A ROBUST MAXIMUM LIKELIHOOD MULTIUSER DETECTOR IN THE PRESENCE OF SIGNATURE UNCERTAINTIES

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ABSTRACT

We consider the problem of designing a multiuser detector for synchronous code-division multiple-access (CDMA) systems, where the signature matrix is subject to *structured* uncertainties. We seek the robust multiuser detector that minimizes the worst-case bit error probability (BER) over all possible values of the unknown signature matrix. We first develop the exact robust multiuser detector. Then, based on the framework of robust semidefinite programming (SDP), we suggest an approximation to the robust multiuser detector that can be obtained as a solution to an SDP, which can be solved efficiently using standard software packages. We then demonstrate through an example that by taking the structure of the uncertainty into account we can increase the detector performance over standard detection methods that do not consider the signature uncertainty.

1. INTODUCTION

Multiuser receivers for detection of code-division multiple-access (CDMA) signals try to mitigate the effect of the multiple-access interference (MAI) and the background noise. These include the optimal maximum likelihood (ML) detector and the linear mean-squared error (MMSE) detector [1]. The ML detector is optimal in the sense that it provides the minimum bit error probability (BER) in jointly detecting the data symbols of all users. Unfortunately, to implement the ML detector, it is necessary to solve a difficult combinatorial optimization problem. The ML detection problem can be solved by an exhaustive search in which the log likelihood function is evaluated for all possible combinations of the data symbols. However, the exhaustive search method is prohibitive for large numbers of users because of its exponential computational complexity.

Due to the intrinsic difficulty in solving the ML detection optimization problem, there has been much interest in the development of suboptimal but computationally efficient ML detectors [2]. Recently, an approximation based on semi-definite relaxation (SDR) was suggested [3],which does not suffer from local maxima, and was shown to have BER performance close to the BER performance of the true ML detector. Both the ML detector and the SDR approximation require the precise knowledge of the channel parameters, namely, the received amplitudes of the user's signals and the signature matrix. There are many scenarios, in practice, where these parameters may not be known or may be changing over time [4]. In these cases the ML and the SDR detectors may not achieve the designated optimality.

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Recently, robust multiuser techniques, that take into design consideration the effect of signature mismatch at the receiver, have attracted a great interest. These methods include the robust minimum output energy (MOE) linear detector [5], and a worst case performance optimization of the MOE multiuser detector [6, 7]. The robust MOE is a linear receiver, which requires multiple signal interval observations, and aims to optimize an energy based criterion while ensuring that the desired user response is distortionless.

In this paper we propose a new robust nonlinear multiuser detector which minimizes the worst-case (WC) BER across all possible channel parameters in the region of uncertainty, and which is based on a single observation signal interval. The suggested nonlinear WC ML detector is based on the ML criterion, i.e., this detector is superior, in the sense of BER, to any linear robust detector. The uncertainty model, which is in the base of WC ML detector design, is quite general, and includes a wide variety of practical cases. Similarly to the standard ML detector, the computational complexity of the WC ML detector grows exponentially with the number of users and the number of uncertain parameters. Therefore, we also suggest an efficient approximation to the WC ML detector, which is based on the SDR method. We then demonstrate, through simulation, that in the presence of uncertainty, the proposed robust multiuser detector outperforms the ML multiuser detector that does not take the uncertainty into account.

The paper is organized as follows. In Section 2, we formulate the robust ML detection problem. The exact robust ML multiuser detector is developed in Section 3. In Section 4, we suggest an efficient approximation of the robust ML multiuser detector. In Section 5, we demonstrate its advantages over the ML detector.

2. PROBLEM FORMULATION

Before proceeding to the detailed development of the WC ML multiuser detector, in this section we provide a formulation and overview of our problem.

Consider an m-user white Gaussian synchronous CDMA system, where each user transmits information by modulating a signature sequence. The received signal over one symbol duration can be modelled as:

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + w(t), \quad t \in \tau, \tag{1}$$

where b_k is the information symbol transmitted by the kth user, $s_k(t)$ is the kth user spreading-code waveform, $A_k \geq 0$ is the received amplitude of the kth user's signal, $w(t)$ is a zero-mean additive white Gaussian noise, K is the number of users, and τ is the observation interval. For concreteness, we assume that $b_k \in \{1, -1\}$. At the receiver, the received signal $r(t)$ is first filtered by a chip-matched filter and then sampled at the chip rate. In general, when the chip shaping is unknown, the filtering is done using suboptimal filters. The output is sampled at chip rate and digitally whitened. The equivalent discrete time white noise model [8] can be expressed in vector form as

$$\mathbf{r} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{w}, \tag{2}$$

where $\mathbf{S}$ is the $n \times m$ matrix of columns $\mathbf{s}_k$, where the vector $\mathbf{s}_k$ contains the corresponding samples, at the chip rate, of the kth user's signature waveform $s_k(t)$ filtered by the chip-matched filter, $\mathbf{A}$ is the diagonal matrix with diagonal elements $A_k > 0$, $\mathbf{b}$ is the data vector with components b_k, and the vector $\mathbf{w}$ contains the corresponding samples of the noise process.

The purpose of multiuser detection is to detect the symbols $\{b_k\}$ given the observed signal $\mathbf{r}$. The ML detector is optimal in the sense that the probability of incorrectly detecting $\{b_k\}$ is minimized, under the standard assumptions that the diagonal matrix $\mathbf{A}$ and the signature matrix $\mathbf{S}$ are known precisely at the receiver.

In practice, the signature vectors $\mathbf{s}_k$ and the diagonal matrix $\mathbf{A}$ may not be known exactly, for example, because of channel distortion [4]. Since the distorted $\mathbf{A}$ can be directly translated to an appropriate signature distortion, without lost of generality, we will focus on the case of signature mismatch. To model the uncertainty in the signature matrix $\mathbf{S}$, we assume that

$$\mathbf{S} = \mathbf{S}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{S}_i, \tag{3}$$

where the matrices $\mathbf{S}_i$, $0 \le i \le \ell$ are known, and δ_i are perturbations that are only known to lie in some perturbation set $\mathscr{D}$. Here, we consider the case in which $\mathscr{D}$ consists of block vectors, where each block satisfies a norm constraint. Denoting by $\mathbf{d}$ the vector with components δ_i, it follows that we can express $\mathbf{d}$ as a concatenation of N vectors $\mathbf{d}_k$, where each subvector $\mathbf{d}_k$ has length n_k, and satisfies a constraint of the form $\|\mathbf{d}_k\| \le \rho_k, 1 \le k \le N$, for a set of nonnegative numbers ρ_k, that determine the size of the uncertainty and can be estimated from the received signal. Thus,

$$\mathscr{D} = \left\{ \mathbf{d} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_\ell \end{bmatrix} \middle| \mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_N \end{bmatrix}, \mathbf{d}_k \in \mathbb{R}^{n_k}, \ \|\mathbf{d}_k\| \le \rho_k \right\}, \tag{4}$$

where $\sum_{k=1}^{N} n_k = \ell$, and $\nu_k = \sum_{s=1}^{k} n_s$. The uncertainty set $\mathscr{D}$ is quite general, and includes several cases of practical interest. For example, $\mathscr{D}$ includes the cases, considered in [7, 5], in which each of the signature vectors is of the form of

$$\mathbf{s}_k = \mathbf{s}_k^0 + \mathbf{a}_k, \tag{5}$$

where $\mathbf{s}_k^0$ is the nominal signature vector and $\mathbf{a}_k$ is the mismatch error vector satisfying $\|\mathbf{a}_k\| \le \rho_k$.

In the case of uncertainty of the form of (3), the received signal can be written as,

$$\mathbf{r} = \left[\mathbf{S}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{S}_i\right] \mathbf{A}\mathbf{b} + \mathbf{w} = \left[\mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{H}_i\right] \mathbf{b} + \mathbf{w}, \tag{6}$$

where $\mathbf{H}_i = \mathbf{S}_i\mathbf{A}$. Since δ_i are not known precisely, we cannot directly minimize the error probability. Instead, we seek the ML detector that minimizes the *worst-case error probability* over all possible values of δ_i. Thus, we seek the symbols that are the solution to the problem

$$\hat{\mathbf{b}}_{WCML} = \arg \min_{\mathbf{b} \in \{1,-1\}^N} \max_{\delta \in \mathscr{D}} \Delta(\mathbf{H}(\delta), \mathbf{b}), \tag{7}$$

where

$$\Delta(\mathbf{H}(\delta), \mathbf{b}) = \|\mathbf{r} - \mathbf{H}(\delta)\mathbf{b}\|^2. \tag{8}$$

We note that previously suggested robust detectors [5, 6, 7], are limited by their inherent linearity, require few symbol periods observation time and aim to minimize the WC BER indirectly. The suggested robust detector requires only one symbol period observation time, and directly minimizes the WC BER.

3. ROBUST ML DETECTOR

To develop a solution to (7), we note that (8) is a square norm of an affine function of δ, hence, (8) is a convex function of δ. We may therefore solve (7) by relying on the following lemma:

Lemma 1 (Maxima of a convex function). *Let $\mathbf{f}$ be a convex function on $\mathbb{R}^n$, and let $\mathbf{X} \in \mathbb{R}^n$ be a convex set with $\mathbf{M}$ extreme points $\mathbf{x}_j \in \mathbf{X}_e$, $1 \le j \le M$. Then*

$$\max_{\mathbf{x} \in \mathbf{X}} \mathbf{f}(\mathbf{x}) = \max_{j} \mathbf{f}(\mathbf{x}_j), \quad \mathbf{x}_j \in \mathbf{X}_e. \tag{9}$$

Equation (8) can be rewritten as

$$\begin{aligned} \Delta(\mathbf{H}(\delta), \mathbf{b}) &= \|\mathbf{r} - \mathbf{H}_0\mathbf{b} - \sum_{i=1}^{\ell} \delta_i \mathbf{H}_i \mathbf{b}\|^2 \\ &= \left\| \mathbf{r} - \mathbf{H}_0\mathbf{b} - \sum_{k=1}^{N} \mathbf{G}_k \mathbf{d}_k \right\|^2, \end{aligned} \tag{10}$$

where

$$\mathbf{G}_k = \left[\mathbf{H}_{\nu_{k-1}+1}\mathbf{b}, \mathbf{H}_{\nu_{k-1}+2}\mathbf{b}, ..., \mathbf{H}_{\nu_{k-1}+n_k}\mathbf{b}\right]. \tag{11}$$

By definition, a point $\hat{\mathbf{d}}_k \in \mathscr{D}$ is an extreme point of $\mathscr{D}$, if there is no positive length segment in the set $\mathscr{D}$ for which $\hat{\mathbf{d}}_k \in \mathscr{D}$ is an interior point. Hence the set $\mathscr{D}_e$ of extreme points of $\mathscr{D}$ consists of all the vectors $\hat{\mathbf{d}}_k^j \in \mathscr{D}$, where the only nonzero element is the jth element with the value of $\pm\rho_k$, i.e., vectors of the form

$$\hat{\mathbf{d}}_k^j = \begin{bmatrix} 0 \\ \vdots \\ \pm\rho_k \\ \vdots \\ 0 \end{bmatrix}, \quad 1 \le j \le n_k. \tag{12}$$

From Lemma 1 it therefore follows that the maximum of the convex function (10) on the convex set $\mathscr{D}$ is attended at one of the points (12), so that the optimization problem (7) can be reformulated as

$$\hat{\mathbf{b}}_{WCML} = \arg \min_{\mathbf{b} \in \{1,-1\}^N} \max_{\mathbf{d} \in \mathscr{D}_e} \Delta(\mathbf{H}(\mathbf{d}), \mathbf{b}). \tag{13}$$

The size of the set $\mathscr{D}$ of extreme points is ℓ. Hence the robust optimization problem, (13), can be solved exactly using an exhaustive search with a complexity of $2^{K+\ell}$.

4. AN APPROXIMATE ROBUST ML DETECTOR

We now develop a computationally efficient approximation of the robust ML detector of (13). To this end, we express the robust optimization problem (7) in an alternative form as

$$\hat{\mathbf{b}}_{WCML} = \arg \min_{\mathbf{b} \in \{1,-1\}^N} \max_{\delta \in \mathscr{D}} \left\{ \widetilde{\mathbf{b}}^* \mathbf{R}(\delta) \widetilde{\mathbf{b}} \right\}, \tag{14}$$

where $\widetilde{\mathbf{b}} = [\mathbf{b}, 1]$ and $\mathbf{R}(\delta)$ is given by

$$\mathbf{R}(\delta) = \begin{bmatrix} \mathbf{Q}(\delta) & -\mathbf{F}^*(\delta) \\ -\mathbf{F}(\delta) & 0 \end{bmatrix}. \tag{15}$$

Here

$$\mathbf{Q}(\delta) = \left[\mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{H}_i \right]^* \left[\mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{H}_i \right], \tag{16}$$

and

$$\mathbf{F}(\delta) = \mathbf{F}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{F}_i = \mathbf{r}^* \mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{r}^* \mathbf{H}_i. \tag{17}$$

The above WC ML multiuser detection problem can be simplified by assuming that $\delta_i \ll 1$ and neglecting the sum of the second order terms in $\mathbf{Q}(\delta)$, i.e., from (16)

$$\begin{aligned} \mathbf{Q}(\delta) &\approx \mathbf{H}_0^* \mathbf{H}_0 + \sum_{i=1}^{\ell} \delta_i \left[\mathbf{H}_i^* \mathbf{H}_0 + \mathbf{H}_0^* \mathbf{H}_i \right] \\ &= \mathbf{Q}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{Q}_i, \end{aligned} \tag{18}$$

where $\mathbf{Q}_0 = \mathbf{H}_0^* \mathbf{H}_0$ and $\mathbf{Q}_i = \mathbf{H}_i^* \mathbf{H}_0 + \mathbf{H}_0^* \mathbf{H}_i$. Inserting (18) into (15) yields

$$\begin{aligned} \mathbf{R} &= \begin{bmatrix} \mathbf{Q}_0 & -\mathbf{F}_0^* \\ \mathbf{F}_0 & 0 \end{bmatrix} + \sum_{i=1}^{\ell} \delta_i \begin{bmatrix} \mathbf{Q}_i & -\mathbf{F}_i^* \\ \mathbf{F}_i & 0 \end{bmatrix} \\ &\triangleq \mathbf{R}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{R}_i. \end{aligned} \tag{19}$$

Since $\widetilde{\mathbf{b}}^* \mathbf{R} \widetilde{\mathbf{b}} = \mathrm{Tr}\left\{ \widetilde{\mathbf{b}} \widetilde{\mathbf{b}}^* \mathbf{R} \right\}$, the problem (14) is equivalent to

$$\begin{aligned} \min_{\mathbf{B}} \max_{\delta \in \mathscr{D}} \quad & \mathrm{Tr}(\mathbf{B}\mathbf{R}(\delta)) \\ \text{s.t.} \quad & \mathbf{B} = \widetilde{\mathbf{b}} \widetilde{\mathbf{b}}^*, \quad \widetilde{\mathbf{b}} \in \mathbb{R}^{K+1} \\ & B_{ii} = 1, \quad i = 1, \ldots, K+1. \end{aligned} \tag{20}$$

The constraint $\mathbf{B} = \widetilde{\mathbf{b}} \widetilde{\mathbf{b}}^*$ implies that $\mathbf{B}$ is rank-1, symmetric and positive semidefinite (PSD). Following [3], we will remove the rank-1 constraint from (20), to obtain the following relaxed optimization problem:

$$\begin{aligned} \min_{\mathbf{B}} \max_{\delta \in \mathscr{D}} \quad & \mathrm{Tr}(\mathbf{B}\mathbf{R}(\delta)) \\ \text{s.t.} \quad & \mathbf{B} \succeq 0, \\ & B_{ii} = 1, \quad i = 1, \ldots, K+1. \end{aligned} \tag{21}$$

Denoting by $\mathbf{v} = \mathrm{vec}(\mathbf{B})$ and $\mathbf{r} = \mathbf{r}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{r}_i = \mathrm{vec}\left(\mathbf{R}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{R}_i\right)$, where $\mathbf{m} = \mathrm{vec}(\mathbf{M})$ denotes the vector obtained by stacking the columns of $\mathbf{M}$, (21) can be reformulated as an SDP with a linear constraint which has affine uncertainty,

$$\begin{aligned} \min_{\alpha, \mathbf{B}} \quad & \alpha \\ \text{s.t.} \quad & \mathbf{v}^* \left[\mathbf{r}_0 + \sum_{i=1}^{\ell} \delta_i \mathbf{r}_i \right] \leq \alpha, \quad \forall \delta_i \in \mathscr{D} \\ & \mathbf{B} \succeq 0, \\ & B_{ii} = 1, \quad i = 1, \ldots, K+1. \end{aligned} \tag{22}$$

The linear constraint (22), can be reformulated as

$$\mathbf{v}^* \mathbf{r}_0 + \mathbf{u}^* \delta \leq \alpha, \quad \forall \delta \in \mathscr{D}, \tag{23}$$

where the ith component of $\mathbf{u}_i$ is $\mathbf{r}_i^* \mathbf{v}$. Now, for any $\delta \in \mathscr{D}$,

$$\mathbf{v}^* \mathbf{r}_0 + \mathbf{u}^* \delta \leq \mathbf{v}^* \mathbf{r}_0 + \|\mathbf{u}^*\| \, \|\delta\| \leq \mathbf{v}^* \mathbf{r}_0 + \|\mathbf{u}^*\| \sqrt{\sum_{k=1}^{N} \rho_k^2}, \tag{24}$$

where, clearly, the upper bound in (24) can be achieved with equality. Thus, the robust optimization problem (22) is equivalent to the optimization problem

$$\begin{aligned} \min_{\alpha, \mathbf{B}} \quad & \alpha \\ \text{s.t.} \quad & \mathbf{v}^* \mathbf{r}_0 + \sqrt{\sum_{k=1}^{N} \rho_k^2} \sqrt{\sum_{i=1}^{\ell} \left| \mathbf{r}_i^* \mathbf{v} \right|^2} \leq \alpha, \\ & \mathbf{B} \succeq 0, \\ & B_{ii} = 1, \quad i = 1, \ldots, K+1. \end{aligned} \tag{25}$$

The problem of (25) is a semidefinite optimization problem with second order cone (SOC) constraints, that can be solved efficiently using standard optimization packages [9]. A matrix $\hat{\mathbf{B}}$, which is a solution to the above SDR (25), has to be converted to the approximate symbol vector $\hat{\mathbf{b}}$. In [3, 10] the suggested conversion is based on a randomization method which has a high computational complexity. Here we seek a computationally efficient method. Since, $\mathbf{B} = \widetilde{\mathbf{b}} \widetilde{\mathbf{b}}^*$, we can extract $\hat{\mathbf{b}}$ from the optimal solution $\hat{\mathbf{B}}$

$$\hat{\mathbf{b}} = sign(\widetilde{\mathbf{b}}_{K+1}), \tag{26}$$

where $\widetilde{\mathbf{b}}_{K+1}$ is the vector consisting of the first K elements of the $(K+1)$'th column of $\hat{\mathbf{B}}$.

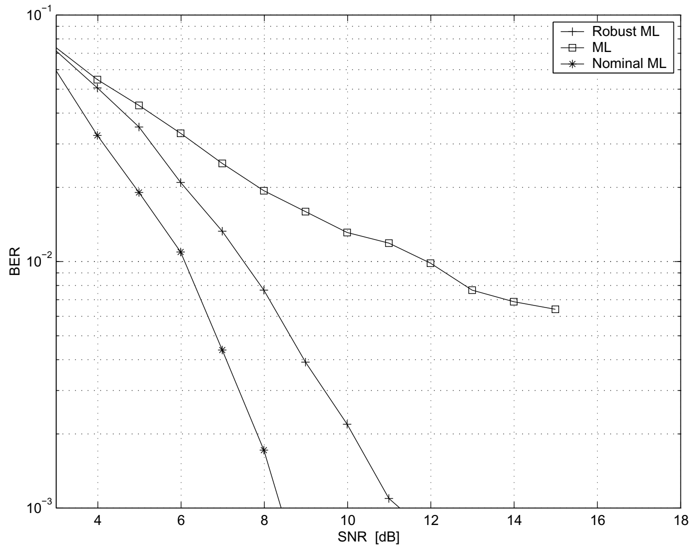

Fig. 1. BER versus SNR performance for 3 different detectors: nominal ML, ML and WCML with mismatched knowledge for 8 user system, and $\rho = 0.2$.

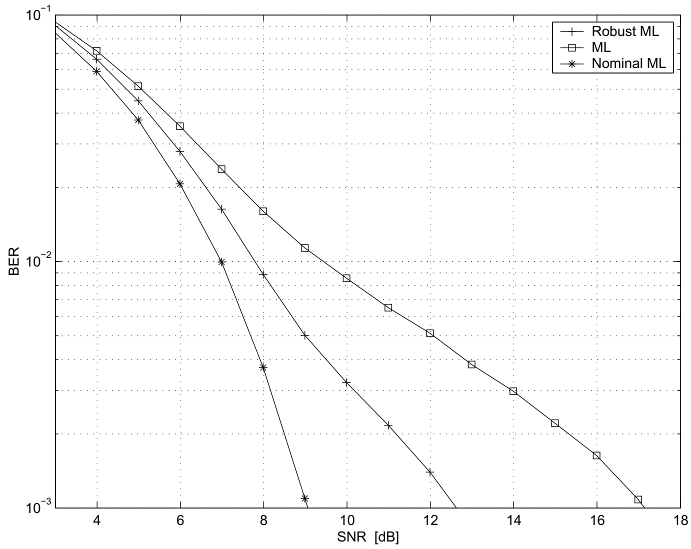

Fig. 2. BER versus SNR performance for 3 different detectors: nominal ML, ML and WCML with mismatched knowledge for 6 user system, and $\rho = 0.2$.

5. SIMULATIONS

In this section, we provide simulation results which demonstrate the performance of the WC ML multiuser detector. The simulations are based on the model

$$\mathbf{r} = (\mathbf{H}_0 + \delta \mathbf{H}_1)\mathbf{b} + \mathbf{w}, \quad |\delta| < \rho, \tag{27}$$

where

$$\mathbf{H}_0 = \begin{bmatrix} 1 & \beta & \beta \\ \beta & \ddots & \beta \\ \beta & \beta & 1 \end{bmatrix}, \quad \mathbf{H}_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & \ddots & 1 \\ 1 & 1 & 0 \end{bmatrix}. \tag{28}$$

In the simulations, we measured the BER performance of the ML detector compared to the BER performance of the WC ML detector as a function of the received signal to noise ratio (SNR) for $\rho = 0.2$ and $\beta = 0.3$. The simulation compare the performance of the ML detector for the case of perfect knowledge, i.e, given $\mathbf{H}_0$, the performance of the ML detector with mismatched knowledge, i.e., given $\mathbf{H} = \mathbf{H}_0 + \delta \mathbf{H}_1$, and the performance of the WC ML given the mismatched knowledge. The simulations are repeated for two system sizes, 8 and 6. The results for the two system sizes are shown in Figs. 1 and 2, respectively. Observing the performance of the ML detector with inaccurate knowledge, it is obvious that the performance degrades by about 6 dB at a BER of 10^{-2}. Comparing the robust ML performance we can see that at BER of 10^{-2} the WC ML detector gains about 4 dB compared to the mismatched true ML detector. In Fig. 2, it can be seen that at a BER of 10^{-3} the performance of the ML detector with inaccurate knowledge degrades severely by about 8 dB. Comparing the robust ML performance we can see that at a BER of 10^{-3} the WC ML detector gains about 4.5 dB compared to the mismatched ML detector.

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