched at fixed time intervals).

To conclude, we have proposed soliton-based complex networks, with some of their unique features. Perhaps it is a little early to expect an experimental construction with a large soliton-based network, but certainly, a small network based on Manakov solitons (as in Fig. 1b) can be constructed today [10] The ideas presented here should be viewed in the context of statistical mechanics: **considering complex networks whose interaction carriers are fields is similar to ascribing a wavefunction to each (deterministic) particle, which has led to the quantum generalization of statistical mechanics.**

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