Detection of an Ensemble Mixed with Unknown States

Noam Elron* and Yonina C. Eldar†

Department of Electrical Engineering

Technion - Israel Institute of Technology

Technion City, Haifa 32000, Israel

(Dated: November 18, 2004)

Abstract

We address the problem of distinguishing among a finite collection of quantum states, when the states are not entirely known. For completely specified states, necessary and sufficient conditions on a quantum measurement minimizing the probability of a detection error have been derived. In this work, we assume that each of the states in our collection is a mixture of a known state ρ_i^0 and an unknown state ρ_i^1, so that $\rho_i = q_i\rho_i^0 + (1-q_i)\rho_i^1$ where $0 \leq q_i \leq 1$ are known. We investigate two criteria for optimality. The first is minimization of the *worst-case* probability of a detection error. For the second we assume a probability distribution for the unknown states, and seek minimization of the *expected* probability of a detection error.

We derive necessary and sufficient conditions for optimality under both criteria, and interpret them in light of the conditions in the nominal case. We also explore some special cases.

*nelron@tx.technion.ac.il

†yonina@ee.technion.ac.il