ROBUST DOWNLINK BEAMFORMING USING COVARIANCE CHANNEL STATE INFORMATION

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ABSTRACT

The problem of multiuser downlink beamforming is studied under the assumption that the transmitter has erroneous covariance-based channel state information (CSI). The goal is to minimize the transmit power under the worst-case quality-of-service (QoS) constraints. Previous convex optimization-based solutions to this problem involve several coarse approximations of the original problem. In our proposed solution, such coarse approximations are avoided and an exact representation of the worst-case solution is obtained using Lagrange duality. The so-obtained problem is then converted to a convex form using semidefinite relaxation (SDR). Computer simulations show that the SDR step does not involve any approximation as the resulting solution is always rank-one. Simulation results demonstrate substantial performance improvements over earlier worst-case optimization-based downlink beamforming techniques.

Index Terms— Convex optimization, downlink beamforming, user quality-of-service

1. INTRODUCTION

Employing multiple antennas at base stations in cellular networks offers substantial improvements in performance and achievable throughput as compared to cellular networks with single-antenna base stations. In the downlink mode, multi-antenna transmitters can efficiently exploit the available channel state information (CSI) to mitigate multi-user interference and improve the user quality-of-service (QoS) [1].

The problem of multiuser downlink beamforming has been extensively studied during the last decade. In [1] and [2], downlink beamforming techniques using perfect instantaneous and covariance-based CSI have been considered. In practice, the CSI available at the transmitter is subject to errors caused by limited (quantized / erroneous / delayed) channel state feedback, estimation errors, short channel coherence time, etc. Since the methods developed for the perfect CSI case are quite sensitive to such channel uncertainties, several recent works have focused on designing downlink beamforming techniques that are robust to CSI errors; see [3]-[6] and references therein. Most of these papers assume imperfections in the instantaneous CSI. Only a few methods have been developed for imperfect covariance-based CSI [3], [4]. However, the use of covariance CSI is more practical due to significantly reduced feedback requirements, and is unavoidable in fast fading channel scenarios.

The work of Y. C Eldar was supported by the European Commission FP7 Network of Excellence in Wireless COMmunications NEWCOM++. The work of A. B. Gershman was supported by the European Research Council under Advanced Investigator Grant Scheme.

The robust technique of [3] uses convex optimization to solve the problem of worst-case optimization-based downlink beamformer design. The method of [4] develops a simplified version of this approach where robustness is only incorporated in the power allocation problem for different transmit weight vectors. However, there are several coarse approximations used in [3] and [4]. In particular, the minimum of the signal-to-interference-plus-noise ratio (SINR) with respect to the norm-bounded channel uncertainty is approximated as the ratio of the minimum of the numerator and the maximum of the denominator. Clearly, such an approximation can be very coarse in a number of practical scenarios when the uncertainty matrices in the numerator and denominator are the same (as in the single-cell case) or are dependent (as in the multiple-cell case). Moreover, the approaches of [3] and [4] ignore positive semidefinitness constraints on the downlink channel covariance matrices.

In this paper, we propose a new robust worst-case optimization-based downlink beamforming technique that avoids the approximations of [3] and [4]. In particular, our formulation explicitly takes into account the positive-semidefinite property of the downlink covariance matrices. We consider minimizing the transmitted power subject to the worst-case QoS constraints. We first show using Lagrange duality that this minimax problem can be formulated exactly as a single minimization problem. The resulting problem has a linear objective, a set of convex constraints, and an additional rank-one constraint. The latter is the only non-convex constraint. By dropping it, we obtain a semidefinite relaxation (SDR) of the original minimax beamforming problem. Similar to the robust beamformer of [3], the proposed technique is solved using convex semidefinite programming (SDP). Computer simulations show that the SDR step does not actually involve any approximation as the resulting solution of the final SDP problem is always rank-one. Simulation results demonstrate substantial performance improvements of the proposed beamformer relative to that of [3].

2. PROBLEM FORMULATION

Let us consider a single-cell flat-fading scenario with K decentralized single-antenna receivers (users) and a single N-antenna base station. Note that the extension of our approach to multiple cells / base stations is straightforward; the single-cell case is only assumed for the sake of notational simplicity. At time t, the base station transmits the $N \times 1$ vector

$$\mathbf{x}(t) = \sum_{k=1}^{K} s_k(t)\mathbf{w}_k \tag{1}$$

where $s_k(t)$ is the signal intended for the kth user and $\mathbf{w}_k$ is the $N \times 1$ complex weight vector for the kth user. The signal received by the ith user is given by

$$y_i(t) = \mathbf{h}_i^H \mathbf{x}(t) + n_i(t) \tag{2}$$

where $\mathbf{h}_i$ is the $N \times 1$ channel vector between the base station and the ith user, $n_i(t)$ is the noise of ith user, and $(\cdot)^H$ is the Hermitian transpose. The noise waveforms are assumed to be zero-mean circularly symmetric white Gaussian with the variance σ_i^2. The received SINR of the ith user can be expressed as [3]

$$\text{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{l=1, l\neq i}^{K} \mathbf{w}_l^H \mathbf{R}_i \mathbf{w}_l + \sigma_i^2} \tag{3}$$

where $\mathbf{R}_i = \text{E}\{\mathbf{h}_i \mathbf{h}_i^H\}$ is the downlink channel covariance matrix for the ith user and $\text{E}\{\cdot\}$ denotes the statistical expectation.

2.1. The Non-Robust Downlink Beamforming Problem

In [2], the channel covariance matrices are assumed to be perfectly known. The downlink beamforming problem is then stated as that of minimizing the total transmitted power subject to the user QoS constraints:

$$\min_{\{\mathbf{w}_k\}} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \quad \text{s.t.} \quad \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{l=1, l\neq i}^{K} \mathbf{w}_l^H \mathbf{R}_i \mathbf{w}_l + \sigma_i^2} \geq \gamma_i \quad i = 1, \ldots, K \tag{4}$$

where γ_i is the minimal acceptable SINR for the ith user and $\|\cdot\|$ denotes the Euclidean norm of a vector or the Frobenius norm of a matrix. The authors of [2] then have shown that this non-convex problem can be transformed into a convex SDP using an SDR approach. The have also proved that the SDR step involves no approximation of the original problem, as the resulting solution of the final SDP problem is always rank-one.

2.2. The Robust Downlink Beamforming Problem

If the available CSI at the transmitter is erroneous (that is, if the covariance matrices $\mathbf{R}_i$, $i = 1, \ldots, K$ are not known perfectly), then the approach based on (4) can be very sensitive to CSI imperfections. To improve the robustness of the design in (4), the authors of [3] proposed a robust modification. Specifically, they model the true channel covariance matrices as $\mathbf{R}_i + \mathbf{\Delta}_i$ ($i = 1, \ldots, K$) where $\mathbf{R}_i$ is the available estimate of the channel covariance matrix of the ith user and $\mathbf{\Delta}_i$ is the error in this estimate. The Frobenius norm of the latter error matrix is assumed in [3] to be upper-bounded by a known constant ϵ_i as $\|\mathbf{\Delta}_i\| \leq \epsilon_i$. The robust downlink beamforming problem is then formulated as

$$\begin{aligned} &\min_{\{\mathbf{w}_k\}} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \\ &\text{s.t.} \quad \min_{\|\mathbf{\Delta}_i\| \leq \epsilon_i} \frac{\mathbf{w}_i^H (\mathbf{R}_i + \mathbf{\Delta}_i) \mathbf{w}_i}{\sum_{l=1, l\neq i}^{K} \mathbf{w}_l^H (\mathbf{R}_i + \mathbf{\Delta}_i) \mathbf{w}_l + \sigma_i^2} \geq \gamma_i \\ &\qquad i = 1, \ldots, K. \end{aligned} \tag{5}$$

The difference between (4) and (5) is in that (5) uses the worst-case QoS constraints rather than the conventional QoS constraints.

In [3], the following approximation of (5) is used:

$$\begin{aligned} &\min_{\{\mathbf{w}_k\}} \sum_{i=1}^{K} \|\mathbf{w}_k\|^2 \\ &\text{s.t.} \quad \frac{\mathbf{w}_i^H (\mathbf{R}_i - \epsilon_i \mathbf{I}) \mathbf{w}_i}{\sum_{l=1, l\neq i}^{K} \mathbf{w}_l^H (\mathbf{R}_i + \epsilon_i \mathbf{I}) \mathbf{w}_l + \sigma_i^2} \geq \gamma_i \\ &\qquad i = 1, \ldots, K. \end{aligned} \tag{6}$$

The latter problem is mathematically similar to (4) and, therefore, can be solved in the same way as (4), i.e., using the SDR technique.

Unfortunately, the transition between (5) and (6) involves several conservative approximations that may affect the performance of the beamformer in (6). In particular, the worst-case QoS constraints in (5) are replaced by

$$\frac{\min_{\|\mathbf{\Delta}_i\| \leq \epsilon_i} \mathbf{w}_i^H (\mathbf{R}_i + \mathbf{\Delta}_i) \mathbf{w}_i}{\max_{\|\mathbf{\Delta}_i\| \leq \epsilon_i} \sum_{l=1, l\neq i}^{K} \mathbf{w}_l^H (\mathbf{R}_i + \mathbf{\Delta}_i) \mathbf{w}_l + \sigma_i^2} \geq \gamma_i. \tag{7}$$

Clearly, as the same uncertainty matrix $\mathbf{\Delta}_i$ is used in the numerator and denominator of the ith worst-case QoS constraint in (5), the transition from the latter constraint to (7) can be very loose (i.e., the constraint in (7) can be much more conservative than that used in (5)). The same remark applies to the multiple-cell case where the dependence between the numerator and denominator of the worst-case QoS constraint is caused by the fact that multiple co-channel users can be assigned to one base station.

Another coarse approximation that is used in (5) (and, respectively, that affects the constraint in (6)) is that the positive semidefiniteness of the covariance matrices $\mathbf{R}_i + \mathbf{\Delta}_i$, $i = 1, \ldots, K$ is ignored in (5). This approximation is also quite conservative, as it further strengthens the QoS constraints.

As a result of these two approximations, the technique of [3] can be overly conservative, which may lead to infeasibility of the robust design and to an unnecessary increase of the total transmitted power. Below, we develop an alternative approach to solve the robust worst-case beamforming problem that avoids these conservative approximations and, hence, offers a more flexible robust beamformer design. As we will see, we solve the inner minimization in (5) exactly, and also take the positive semidefitness of the downlink covariance matrices into account.

3. THE PROPOSED ROBUST DOWNLINK BEAMFORMER

The aim of this section is to solve the robust downlink beamforming problem (5), also taking into account the positive semidefiniteness constraints for the matrices $\mathbf{R}_i + \mathbf{\Delta}_i$, $i = 1, \ldots, K$.

First, let us introduce the matrices

$$\mathbf{A}_i \triangleq \gamma_i \sum_{l=1, l\neq i}^{K} \mathbf{w}_l \mathbf{w}_l^H - \mathbf{w}_i \mathbf{w}_i^H. \tag{8}$$

Then, using the property $\text{Tr}\{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i\} = \text{Tr}\{\mathbf{R}_i \mathbf{w}_i \mathbf{w}_i^H\}$ where $\text{Tr}\{\cdot\}$ is the trace of a matrix, the ith constraint in (5) can be written as

$$-\text{Tr}\{\mathbf{R}_i \mathbf{A}_i\} - \sigma_i^2 \gamma_i \geq \max_{\|\mathbf{\Delta}_i\| \leq \epsilon_i} \text{Tr}\{\mathbf{\Delta}_i \mathbf{A}_i\} \tag{9}$$

or, equivalently,

$$\min_{\|\mathbf{\Delta}_i\| \leq \epsilon_i} -\left(\text{Tr}\{\mathbf{\Delta}_i \mathbf{A}_i\} + \text{Tr}\{\mathbf{R}_i \mathbf{A}_i\} + \sigma_i^2 \gamma_i\right) \geq 0. \tag{10}$$

Adding the positive semidefinite constraint on $\mathbf{R}_i + \mathbf{\Delta}_i$, the QoS constraints correspond to the following optimization problem:

$$\begin{aligned}\min_{\mathbf{\Delta}_i} & -\big(\mathrm{Tr}\{\mathbf{\Delta}_i\mathbf{A}_i\} + \mathrm{Tr}\{\mathbf{R}_i\mathbf{A}_i\} + \sigma_i^2\gamma_i\big)\\ \text{s.t.} \quad & \|\mathbf{\Delta}_i\| \le \epsilon_i \\ & -\mathbf{R}_i - \mathbf{\Delta}_i \preceq 0. \end{aligned} \tag{11}$$

For given $\mathbf{A}_i$, the problem (11) is convex. Therefore, it can be replaced by its dual value. The Lagrange dual function can be written as

$$g(\lambda_i, \mathbf{Z}_i) = \inf_{\mathbf{\Delta}_i} f(\lambda_i, \mathbf{\Delta}_i, \mathbf{Z}_i) \tag{12}$$

where

$$\begin{aligned} f(\lambda_i, \mathbf{\Delta}_i, \mathbf{Z}_i) = \Big(& -\mathrm{Tr}\{\mathbf{\Delta}_i\mathbf{A}_i\} - \mathrm{Tr}\{\mathbf{R}_i\mathbf{A}_i\} - \sigma_i^2\gamma_i \\ & + \lambda_i(\|\mathbf{\Delta}_i\|^2 - {\epsilon_i}^2) - \mathrm{Tr}\{(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{Z}_i\}\Big). \end{aligned} \tag{13}$$

Differentiating with respect to $\mathbf{\Delta}_i$ and setting the derivative to zero, the optimal value of $\mathbf{\Delta}_i$ is

$$\mathbf{\Delta}_i = \frac{1}{2\lambda_i}(\mathbf{A}_i + \mathbf{Z}_i). \tag{14}$$

Therefore, the Lagrange dual function (12) becomes

$$g(\lambda_i, \mathbf{Z}_i) = -\frac{\|\mathbf{A}_i + \mathbf{Z}_i\|^2}{4\lambda_i} - \lambda_i{\epsilon_i}^2 - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \tag{15}$$

and the corresponding dual problem can be written as

$$\begin{aligned}\max_{\lambda_i, \mathbf{Z}_i} & -\frac{\|\mathbf{A}_i + \mathbf{Z}_i\|^2}{4\lambda_i} - \lambda_i{\epsilon_i}^2 - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \\ \text{s.t.} \quad & \lambda_i \ge 0,\ \mathbf{Z}_i \succeq 0. \end{aligned} \tag{16}$$

Maximizing the objective function in (16) with respect to λ_i leads to the following problem:

$$\begin{aligned}\max_{\mathbf{Z}_i} & -\epsilon_i\|\mathbf{A}_i + \mathbf{Z}_i\| - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \\ \text{s.t.} \quad & \mathbf{Z}_i \succeq 0. \end{aligned} \tag{17}$$

Recalling that (17) is equivalent to the minimization in (10), our problem becomes

$$\begin{aligned}\min_{\{\mathbf{w}_k\}} & \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \\ \text{s.t.} \ & \max_{\mathbf{Z}_i \succeq 0}\big(-\epsilon_i\|\mathbf{A}_i + \mathbf{Z}_i\| - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i\big) \ge 0 \\ & \forall\, i = 1, \ldots, K. \end{aligned} \tag{18}$$

Clearly, the constraint in (18) will be satisfied if there exists some $\mathbf{Z}_i$ for which it is satisfied. Our problem therefore reduces to

$$\begin{aligned}\min_{\{\mathbf{w}_k, \mathbf{Z}_i\}} & \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \\ \text{s.t.} \quad & -\epsilon_i\|\mathbf{A}_i + \mathbf{Z}_i\| - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \ge 0 \\ & \mathbf{Z}_i \succeq 0, \quad \forall\, i = 1, \ldots, K. \end{aligned} \tag{19}$$

Introducing a new variable $\mathbf{W}_i \triangleq \mathbf{w}_i\mathbf{w}_i^H$, the variable $\mathbf{A}_i$ can be rewritten as

$$\mathbf{A}_i = \gamma_i \sum_{l=1, l\neq i}^{K} \mathbf{W}_l - \mathbf{W}_i$$

The problem (19) can be rewritten in terms of the new variables as

$$\begin{aligned}\min_{\{\mathbf{W}_k, \mathbf{Z}_i\}} & \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\} \\ \text{s.t.} \quad & -\epsilon_i\|\mathbf{A}_i + \mathbf{Z}_i\| - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \ge 0 \\ & \mathbf{Z}_i \succeq 0,\ \ \mathbf{W}_i \succeq 0,\ \ \mathrm{rank}(\mathbf{W}_i) = 1,\ \ \forall\, i = 1, \ldots, K. \end{aligned} \tag{20}$$

Now, the objective function in (20) is linear and all the constraints except the rank-one constraint are convex. Therefore, following the SDR approach, we will drop this non-convex constraint. The resulting problem becomes

$$\begin{aligned}\min_{\{\mathbf{W}_k, \mathbf{Z}_i\}} & \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\} \\ \text{s.t.} \quad & -\epsilon_i\|\mathbf{A}_i + \mathbf{Z}_i\| - \mathrm{Tr}\{\mathbf{R}_i(\mathbf{Z}_i + \mathbf{A}_i)\} - \sigma_i^2\gamma_i \ge 0 \\ & \mathbf{Z}_i \succeq 0,\ \ \mathbf{W}_i \succeq 0,\ \ \forall\, i = 1, \ldots, K. \end{aligned} \tag{21}$$

The problem (21) is a convex SDP problem and can be solved in polynomial time using interior-point algorithms [7], [8].

The solutions to SDR-based problems are not rank-one in general. In such cases, dropping the rank-one constraint is an approximation, and the so-called *randomization techniques* [9] have to be used to obtain an approximate solution of the original problem from the solution of the relaxed problem. However, in the case of (21), our simulation results show that we always obtain rank-one solutions for $\mathbf{W}_i$. Therefore, the SDR step does not cause any approximation, and the weight vector $\mathbf{w}_i$ can be retrieved from $\mathbf{W}_i$ exactly, from the principal eigenvector of $\mathbf{W}_i$.

4. SIMULATION RESULTS

In our simulations, the same scenario as in [2] is considered. The base station is equipped with a uniform linear array of $N = 8$ sensors spaced half a wavelength apart from each other and $K = 3$ singe-antenna users are assumed. One of the users is located at $\theta_1 = 10^\circ$ relative to the array broadside, while the other two are located at $\theta_{2,3} = 10^\circ \pm \phi$, where ϕ is varied from 5° to 10°. The users are assumed to be surrounded by a large number of local scatterers corresponding to a spread angle of $\sigma_\theta = 2^\circ$, as seen from the base station. The channel covariance matrices $\mathbf{R}_i, i = 1, \ldots, K$ are calculated in the same way as used in [2].

The user noises are assumed to be additive white Gaussian with variances $\sigma_i^2 = 1, i = 1, \ldots, K$. For each channel covariance matrix $\mathbf{R}_i$, the corresponding Hermitian error matrix $\mathbf{\Delta}_i$ is uniformly randomly generated in a sphere centered at zero with radius ϵ_i. It is assumed that $\epsilon_i = \epsilon$ for all $i = 1, \ldots, K$ and $\gamma_i = \gamma$ for all $i = 1, \ldots, K$. Throughout our simulations, the proposed robust technique is compared to the non-robust and robust techniques of [2] and [3], respectively.

Fig. 1 shows the total transmitted power versus the angular separation ϕ for $\gamma = 5$ dB and different values of ϵ. It can be seen from the figure that the larger the error bound ϵ, the more transmitted power is required for the robust beamformers. Also, the reduction of ϕ leads to an increase in the transmitted power, until the problem becomes infeasible. As expected, the proposed robust technique outperforms the robust approach of [3] in terms of the transmitted power. This performance gain becomes more pronounced when either ϕ is small, or ϵ is increased. As a reference, we have also plotted the transmitted power for the non-robust approach of [2].

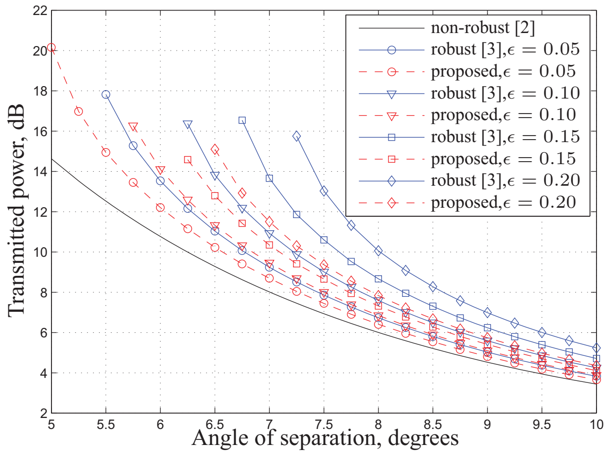

Fig. 1. Total transmitted power versus angular separation ϕ.

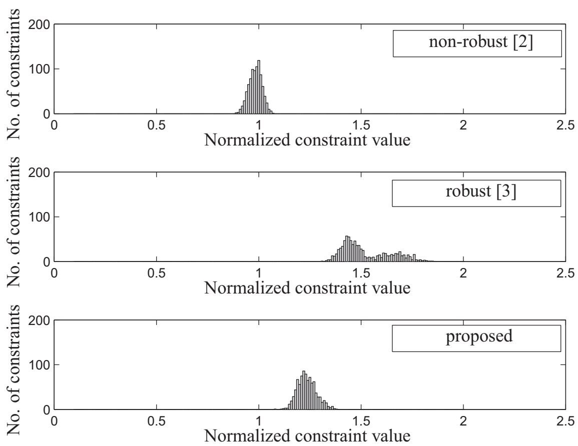

Fig. 2. Histogram of ζ for $\epsilon = 0.15$ and $\phi = 7°$.

Let us define the normalized constraint value ζ_i as

$$\zeta_i = \frac{\mathbf{w}_i^H(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{w}_i}{\gamma_i(\sum_{l=1,l\neq i}^{K} \mathbf{w}_l^H(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{w}_l + \sigma_i^2)}.$$

If $\zeta_i \geq 1$, then the corresponding constraint is satisfied. Otherwise, it is violated.

Fig. 2 shows the histograms of ζ_i in the case $\epsilon = 0.15$ and $\phi = 7°$. It can be seen that the non-robust technique of [2] violates about 50% of the constraints. The robust technique of [3] significantly oversatisfies the constraints as compared to the proposed robust approach.

Fig. 3 plots the total transmitted power versus γ for $\phi = 7°$ and different values of ϵ. Again, the proposed robust technique substantially outperforms the robust technique of [3] in terms of the transmitted power.

As can be seen from Figs. 1 and 3, different curves end at different values of ϕ and SINR, respectively. Missing points in these curves correspond to the values of ϕ and SINR at which the corresponding problem becomes infeasible, that is, when the QoS constraints of this problem cannot be satisfied.

It clearly follows from Figs. 1 and 3 that the proposed method maintains feasibility for larger values of γ and ϵ (and smaller values

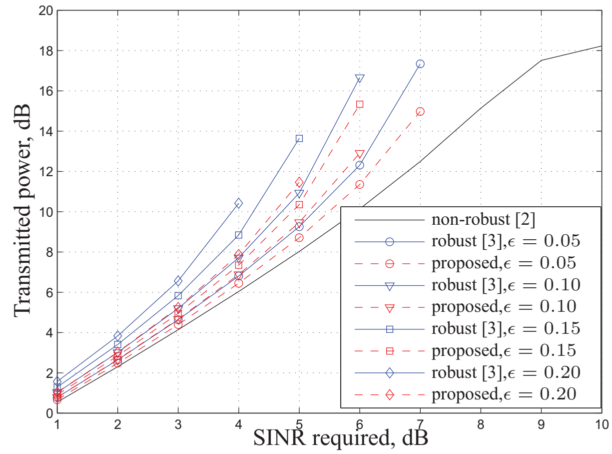

Fig. 3. Total transmitted power versus required SINR γ.

of ϕ) than the robust technique of [3]. Therefore, apart from substantial improvements in the transmitted power, our method also offers better feasibility than the approach of [3].

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