Detection of an Ensemble Mixed with Unknown States

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Abstract

We address the problem of distinguishing among a finite collection of quantum states, when the
states are not entirely known. For completely specified states, necessary and sufficient conditions
on a quantum measurement minimizing the probability of a detection error have been derived. In
this work, we assume that each of the states in our collection is a mixture of a known state $\rho_i^0$ and
an unknown state $\rho_i^1$, so that $\rho_i = q_i \rho_i^0 + (1 - q_i) \rho_i^1$ where $0 \leq q_i \leq 1$ are known. We investigate two
criteria for optimality. The first is minimization of the worst-case probability of a detection error.
For the second we assume a probability distribution for the unknown states, and seek minimization
of the expected probability of a detection error.

We derive necessary and sufficient conditions for optimality under both criteria, and interpret
them in light of the conditions in the nominal case. We also explore some special cases.

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