Covariance Shaping Multiuser Detection

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I. Introduction

The discrete-time model for the received signal $\mathbf{r}$ in an M-user white Gaussian synchronous CDMA system is given by

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n}, \tag{1}$$

where $\mathbf{S}$ is the matrix of length-N signatures $\mathbf{s}_m$, $\mathbf{A} = \mathrm{diag}(A_1, \ldots, A_M)$ is the matrix of received amplitudes $A_m > 0$, $\mathbf{b}$ is the vector of transmitted bits $b_m \in \{1, -1\}$, and $\mathbf{n}$ is a zero-mean Gaussian noise vector with covariance $\mathbf{C}_n = \sigma^2 \mathbf{I}$.

Multiuser receivers for CDMA detection try to mitigate the effect of the multiple-access interference (MAI) and the background noise [1]. We restrict our attention to linear receivers that do not require knowledge of the channel parameters A_m and σ^2. The receiver estimates $\mathbf{x} = \mathbf{Ab}$ as $\hat{\mathbf{x}} = \mathbf{Q}^*\mathbf{r}$ for some matrix $\mathbf{Q}$. The mth user's bit is then detected as $\hat{b}_m = \mathrm{sgn}\ (\hat{x}_m)$ where $\hat{x}_m$ is the mth component of $\hat{\mathbf{x}}$.

The matched filter (MF) results from choosing $\mathbf{Q} = \mathbf{S}$. This receiver optimally compensates for the white noise but ignores the MAI. The decorrelator results from using a least-squares estimator of $\mathbf{x}$, and corresponds to $\mathbf{Q} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^\dagger$ where $(\cdot)^\dagger$ denotes the pseudoinverse. The decorrelator optimally rejects the MAI for linearly independent signatures; however, the inverse operation may lead to an output noise component with large variance and a covariance structure with a very high dynamic range, resulting in degraded performance.

II. The Covariance Shaping Multiuser Receiver

To approach the performance of the MMSE receiver without requiring knowledge of the channel parameters we propose the *covariance shaping multiuser (CSMU) receiver*, which is based on the recently proposed covariance shaping least-squares (CSLS) estimator [2]. The CSMU receiver mitigates both the MAI and the white noise, by optimally shaping the noise in the output of the receiver prior to detection.

The CSLS estimate of $\mathbf{x}$, denoted $\hat{\mathbf{x}}_{\mathrm{CSLS}}$, is chosen to minimize the total variance of the weighted error between $\hat{\mathbf{r}} = \mathbf{S}\hat{\mathbf{x}}_{\mathrm{CSLS}} = \mathbf{SQ}^*\mathbf{r}$ and $\mathbf{r}$, subject to the constraint that the covariance of the noise component in $\hat{\mathbf{x}}_{\mathrm{CSLS}}$ is equal to $\sigma^2\mathbf{R}$ for some given covariance matrix $\mathbf{R}$, so that we control the dynamic range and spectral shape of the output noise.

From the general definition of the CSLS estimator, the CSMU receiver corresponds to choosing

$$\mathbf{Q} = \mathbf{SR}((\mathbf{S}^*\mathbf{SR})^{1/2})^\dagger, \tag{2}$$

for some covariance matrix $\mathbf{R}$ with null space $\mathcal{N}(\mathbf{R}) = \mathcal{N}(\mathbf{S})$.

III. Spectral Efficiency of the CSMU Receiver

Theorem 1. *Let the elements of $\mathbf{S}$ be independent $\mathcal{CN}(0, 1/N)$, let $\mathbf{A} = A\mathbf{I}$ and let $\mathbf{R}$ be a covariance matrix that commutes with $\mathbf{S}^*\mathbf{S}$ and with eigenvalues $t(\lambda_m)$ where λ_m are the eigenvalues of $\mathbf{S}^*\mathbf{S}$ and $t(\cdot)$ satisfies $t(x) < \infty$ for $x = 0$ and $x \in [\eta_1, \eta_2]$ with $\eta_{1,2} = (1 \mp \sqrt{\beta})^2$, and $E(t^2(x)) < \infty$ where the expectation is taken with respect to the pdf*

$$f_\beta(x) = [1 - \beta^{-1}]^+ \delta(x) + \frac{\sqrt{[x - \eta_1]^+ [\eta_2 - x]^+}}{2\pi\beta x}. \tag{3}$$

Here $[u]^+ \triangleq \max\{0, u\}$. Then in the limit as $M \to \infty$ with $\beta \triangleq M/N$ held constant,

1. *The SINR at the CSMU receiver output converges in MSE to $\gamma = \alpha/(1 - \alpha)$ where $\alpha = E^2\left((t(\lambda)\lambda)^{1/2}\right)/E\left(t(\lambda)(\lambda + \zeta)\right)$, with $1/\zeta = A^2/\sigma^2$ and the expectation is evaluated according to $f_\beta(x)$;*
2. *The spectral efficiency of the CSMU receiver converges in MSE to $\mathsf{C} = (\beta/2)\log(1 + \gamma)$.*

For equal power users, the MMSE receiver is equal to a CSMU receiver with output covariance $\sigma^2\mathbf{R}^{\mathsf{mmse}}$ where $\mathbf{R}^{\mathsf{mmse}} = (\mathbf{S}^*\mathbf{S} + \zeta\mathbf{I})^{-1}\mathbf{S}^*\mathbf{S}(\mathbf{S}^*\mathbf{S} + \zeta\mathbf{I})^{-1}$. If the SNR is unknown, then this receiver cannot be implemented. Instead we choose $\mathbf{R} = (\mathbf{S}^*\mathbf{S} + \delta\mathbf{I})^{-1}\mathbf{S}^*\mathbf{S}(\mathbf{S}^*\mathbf{S} + \delta\mathbf{I})^{-1}$ for some δ. From Theorem 1 the spectral efficiency of the resulting receiver is $\mathsf{C} = \beta/2\log(T)$ where

$$T = \frac{1 + \beta + \zeta - \frac{1}{2}\delta\mathcal{F}(1/\delta, \beta)}{\left(1 + \beta + \delta - \frac{1}{2}\delta\mathcal{F}(1/\delta, \beta)\right)\left(1 + \frac{1}{\delta} - \frac{1}{4}\mathcal{F}(1/\delta, \beta)\right)^{-1} + \zeta - \delta}, \tag{4}$$

and $\mathcal{F}(x, \beta) = \left(\sqrt{x\eta_2 + 1} - \sqrt{x\eta_1 + 1}\right)^2$.

Comparing (4) with the spectral efficiency of the MMSE receiver we conclude that when δ and ζ are fixed, for $\beta \gg 1$ or $\beta \ll 1$, there is no loss in spectral efficiency (SE) using the CSMU receiver. For fixed β, when $\delta, \zeta \gg 1$ or $\delta, \zeta \ll 1$ then again there is no loss in SE. When both δ and β are fixed, then for low to intermediate values of SNR there is essentially no loss in SE. For high SNR values, if $1/\delta$ is large or $\beta > 1$, then again there is essentially no loss in SE. For $\beta < 1$, the loss in SE over a wide range of SNR values will be small if we choose δ to be small. In particular, we can always choose δ so that the loss in SE with respect to the MMSE receiver over an SNR range of interest is small. In some cases, this will entail a larger loss in other SNR regimes. However, it seems reasonable that although the receiver may be operating in a changing environment, so that the SNR will fluctuate, there is a range of SNR values over which fluctuations will occur. Over this range, the parameters can be chosen to achieve essentially the same capacity as the MMSE receiver.

References

[1] S. Verdu, *Multiuser Detection*, Cambridge, UK: Cambridge Univ. Press, 1998.

[2] Y. C. Eldar and A. V. Oppenheim, "Covariance shaping least-squares estimation," *IEEE Trans. Signal Processing*, vol. 51, pp. 686–697, Mar. 2003.