Semidefinite Relaxation of the 16-QAM Maximum Likelihood Detector

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Abstract — **We develop a computationally efficient approximation of the maximum likelihood (ML) detector for 16 Quadrature Amplitude Modulation (16-QAM) in multiple input multiple output (MIMO) systems. The detector is based on a semi definite relaxation (SDR) of the ML problem. The resulting optimization is a semi definite program (SDP) which can be solved in polynomial time with respect to the number of inputs in the system. Simulation results show that the SDR outperforms the conventional decorrelator detector by about 2.5dB.**

I. Introduction

One of the most promising suboptimal MIMO detectors is the SDR detector which recently gained considerable attention [1]. The SDR is an attempt to approximate the non convex ML detector using a convex program which can be efficiently solved in polynomial time. The SDR was initially derived for Binary Phase Shift Keying (BPSK) signaling in [1]. Later, in [2], it was extended to the detection of M-Phase Shift Keying (M-PSK) signaling. Here, we propose extending these results to the detection of other constellations used in digital communications. The key observation is that any finite alphabet constraint can be replaced by polynomial constraints. Then, we can convexify these constraints using standard relaxation techniques.

For illustration, we now derive the SDR detector for the well used 16-QAM constellation set. The standard ML detector for 16-QAM signaling over MIMO channels is

$$\text{ML}: \left\{ \begin{array}{lll} \min_{\mathbf{s}} & \|\mathbf{y}-\mathbf{H}\mathbf{s}\|^2, & \\ \text{s.t.} & \mathbf{s}_i \in \{\pm 1, \pm 3\}, & i=1,\cdots,K. \end{array} \right. \tag{1}$$

By introducing slack variables, the constraints can be expressed using quadratic polynomials:

$$\left\{ \begin{array}{lll} \min_{\mathbf{s},\mathbf{t}} & \|\mathbf{y}-\mathbf{H}\mathbf{s}\|^2 & \\ \text{s.t.} & s_i^2 - t_i = 0, & i=1,\cdots,K; \\ & t_i^2 - 10t_i + 9 = 0, & i=1,\cdots,K. \end{array} \right. \tag{2}$$

Now, there are two approaches for deriving the SDR based on (2). The first approach is to reformulate the problem in a higher dimension (the semidefinite matrix cone), and then relax the non convex constraints. Alternatively, the SDR detector can be derived as the Lagrange bidual program of (2), i.e., the dual program of the dual of the ML problem. The resulting program in both of these approaches is the following

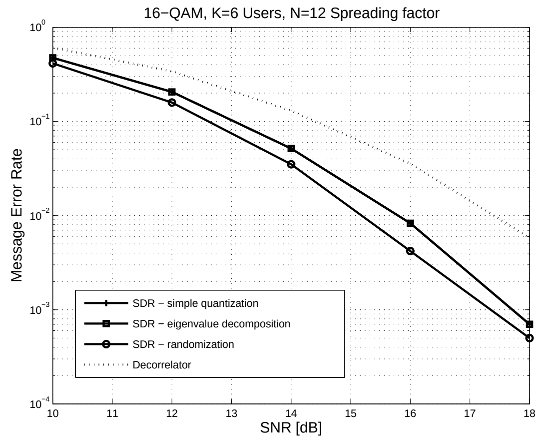

Fig. 1: Message error rates of a random MIMO system with $K=6$ and $N=12$ using 16-QAM.

SDP which can be efficiently solved in polynomial time:

$$\text{SDR}: \left\{ \begin{array}{ll} \min_{\mathbf{W}} & \text{Tr}\left\{ \mathbf{W} \left[\begin{array}{ccc} \mathbf{H}^T\mathbf{H} & \mathbf{0} & -\mathbf{H}^T\mathbf{y} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{y}^T\mathbf{H} & \mathbf{0} & \mathbf{y}^T\mathbf{y} \end{array} \right] \right\}, \\ \text{s.t.} & \text{diag}\{\mathbf{W}_{1,1}\} - \mathbf{W}_{2,3} = 0; \\ & \text{diag}\{\mathbf{W}_{2,2}\} - 10\mathbf{W}_{2,3} + 9\mathbf{1} = \mathbf{0}; \\ & \mathbf{W} \succeq 0; \\ & \mathbf{W}_{3,3} = 1. \end{array} \right. \tag{3}$$

If the optimal argument $\mathbf{W}$ of SDR has rank one, then the relaxation is tight and the ML solution of $\mathbf{s}$ is the first $2K$ elements of the last column of $\mathbf{W}$. Otherwise, SDR is only an approximation of ML, and there is no strict relation between $\mathbf{W}$ and $\mathbf{s}$. Instead, there are a few standard techniques based on the eigenvalue decomposition or a randomization method for approximating $\mathbf{s}$ based on $\mathbf{W}$ [1,2]. These techniques were initially derived for the detection of BPSK signaling, but their generalization to the detection of 16-QAM signaling is straight forward.

Finally, simulation results provided in Fig. 1 show that the proposed SDR outperforms the conventional decorrelator detector by about 2.5dB at high signal to noise ratios.

References

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