Multiuser precoders for fixed receivers

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Abstract— **We consider the problem of designing precoders for multiuser systems using fixed receivers. We first derive a precoder that minimizes the transmitted power subject to signal to interference plus noise ratio (SINR) constraints, and then derive a precoder that maximizes the worst case SINR subject to a power constraint. We show that both problems can be solved using standard optimization packages. In contrast to most of the existing precoders, our precoder design is not limited to full rank systems, and is applicable to systems in which the number of users is smaller, equal or larger than the processing gain. Simulation results in a multiuser system show that the proposed precoders can significantly outperform existing linear precoders.**

I. Introduction

One of the central problems in multiuser systems is interference between users. The traditional way to deal with channel distortion and interference in such systems is receiver optimization. Recently, the search for simple, low complexity receivers, led researchers to optimize the transmitter without modifying the receiver. In this paper, we propose methods for designing multiuser transmit precoders given fixed effective channels, which represent both the suboptimal transmitter, the distorting channel and the suboptimal linear receiver.

One of the first precoders for fixed receivers is the transmit decorrelator presented in [1]. This precoder, as well as most of the precoders that followed it [2]-[7], are based on the common minimum mean squared error (MMSE) criterion. This criterion is usually computationally attractive, but does not guarantee optimality in any of the practical performance metrics, such as bit error rate (BER), throughput, or multiuser efficiency. On the other hand, in practical systems, such as systems using error correcting codes, these metrics are directly related to the output signal to interference plus noise ratios (SINR), and, in particular, to the worst SINR[1]. There is an obvious trade off between maximizing the SINRs and minimizing the averaged transmitted power. Therefore, we propose two design strategies. In each strategy, we optimize one of the two parameters, subject to a constraint on the other.

Another major drawback of most of the previous precoders is their assumption of a full rank effective channel. For example, one cannot decorrelate the channel in [1] if the channel is rank deficient, as is the case when the number of users is larger than the spreading factor. Our precoder is applicable to such scenarios too. Following [8] which addressed this problem in the context of optimal sequences design, we provide an upper bound for the maximal feasible SINRs in this case, and illustrate using an example when they can be achieved.

[1]Note that this is in contrast to problems where the receiver is not fixed but jointly designed with the precoder, in which case the MMSE and SINR criteria coincide.

Our precoder design is based on the powerful framework of convex optimization theory [9], which allows efficient numerical solutions using standard optimization packages. In the sequel, we show that our design problems can be solved using standard conic optimization packages, such as Second Order Cone (SOC) Programming, or Linear Matrix Inequalities (LMI) programming. We will also establish the connection between our problem and the Generalized Eigenvalue Problem (GEVP) [10]. Note that similar results were derived independently in the context of beamforming [11]-[13]. However, to the best of our knowledge, the use of the GEVP formulation has never been addressed before in this context.

The paper is organized as follows. In Section II we formulate our problem. Next, in Sections III-IV, we express our two design problems as solutions of standard optimization problems. Finally, in Section V, we analyze the performance of the proposed precoders using simulations in different scenarios.

The following notation is used: $[\mathbf{X}]_{i,j}$ denotes the (i,j)th element of the matrix $\mathbf{X}$, $\mathrm{diag}\{x_i\}$ denotes a diagonal matrix with the elements x_i, $\mathrm{vec}(\mathbf{X})$ denotes the vector obtained from stacking the columns of $\mathbf{X}$, $\mathbf{e}_i$ is a vector of zeros with a one on the ith element, $\mathbf{1}$ is a vector of ones, and $\mathbf{X} \succeq 0$ denotes a positive semidefinite matrix $\mathbf{X}$. Finally, $(\cdot)^H$, $(\cdot)^\dagger$, $\mathrm{Tr}\{\cdot\}$, $E_X[\cdot]$, and $\otimes$, $\|\cdot\|$, denote the Hermitian transpose, the Moore Penrose pseudoinverse, the trace, the expectation with respect to X, the Kronecker product, and the Euclidean norm, respectively.

II. Problem formulation

Consider a general multiuser communication system with a centralized transmitter. At each time instant, a block of symbols is precoded, modulated, and transmitted over a channel. The signal at the output of the receiver can be expressed as

$$\underbrace{\begin{bmatrix}\mathbf{y}_1\\ \vdots\\ \mathbf{y}_M\end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{Rx},1}\mathbf{H}_{\mathrm{Ch},1}\\ \vdots\\ \mathbf{H}_{\mathrm{Rx},M}\mathbf{H}_{\mathrm{Ch},M}\end{bmatrix}}_{\mathbf{H}} \mathbf{H}_{\mathrm{Tx}}\mathbf{T}\mathbf{b} + \underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{Rx},1}\mathbf{w}_1\\ \vdots\\ \mathbf{H}_{\mathrm{Rx},M}\mathbf{w}_M\end{bmatrix}}_{\mathbf{w}}, \quad (1)$$

where $\mathbf{y}_j$, $\mathbf{H}_{\mathrm{Rx},j}$, $\mathbf{H}_{\mathrm{Ch},j}$ and $\mathbf{w}_j$ are the received vector, receiver matrix, channel matrix and noise vector of the j'th user, respectively. The matrices $\mathbf{T}$ and $\mathbf{H}_{\mathrm{TX}}$ denote the centralized precoder and transmitter, respectively, and $\mathbf{b}$ is a vector of independent, unit variance symbols. By stacking the matrices of all the users, we arrive at the following simplified model:

$$\mathbf{y} \;=\; \mathbf{HTb} + \mathbf{w}. \tag{2}$$

where $\mathbf{y}$, $\mathbf{b}$ and $\mathbf{w}$ are length K vectors, and the matrices $\mathbf{H}$ and $\mathbf{T}$ are both of size $K \times K$. Our goal is to design the precoder $\mathbf{T}$ so that it will improve the performance of the system without modifying this fixed structure. It is well known that the performance metrics of a communication system (BER, throughput, etc.) are highly related to the expected output SINR of each sub channel:

$$\begin{aligned}\gamma_i &= \frac{E_{\mathbf{b}}\left\{\left|[\mathbf{HT}]_{i,i}\,\mathbf{b}_i\right|^2\right\}}{E_{\mathbf{b}}\left\{|[\mathbf{HTb}]_i|^2 - \left|[\mathbf{HT}]_{i,i}\,\mathbf{b}_i\right|^2\right\} + E_{\mathbf{w}}\left\{[\mathbf{w}]_i^2\right\}} \\ &= \frac{|[\mathbf{HT}]_{ii}|^2}{[\mathbf{HTT}^H\mathbf{H}^H]_{ii} - |[\mathbf{HT}]_{ii}|^2 + \sigma_i^2}, \quad i = 1\cdots K, \end{aligned} \tag{3}$$

where σ_i^2 denotes the variance of the i'th element in $\mathbf{w}$. A conflicting performance metric is the average transmitted power:

$$\begin{aligned} P &= E_{\mathbf{b}}\left[\mathrm{Tr}\left\{\mathbf{H}_{\mathrm{Tx}}\mathbf{Tbb}^H\mathbf{T}^H\mathbf{H}_{\mathrm{Tx}}^H\right\}\right] \\ &= \mathrm{Tr}\left\{\mathbf{T}^H\mathbf{H}_{\mathrm{Tx}}^H\mathbf{H}_{\mathrm{Tx}}\mathbf{T}\right\}. \end{aligned} \tag{4}$$

Thus, we propose two opposite criteria for the design. The first designs the precoder to maximize the minimum SINR, subject to a power constraint:

$$\mathrm{PCO}(P_o) : \left\{ \begin{array}{ll} \max_{\mathbf{T}} & \min_i \gamma_i \\ \text{s.t.} & P \le P_o. \end{array} \right. \tag{5}$$

The second criterion designs the precoder to minimize the required power subject to SINR constraints[2]:

$$\mathrm{SCO}(\gamma_o) : \left\{ \begin{array}{ll} \min_{\mathbf{T}} & P \\ \text{s.t.} & \gamma_i \ge \gamma_o, \quad i = 1, \cdots, K. \end{array} \right. \tag{6}$$

Optimization problems PCO and SCO are closely related. It is easy to show that the optimal value of PCO is continuous, and strictly monotonically increasing in P, and that the optimal value of SCO is continuous, and strictly monotonically increasing in γ. Furthermore, if γ is the optimal value of PCO(P), then P is the optimal value of SCO(γ), and vice versa. An attractive property of the optimization problems is that the optimal solutions promise equal SINRs and fairness among all the outputs.

III. SINR CONSTRAINED PROGRAM

The first important property of an optimization problem is its feasibility, i.e., whether a solution exists. Unfortunately, the SCO programs are not always feasible. If the effective channel $\mathbf{H}$ is full rank, a solution always exists. The channel can be inverted by the precoder, eliminating all interference and producing a noise limited system in which any SINR can be attained by increasing the transmitted power. Otherwise, when the effective channel is rank deficient, it is clear that the interference cannot be eliminated, and there is an upper bound on the SINRs even in a zero noise environment. In [8], it was shown that even when an optimal linear receiver is used, the problem is feasible if and only if the following bound is satisfied:

$$\gamma_i \le \frac{1}{\frac{K}{\mathrm{rank}(\mathbf{H})} - 1}, \quad i = 1\cdots K. \tag{7}$$

Obviously, when sub optimal receivers which are not optimized are used, the maximal SINRs can only decrease. In degenerated channels, e.g., when $\mathbf{H}$ has two identical rows, the bound cannot be achieved. However, in almost all practical channels this bound can be achieved even for a fixed sub optimal receiver. As an example, in the appendix we construct a precoder that attains the bound for any non degenerated rank $K-1$ channel.

We will now show that the SCO program can be represented as a standard optimization problem. Without loss of generality, we can choose $[\mathbf{HT}]_{i,i} \ge 0$, i.e., non negative real part and zero imaginary part. The SCO program can be formulated as:

$$\begin{aligned} \min_{\mathbf{T},\sqrt{P_o}} \quad & \sqrt{P_o} \\ \text{s.t.} \quad & \beta\,[\mathbf{HT}]_{i,i} \ge \left\| \begin{array}{c} \mathbf{T}^H\mathbf{H}^H\mathbf{e}_i \\ \sigma_i \end{array} \right\|, \quad i = 1, \cdots, K; \\ & \sqrt{P_o} \ge \|\mathrm{vec}(\mathbf{H}_{\mathrm{Tx}}\mathbf{T})\|, \end{aligned} \tag{8}$$

where $\beta = \sqrt{1+\gamma_o^{-1}}$. The constraints in (8) are of the form $z_1(\mathbf{x}) \ge \| \boldsymbol{z}_2(\mathbf{x}) \|$, where the scalar $z_1(\mathbf{x})$ and the vector $\boldsymbol{z}_2(\mathbf{x})$ depend affinely on the optimization variables $\mathbf{x}$. Such inequalities define convex sets which are called SOC. Thus, the program in (8) can be solved efficiently using any standard SOC package [14]. Such solvers can also numerically determine the feasibility of the problem.

Moreover, any SOC can also be represented as an LMI, i.e., a cone obeying $\mathbf{Z}(\mathbf{x}) \succeq 0$, where the matrix $\mathbf{Z}(\mathbf{x})$ depends affinely on the optimization variables $\mathbf{x}$. For example, the expected SINR and power inequalities are also equivalent to

$$\mathbf{A}_i(\mathbf{T}) = \left[\begin{array}{cc} \beta\,[\mathbf{HT}]_{i,i} & \left[\begin{array}{cc} \mathbf{e}_i^H\mathbf{HT} & \sigma_i \end{array}\right] \\ \left[\begin{array}{c} \mathbf{T}^H\mathbf{H}^H\mathbf{e}_i \\ \sigma_i \end{array}\right] & \beta\,[\mathbf{HT}]_{i,i}\,\mathbf{I} \end{array} \right] \succeq 0, \tag{9}$$

for $i = 1\cdots K$, and

$$\mathbf{B}(\mathbf{T}) = \left[\begin{array}{cc} \sqrt{P_o} & \mathrm{vec}^H(\mathbf{H}_{\mathrm{Tx}}\mathbf{T}) \\ \mathrm{vec}(\mathbf{H}_{\mathrm{Tx}}\mathbf{T}) & \sqrt{P_o}\mathbf{I} \end{array} \right] \succeq 0. \tag{10}$$

Thus, the SCO programs can also be solved using standard LMI packages [15]. However, SOC solvers have a much better worst case computational complexity than LMI solvers for these problems.

IV. POWER CONSTRAINED PROGRAM

Let us now turn to the PCO program. It is easy to verify that the PCO program is always feasible, since we can always scale $\mathbf{T}$ so that it will satisfy the power constraint. At first

[2] In general, each output can be constrained to a different value, but in this paper we consider the case of equal values.

glance, (5) seems similar to (6). However, it turns out to be considerably more complicated. This is because the matrix inequalities in (9) are linear in β or in $\mathbf{T}$, but not in both simultaneously. Thus, when β is an optimization variable and not a parameter, these constraints are no longer LMIs. In fact, the sets which they define are not convex[3]. Nonetheless, if we rewrite (9) and separate out the terms which are linear, we have

$$\mathbf{A}_i\left(\mathbf{T}\right) = \beta \mathbf{A}_i^1\left(\mathbf{T}\right) - \mathbf{A}_i^2\left(\mathbf{T}\right), \tag{11}$$

where $\mathbf{A}_i^1\left(\mathbf{T}\right)$ and $\mathbf{A}_i^2\left(\mathbf{T}\right)$ depend affinely on $\mathbf{T}$:

$$\begin{aligned}
\mathbf{A}_i^1\left(\mathbf{T}\right) &= \begin{bmatrix} [\mathbf{HT}]_{i,i} & \mathbf{0} \\ \mathbf{0} & [\mathbf{HT}]_{i,i}\,\mathbf{I} \end{bmatrix}; & (12)\\
\mathbf{A}_i^2\left(\mathbf{T}\right) &= \begin{bmatrix} 0 & -\begin{bmatrix} \mathbf{e}_i^H\mathbf{HT} & \sigma_i \end{bmatrix} \\ -\begin{bmatrix} \mathbf{T}^H\mathbf{H}^H\mathbf{e}_i \\ \sigma_i \end{bmatrix} & \mathbf{0} \end{bmatrix}.
\end{aligned}$$

Using (11) we can express PCO as

$$\begin{cases} \min_{\mathbf{T},\beta} & \beta \\ \text{s.t.} & \beta\mathbf{A}_i^1\left(\mathbf{T}\right) \succeq \mathbf{A}_i^2\left(\mathbf{T}\right), \quad i = 1,\cdots,K; \\ & \mathbf{A}_i^1\left(\mathbf{T}\right) \succeq 0, \quad i = 1,\cdots,K; \\ & \mathbf{B}\left(\mathbf{T}\right) \succeq 0. \end{cases} \tag{13}$$

Although not convex, problems with the structure in (11) have been investigated in the context of control theory, and are known as GEVP, i.e., minimizing the maximum generalized eigenvalue of a pencil of matrices $\mathbf{A}_i^1\left(\mathbf{T}\right)$ and $\mathbf{A}_i^2\left(\mathbf{T}\right)$ that depend affinely on the optimization variables (for more details see [10] and references within). Such problems can be solved using appropriate software, e.g., the GEVP command in the LMI toolbox [15].

A different approach for solving PCO, that does not require a dedicated GEVP software, exploits the connection between the PCO program and the SCO program. Specifically, we can solve PCO(P) by iteratively solving its convex counterpart $\widetilde{P}$ =SCO($\widetilde{\gamma}$) for different $\widetilde{\gamma}$'s until we find a solution in which $\widetilde{P} = P$:

PCO(P)
$\gamma_{\max} \leftarrow \mathrm{MaxSINR}$
$\gamma_{\min} \leftarrow \mathrm{MinSINR}$
repeat
  $\gamma \leftarrow \left(\gamma_{\min} + \gamma_{\max}\right)/2$
  $[\mathbf{T}, \widehat{P}] \leftarrow \mathrm{SCO}(\gamma)$
  if $\widehat{P} \leq P$
   then $\gamma_{\min} \leftarrow \gamma$
   else $\gamma_{\max} \leftarrow \gamma$
 until $\widehat{P} = P$
return $\mathbf{T}, \gamma$

Due to the strict monotonicity of SCO and continuity, the algorithm will converge.

[3]The exact definition of such sets is quasi convex [9].

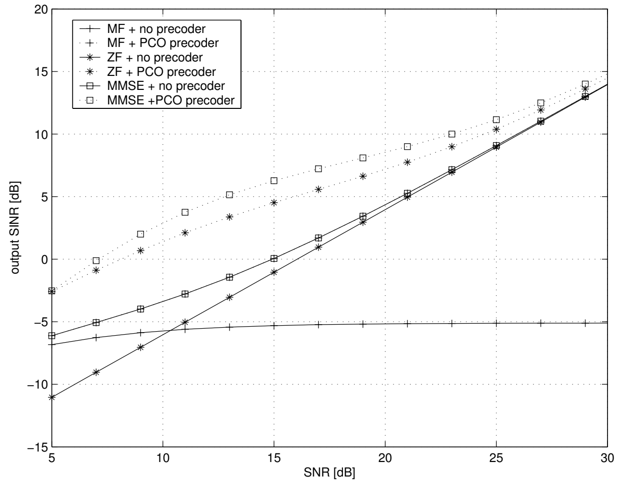

Fig. 1. SINR of a symmetric K=3 users system.

V. Simulation results

In this section, we analyze the performance of the proposed precoders in a multiuser downlink system through several simulations. At each symbol's period the base station transmits K symbols using an $N \times K$ signature matrix $\mathbf{H}_{\mathrm{Tx}} = \mathbf{S}$. The signatures are normalized so that $\left[\mathbf{S}^H\mathbf{S}\right]_{i,i} = 1$, and the cross correlations are denoted by $\left[\mathbf{S}^H\mathbf{S}\right]_{i,j} = \rho_{i,j}$. We assume ideal channels $\mathbf{H}_{\mathrm{Ch},i} = \mathbf{I}$ and equal noise variances σ^2. Each user detects its symbols using one of the standard linear receivers:

- MF receiver, $\mathbf{H}_{\mathrm{Rx},i} = \mathbf{e}_i^H\mathbf{S}^H$.
- Decorrelator receiver (ZF), $\mathbf{H}_{\mathrm{Rx},i} = \mathbf{e}_i^H\left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H$.
- MMSE receiver, $\mathbf{H}_{\mathrm{Rx},i} = \mathbf{e}_i^H\left(\mathbf{S}^H\mathbf{S} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{S}^H$.

We first consider the symmetric case in which $\rho_{i,j} = 0.9$. In Fig. 1 we plot the output SINRs for the three linear receivers. For comparison, we also plot the output SINRs that result from similar systems without a precoder. From the figure, we see that our precoder using a MF receiver attains the performance of an MMSE receiver without a precoder. Moreover, when our precoder is used with a ZF or an MMSE receiver, the output SINRs improve even more.

As a second example, we consider an equal power system with unequal cross correlations between the users signatures. Following [1], we simulate the cross correlations $\rho_{12} = 0.8$, $\rho_{13} = 0.9$, and $\rho_{23} = 0.7$. Each user uses an MF receiver. For comparison, we provide BER results of the decorrelator precoder [1], and the PCO precoder. In addition, a system without a precoder using a ZF receiver is also examined. The results are provided in Fig. 2. Due to the asymmetry, each of the three users performs differently without the precoders. On the other hand, the precoders promise fairness and equal BERs for all the users. Naturally, the performance of the best user degrades, but this is less important from a system prospective because the overall performance is dominated by the worst user. When compared to the decorrelating precoder, the PCO precoder gains up to 1dB.

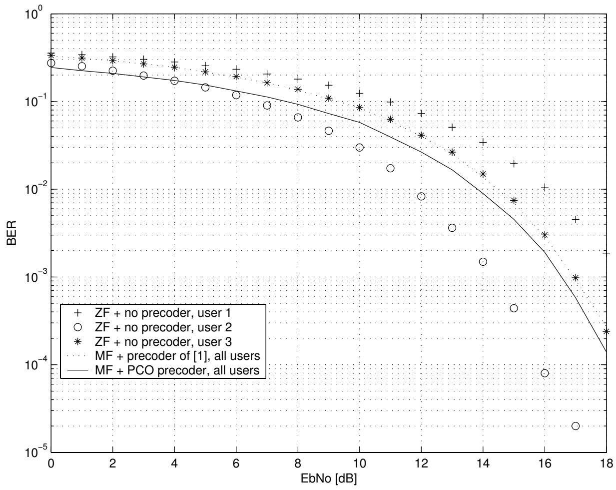

Fig. 2. BERs of a non symmetric K=3 users system.

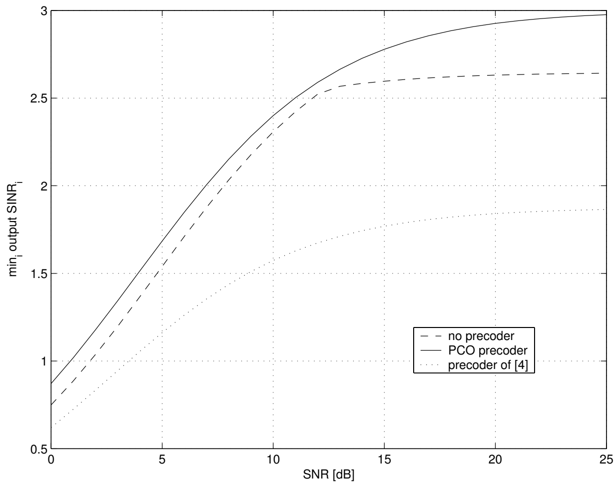

Fig. 3. Worst output SINR in a system with $K = 4$ and $N = 3$.

As explained in the previous sections, one of the main advantages of our precoder over previous ones is its performance in rank deficient systems. We now illustrate this property in a multiuser system with $K = 4$ users and length $N = 3$ sequences. The transmitter uses the optimal sequences of [8], and the receiver uses conventional matched filters. However, we use a distorting channel for the first user, i.e., $\mathbf{H}_{\mathrm{Ch},1}$ is a toeplitz matrix with the first row $[1.0, 0.8, 0.0]$. Due to this channel the sequences are no longer optimal and a precoder should be used. The common decorrelating precoder of [1] cannot be derived in this case as $N < K$. Therefore, we compare our results to the precoder of [4]. The worst output SINRs with and without the precoders are presented in Fig. 3. Our PCO precoder significantly outperforms the precoder of [4]. In addition, the SINRs of the PCO precoder, unlike those of its competitor, asymptotically converge to the bound in (7), i.e. $\gamma_i = \frac{1}{\frac{4}{3}-1} = 3$ for $i = 1 \cdots 4$.

APPENDIX

In this appendix, we construct a precoder $\mathbf{T}$ for a rank $K-1$, size K channel $\mathbf{H}$ that attains the bound in (7). Begin by noting that any $\mathbf{T}$ such that $\mathbf{HT} = \mathbf{DQ}$ where

$$[\mathbf{Q}]_{i,j} = \begin{cases} 1, & i = j; \\ -\frac{1}{K-1}, & i \neq j, \end{cases} \tag{14}$$

and $\mathbf{D}$ is diagonal with non zero elements, will satisfy (7) with equality. Consider the following chain:

$$\begin{aligned} \mathbf{HH}^{\dagger}\mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q} &= \left[\mathbf{I} - \mathbf{uu}^H\right]\mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q} \\ &= \mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q} - \mathbf{uu}^H\mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q} \\ &= \mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q} - \mathbf{u1}^H\mathbf{Q} = \mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q}, \end{aligned} \tag{15}$$

where the first equality holds if we choose $\mathbf{u}$ to be the null vector of $\mathbf{HH}^{\dagger}$. The last equality is due to the fact that $\mathbf{1}$ is in the null space of $\mathbf{Q}$. Hence, if we design the precoder as

$$\mathbf{T} = \mathbf{H}^{\dagger}\mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q}, \tag{16}$$

then $\mathbf{HT} = \mathrm{diag}\{1/\mathbf{u}_i\}\mathbf{Q}$, and the bound in (7) holds with equality. Note that if $\mathbf{u}_i = 0$ for some i, e.g., when $\mathbf{H}$ has two identical rows, then this construction is not possible.

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