Detection of Anomalies in Texture Images using Multi-Resolution Features

Lior Shadhan

Electrical Engineering Department
Technion - Israel Institute of Technology

Supervisor: Prof. Israel Cohen
Outline

1. Introduction
   - Anomaly Detection
   - Texture Segmentation and Classification

2. Research Schemes
   - Anomaly Detection Using MR Gaussian Textural Features
   - Anomaly Subspace Detection Using MR Textural Features

3. Experimental Results
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   - Anomaly Detection Examples

4. Summary

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Motivation

Objective

Locating elements in a scene which are unlikely to be a part of it

Main Challenges

- Large variability of the scene's background clutter
- Appearance of anomalous elements
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Feature Space

The detection process is generally performed with respect to an appropriate feature space in which a clear segregation between the anomalous elements and the rest of the background clutter in the scene is possible.

The feature space is derived using:

- Single resolution spatial analysis
- Multi-resolution analysis (hyper-spectral images, multi-resolution decompositions)
- Integration of both

The Simultaneous Auto-Regressive (SAR) and Gaussian Markov Random Field (GMRF) are common used Random Field Models (RFM) for spatial analysis.
Anomaly detectors often use Bayesian classifiers, utilizing available *a-priori* knowledge and *a-posteriori* parametric statistics of both background clutter and anomalous targets.

Common classifiers are:

- Single Hypothesis Testing (SHT)
- Matched Signal Detector
- Matched Subspace Detector (MSD)
Anomaly detection algorithm for hyper-spectral imagery based on the Eigen Separation Transform, performed using the covariance difference of statistics which are derived from a sliding dual window.

Additive anomalies will not be detected. Sample covariance estimation is performed using a small set of data and therefore is not accurate. Spatial correlation within each hyper-spectral image is disregarded.
Recent Work (continued)

(Bello 1992, Hazel 2000)

Anomaly detection algorithms for detecting regions which appear unlikely with respect to a SAR / GMRF model of the background image

The use of a single resolution spatial analysis introduces high false alarms in semi-homogenous background textures due to deviations from the random field model.
Anomaly detection algorithm for detecting regions which appear unlikely in hyper-spectral images with respect to a GMRF model with a first order 3-D field, derived using small homogenous windows of the clutter.

Anomaly detection algorithm for detecting regions which appear unlikely with respect to a multi-resolution GMRF model of the background image, using a MSD classifier.
The quality of an image which is synthesized from its SAR or GMRF models varies considerably depending on the use of appropriate neighbor set.

The theoretical detection performance of a MSD is highly affected by two main parameters: target subspace dimensionality and SNR.

- Inappropriate choice of a neighbor set increases the prediction error due to SNR degradation, resulting in higher false alarms.
- Non-linearities can be used to increase the SNR in order to achieve improved detection performance.
Anomaly Detection in Synthesized Images
Genuine background texture with additive synthesized anomaly

Texture with additive anomaly

Proposed MSD Scheme

Goldman and Cohen (2005)

Proposed SHT Scheme
Motivation

Objective

Segmentation of a given scene into different textures using a texture classification scheme

Main Challenges

- Images of the same underlying texture can vary significantly
- Textural features must be invariant to image variations and at the same time sensitive to intrinsic spatial structures that define textures
- The distance measure between textures must be robust to the image variations in order to avoid classification errors
- Separation contours - spatial continuity must be preserved

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Recent Work

(Clasui and Yue 2004)

- Unsupervised texture segmentation algorithm using a single-resolution GMRF employed directly on the image

(Mittelman and Porat 2005)

- Unsupervised texture segmentation algorithm using a multi-resolution GMRF employed on the logarithm of the Gaussian Scale Mixture (GSM) hidden multipliers estimates
- Texture classification algorithm using the GSM hidden multipliers estimates (a non-parametric statistical model based on histogram derived dissimilarity measure)
Can it be used for Anomaly Detection?

Texture Segmentation

- Unsupervised texture segmentation schemes perform poorly when used for detecting anomalous targets in a single texture.
- However, these schemes can be used for segmenting a given scene to several textures prior to anomaly detection in each texture.

Texture Classification

- Possibly, but in a supervised scheme.
Proposed Algorithm

Feature Space:

\[ t_j(s) = \log \left( \frac{\sum_{r \in \mathcal{R}_1} y_j^2(s+r)}{|\mathcal{R}_1|} \right) \]

\[ v_j(s) = \frac{\sum_{r \in \mathcal{R}_2} t_j(s+r)}{|\mathcal{R}_2|} \]

Feature Space Statistics (Background Textural Signature):

\[ \hat{\mu}_0 \approx E[v(s)] \]

\[ \hat{\Sigma}_0 \approx E[(v(s) - \hat{\mu}_0)(v(s) - \hat{\mu}_0)^T] \]

Decision Rule (SHT):

\[ d(s) = (v(s) - \mu_0)^T \Sigma_0^{-1} (v(s) - \mu_0) \]

\[ \begin{cases} H_1 & \text{if } d(s) \geq \eta \\ H_0 & \text{otherwise} \end{cases} \]
The logarithm of the GSM hidden multipliers estimates of wavelet coefficients has a Gaussian marginal distribution

⇒ The feature space of the background clutter follows the multivariate Gaussian distribution, therefor:

\[ d(s) |_{H_0} \sim \chi^2_m(0) \quad P_{FA} = 1 - P(\chi^2_m(0) \leq \eta) \]

A special case is when under hypothesis \( H_1 \) the observations are assumed to be a linear combination of the target signature and the background clutter, resulting in the RX algorithm (Reed and Yu 1990):

\[ d(s) |_{H_1} \sim \chi^2_m((\mu_1 - \mu_0)^T \Sigma_0^{-1} (\mu_1 - \mu_0)) \]

\[ P_D = 1 - P(\chi^2_m((\mu_1 - \mu_0)^T \Sigma_0^{-1} (\mu_1 - \mu_0)) \leq \eta) \]
Proposed Algorithm (continued)
Avoiding Miss-Detection of Scaled Background Textures

- Textural signature of a scaled background texture contains an additive bias for all multi-resolution layers
- Bias normalization is required in order to avoid miss-detection

Revised Normalized Feature Space:

\[ q_j(s) = v_j(s) + ([\mu_0]_1 - v_1(s)) \]

\[ q(s) = [q_2(s), ..., q_m(s)]^T \Rightarrow \hat{\mu}_0, \hat{\Sigma}_0 \]

Decision Rule (SHT):

\[ d(s) = (q(s) - \mu_0)^T \Sigma_0^{-1} (q(s) - \mu_0) \]
Main Features

- Detection is based on deviation from a Textural Signature which represents the unique spatial structure of the background texture
- Unsupervised (assuming rare targets)
- Constant False Alarm Rate (CFAR) regardless of the background clutter and anomaly type
- Does not rely on the exhaustive statistical model of the targets, but rather on the multi-resolution statistics of the background clutter $\Rightarrow$ different types of targets can be detected
- Can incorporate a-priori information on the minimal expected size of the targets for achieving improved detection results
Classification Scheme - Proposed Algorithm

Classification is based on the calculated Mahalanobis distance from pre-trained Textural Signatures
Proposed Algorithm

1. Target Subspace Images
2. Interference Subspace Images
3. Input Image with Target Signals

- Generate multi-resolution representation using the RDWT
- Feature Reduction using KLT (Optional)
- Extract Background Subspace using Eigenvectors

- Non-Linearity ($y$)
- Innovation Process Calculation

- Target Subspace Calculation
- Interference Subspace Calculation

- Matched Subspace Detector

- Anomaly Detection
Texture Modeling using a RFM

Each pixel in a given image is represented as a weighted sum of pixels at nearby locations and an additive prediction error, often referred as the innovations process

\[ y(s) = \sum_{r \in \mathcal{R}} \theta(r)y(s + r) + \varepsilon(s) \quad \Rightarrow \quad \mathbf{B(\theta)}\mathbf{y} = \varepsilon \]

\[ \hat{\theta}_{LS} = \left[ \sum_{s \in \Omega_0} \mathbf{w}(s)\mathbf{w}(s)^T \right]^{-1} \left[ \sum_{s \in \Omega_0} y(s)\mathbf{w}(s) \right] \]

\[ \mathbf{w}(s) = \text{col} [y(s + r), r \in \mathcal{R}] \]

The estimated weight coefficients are not affected by the scaling of the image.
The covariance of innovations which are derived from natural textures or from natural textures’ wavelet coefficients does not necessarily follow the SAR or the GMRF models. This is mainly the result of inappropriate choice of neighborhood to be used with a given image.

Taking into consideration introduced correlation between pixels which are not accounted for in the GMRF model moderates the need for a proper choice of neighborhood for each background texture.
The squaring non-linearity achieves a random field model with lower prediction error variance whenever a proper scaling of the modeled image is performed.

\[ y = \delta x \Rightarrow \varepsilon_y = B(\theta_x)y = \delta B(\theta_x)x = \delta \varepsilon_x \]

\[ [z_y]_i = [y]_i^2 - E\left([y]_i^2\right) = \delta^2 [z_x]_i \Rightarrow \varepsilon_{z_y} = B(\theta_{z_x})z_y = \delta^2 B(\theta_{z_x})z_x = \delta^2 \varepsilon_{z_x} \]

\[ [\text{cov}[\varepsilon_{z_y}]]_{i,i} \leq [\text{cov}[\varepsilon_y]]_{i,i} \iff |\delta| \leq \sqrt{[\text{cov}[\varepsilon_x]]_{i,i} / [\text{cov}[\varepsilon_{z_x}]]_{i,i}} \]
Multi-Resolution Feature Space

**RDWT**

\[ y(v, s) = [y_1(v, s), y_2(v, s), \ldots, y_m(v, s)]^T \]

**KLT**

\[ t(v, s) = K^T y(v, s) = [t_1(v, s), t_2(v, s), \ldots, t_p(v, s)]^T, \]

\[ 1 \leq p \leq m \]

**Squaring**

\[ z(v, s) = [z_1(v, s), z_2(v, s), \ldots, z_p(v, s)]^T, \]

\[ z_k(v, s) = t_k^2(v, s), \quad k = 1, \ldots, p \]

**RFM**

\[ n_k(v) = B(\theta_k) z_k(v) - \mu z_k \left( 1 - \sum_{r \in \mathcal{R}} \theta_k(r) \right) \]

**Feature Space**

\[ \gamma_k(v) = \Lambda_k^{-1/2} n_k(v), \quad \Lambda_k \approx E \left[ n_k(v)n_k(v)^T \right] \]
As a result of the introduced squaring non-linearity:

- The interfering subspace $A_k$ should account for the interfering signals, the interaction among them and the interaction between them and the background clutter.

- The dimensionality of the target subspace $B_k$ highly affects the detector performance $\Rightarrow$ the interaction among target signals, the interaction between them and the background clutter and the interaction between them and the interfering signals are discarded.
The Generalized Likelihood Ratio Test (GLRT) is given by:

\[
L(v) = \sum_{k=1}^{p} \gamma_k(v)^T \left(P_{A_k}B_k - P_{A_k}\right) \gamma_k(v) \begin{cases} \geq \eta & H_1 \\ \leq \eta & H_0 \end{cases}
\]

\[
L(v) \sim \begin{cases} \chi_{q}(0) & , \text{under } H_0 \\ \chi_{q}^2 \left(\sum_{k=1}^{p} [B_k \psi_k(v)]^T (I - P_{A_k}) [B_k \psi_k(v)]\right) & , \text{under } H_1 \end{cases}
\]

The theoretical detection rate and false alarm are given by:

\[
P_D(v) = 1 - P\left(\chi_{q}^2 \left(\sum_{k=1}^{p} \text{SNR}(k, v)\right) \leq \eta\right)
\]

\[
P_{FA}(v) = 1 - P(\chi_{q}^2(0) \leq \eta)
\]
Main Features

- Unsupervised (assuming rare targets) - requires only \textit{a-priori} information on the target and interference characterizing subspaces.
- Based on a multi-resolution RFM which better describes the background clutter in natural images than other models such as the SAR and the GMRF, resulting in enhanced segregation.
- Can incorporate \textit{a-priori} information on multi-resolution layers which are most significant to the detection process for achieving improved detection results.
- Can be formulated to be used in a "Subspace Detector Bank" scheme for detecting various types of anomalies.
Anomaly Detection (SHT)
Affects of the second spatial smoothing stage on calculated distance

![Graph showing the effect of the second spatial smoothing stage on calculated distance.](image)
Anomaly Detection (SHT)

ROC curves using anomaly taken from texture with no evident visual resemblance to the background clutter.

Texture #1 (for background)  Texture #2 (for anomaly)

Typical texture with anomaly

![ROC curve](image.png)
Anomaly Detection (SHT)

ROC curves using anomaly taken from texture with evident visual resemblance to the background clutter

Texture #1 (for background)  Texture #2 (for anomaly)

Detection Rate

False Alarm

M=5  M=10  M=15

Typical texture with anomaly
## Anomaly Detection (SHT)

### Comparison of Classification Results

<table>
<thead>
<tr>
<th></th>
<th>Average [%]</th>
<th>Worst [%]</th>
<th>Best [%]</th>
<th>Std</th>
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<tr>
<td>Proposed</td>
<td>99.94</td>
<td>99.22</td>
<td>100.00</td>
<td>0.13</td>
</tr>
<tr>
<td>Mittelman &amp; Porat</td>
<td>93.39</td>
<td>91.33</td>
<td>94.45</td>
<td>0.53</td>
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<td>Mittelman &amp; Porat (ICIP 2005)</td>
<td>93.74</td>
<td>92.03</td>
<td>95</td>
<td>NA</td>
</tr>
<tr>
<td>Liu &amp; Wang (ICIP 2005)</td>
<td>92.5</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The results suggest that a better segmentation scheme could be achieved using a fusion of the presented textural features and Mittelman & Porat segmentation algorithm (EUSIPCO 2005).
Anomaly Subspace Detection (MSD)

KLT affects on detection performance
Anomaly Subspace Detection (MSD)

ROC curves - compared to recent published detectors
The squaring non-linearity ensures higher GLR values whenever a certain minimum SNR threshold is assured:

\[ \text{SNER} \approx \text{SNR} - 26.3 \, [\text{dB}] \]
Anomaly Subspace Detection (MSD)
MSD performance - compared to recent published detectors

![Detection Rate vs SNR graphs for different False Alarm rates](image-url)
Anomaly Subspace Detection (MSD)
GLR distribution under hypothesis $H_0$

In practice, the background innovations should follow an elliptical multivariate $t$-distribution and the $k$th layer GLR then follows a univariate $F$-distribution.

The resulting GLR then follows a mixture of $p$ F-distributions and a new threshold $\eta$ can be derived to ensure a required false alarm rate.

(Manolakis and Shaw 2002)
Anomaly Detection of Detached Anomalies
Anomaly Detection of Additive Anomalies
Anomaly Detection of Additive Anomalies
Bank of Subspace Detectors, SNER=0[dB]

Background: Fabric, Anomaly 1: Woodgrain, Anomaly 2: Raffia
Anomaly Detection of Additive Anomalies
Bank of Subspace Detectors, SNER=0[dB]

Background: Raffia, Anomaly 1: Fabric, Anomaly 2: Woodgrain
Anomaly Detection of Additive Anomalies
Bank of Subspace Detectors, SNER=0[dB]

Background: Woodgrain, Anomaly 1: Raffia, Anomaly 2: Fabric
Anomaly Detection in Side Scan Sonar Images

Subspace Detector
Summary

Anomaly Detection Using MR Gaussian Textural Features

- We have presented a multi-resolution Gaussian feature space.
- We have proposed a textural signature which uniquely represents the spatial structure of a given background texture.
- We have formulated an unsupervised anomaly detection algorithm with CFAR based on deviations from the proposed textural signature.
- We have formulated a texture classification algorithm with improved classification results based on the proposed textural signature and anomaly detection scheme.
Summary

Anomaly Subspace Detection Using MR Textural Features

- We have proposed a multi-resolution RFM which is less susceptible to its choice of neighbors, achieving a smaller prediction error.

- We have formulated a Multi-resolution feature space with enhanced anomaly segregation with respect to the spatial structure of the background texture, based on the proposed RFM.

- We have formulated an unsupervised anomaly subspace detection algorithm, designated to detect additive anomalies in harsh environments where signal to noise energy ratio values are very low.